TEACHING OF HIGH SCHOOL FUNCTION CONCEPTS FROM THE PERSPECTIVE OF LOGICAL REASONING LITERACY

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ABSTRACT

In the past, most of the research on the concept of functions in high school was from the perspective of mathematical abstract literacy. It is undeniable that for students who have just entered high school, the concept of functions is abstract and difficult to understand. However, if we follow this process to teach the concept of functions that start from the facts, carry out inductive reasoning from the special to the general form, so as to obtain the concept, the teaching of functional concepts can not only develop students' logical thinking, but also cultivate students' ability to solve problems. Based on this step, this paper carries out the teaching design of function concepts, in order to combine core literacy of logical reasoning with function concept teaching in the overall design and reflection.

Keywords: Logical reasoning literacy, The concept of functions, Conceptual teaching.

1. INTRODUCTION

As one of the four main lines running through high school mathematics, function is not only an important mathematical model, but also a widely used mathematical thinking method. The concept of function in high school is based on mapping, and adopts more rigorous set and corresponding language expression, which has stronger abstraction and logic, which is the basis for the learning of the main line of functions. For students who have just entered the first year of high school, their logical thinking ability and abstract generalization ability need to be improved, and it may be difficult to understand the abstract function concept. If teachers can trace back to the roots, start from the background and development process of the concept of function, introduce real-life examples, use logical reasoning proofs, and guide students to derive the content of function concepts step by step, it can not only effectively eliminate students' sense of separation between new and old knowledge, but also infiltrate mathematical culture while developing students' core literacy in mathematics. In addition, the teaching process of function concepts also provides a reference paradigm for the teaching of specific function models in the future.

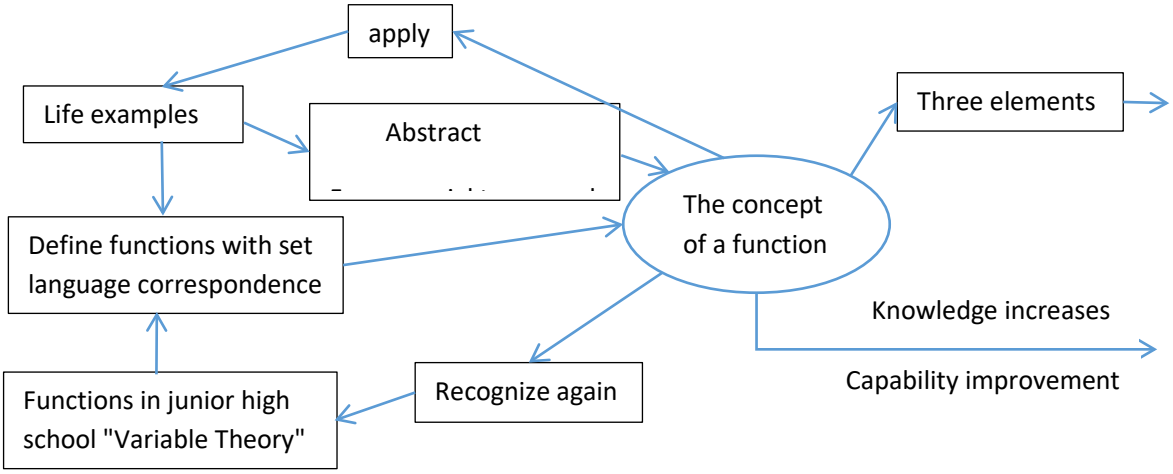
As one of the core literacies of the six major mathematics disciplines, logical reasoning literacy focuses on enabling students to understand the "ins and outs" in teaching, learn to analyze their own ideas, naturally contain problems in the situation, grasp the logical relationship between knowledge and skills, and think and express in logical language. Students experience the process of logical reasoning, experience the method of logical reasoning, and feel the source of knowledge, which plays an important role in their formation of logical thinking mode and mathematical thinking quality. In the teaching of function concepts, students are generally guided to review the definitions of function concepts learned in junior high school, and then summarize the concepts of functions with a few examples and questions. This is undoubtedly the teaching process that is closest to the textbooks and course standards, but it is also necessary to pay more attention to the expansion and extension of the content of the textbooks. The first volume of the compulsory mathematics of the A version of the general high school textbook Renjiao has set up a "Reading and Thinking" section after the section "The Concept of

Functions and Their Representations". This paper describes the development process of the concept of functions, which reflects the infiltration of textbooks into the history and culture of mathematics, and teachers should also pay attention to it in teaching. Therefore, this paper attempts to give the relevant teaching design of function concepts from the perspective of logical reasoning, so that students can experience the formation process of function concepts, deeply understand the concept of functions, and experience the ideas and methods of logical reasoning.

2. Teaching design of high school function concepts from the perspective of logical reasoning literacy

2.1 Analysis of teaching content

Function is the concept that students are most exposed to in middle and high school, and it is also the most commonly used mathematical model to solve real-world problems. As the starting course of the main content of functions in high school, the definition of the concept of function is further abstracted on the basis of the "variable theory" in junior high school, which is a re-understanding of the concept of function, so that the connotation and application scope of function research can be expanded. In teaching, it is necessary to pay attention to infiltrating the mathematical culture, so that students can understand the evolution of the concept of functions, take the actual situation as the introduction, and set up problems to let students learn to think logically, guide students to carry out inductive reasoning, cultivate students' mathematical abstraction, logical reasoning and other core literacy of mathematics, and experience the idea of transformation from special to general. The flowchart of this section is as follows.



2.2 Learning situation analysis

In junior high school, students already have the knowledge of functions, so "functions" are not unfamiliar to students, but to break their existing knowledge system and re-establish more abstract related concepts may cause confusion in knowledge. Teachers should pay attention to starting from the core of the functional concept of "correspondence", so that students can understand the unique certainty of correspondence between set elements.

2.3 Teaching objectives

The content requirements of the General High School Mathematics Curriculum Standards (2017 Edition, 2020 Revision) for the concept of functions are: on the basis of using the dependencies between variables to describe functions in junior high school, students use set language and correspondence to describe functions, establish a complete concept of functions, and experience the role of set language and correspondence in describing the concept of

functions. Understand the elements that make up a function and be able to find the domain of a simple function. It can be further understood as:

(1)With the help of specific problem situations, students can abstract the concept of functions, go through the reasoning process from special to general, experience the role of ensemble language in describing the concept of functions, and develop students' core literacy in mathematical abstraction.

(2)Through step-by-step problem guidance, students can think logically, understand the three elements and meanings of function concepts, and cultivate students' core literacy of logical reasoning.

(3)To show students the development of the concept of functions and to infiltrate the mathematical culture.

2.4 Teaching focus and difficulties

(1)Teaching focus: understand the correspondence between two non-empty sets, and be able to describe this relationship in the language of sets and correspondence.

(2)Teaching difficulties: understand the essence of the concept of functions.

2.5 Teaching process

(1)Review old knowledge and cause conflicts

Question 1: You have been exposed to the concept of functions in junior high school, can you name some functions?

Students give examples of functions such as y=2x,$y = x^2$, $y = \frac{1}{x}$.

Follow-up question 1.1 Can you tell us how we define functions in junior high school?

Follow-up question 1.2: Please think about y=1(x∈R) is a function?

Follow-up question1.3:Is y=x and $y = \frac{x^2}{x}$ the same function?

Teacher and Student Activities: The teacher led the students to recall the concept of "variable theory" in junior high school, and asked the students to explore and discuss whether y=1(x∈R) is a function, and finally formed different views. It was not until the end of the 19th century that the Germany mathematician Cantor's set theory solved this problem, and functions such as y=1(x∈R) were finally included in the scope of functions. At the same time, continue to ask whether y=x is the same function as the same function, and the teacher cannot rush to give a conclusion after the students answer, leaving suspense. Next, by playing a video, students can understand the research process of mathematicians on functions and become interested in how to use set theory to describe the concept of functions.

Design intent: First, the students' existing knowledge is mobilized through question 1 and follow-up question 1.1, and then the follow-up question 1.2 allows students to perceive the lack of functional knowledge learned in junior high school, which leads to cognitive conflicts and intellectual curiosity. Follow-up question 1.3 allows students to revisit the concepts of functions they have learned and have doubts, which leads to the topic of this lesson: the concept of functions. In addition, in the process of teaching, the culture of mathematics is also infiltrated, so that students can feel the rigor of mathematics and experience the spirit of scientists.

(2)Create a situation and introduce a topic

Question 2: (The specific problem situation is shown by PPT)

Scenario 1: A "Fuxing" high-speed train accelerates to 350km/h and then maintains a constant speed for half an hour, and the relationship between the distance S (unit: km) and the running time t (unit: h) during this period.

Scenario 2: An electrical repair company requires workers to work at least 1 day and up to 6 days per week. If the wage standard is 350 yuan per person per day, the salary will be paid once a week. the relationship between a worker's wage w (unit: yuan) and the number of days worked d.
Scenario 3: The change of air quality index in Beijing on November 23, 2016 is presented. the relationship between t h and the value I of the AQI at any time of the day.
Scenario 4: The change of Engel's coefficient of urban residents in a province of China is given. the relationship between Engel's coefficient r and year y.

Follow-up question 2.1 How many variables are involved in each question scenario? How is the range of each variable represented as a set?
Follow-up question 2.2 According to the correspondence between variables, is the correspondence between variables uniquely determined?
Follow-up question 2.3 Does the relationship between variables constitute functions? How to represent?

Teacher and Student Activities: Students observe the four situations and think about the follow-up questions, the group communicates and discusses, after which the teacher asks the group representative to answer, which is supplemented by other groups, and finally the teacher leads the students to summarize together.

Design Intent: Starting with the set representation of variables, by asking students to think about the correspondence between variables, the correspondence between the two sets is naturally derived, and then the uniqueness is used to judge whether the functional relationship is formed. This design is in line with students' cognition, allowing students to follow the question, think about the problem logically, and gradually realize the transformation from natural language to mathematical language, which is conducive to guiding the concept of function from shallow to deep. By setting up specific problem situations, students can more easily mobilize their thinking in familiar situations, which provides an entry point for cultivating students' logical thinking and logical reasoning literacy.
(3)Summarize commonalities and construct concepts

Question 3: What are the common characteristics of the functions in scenarios 1-4 above? Can you summarize the essential characteristics of the concept of functions?
Teacher and Student Activities: The students discussed and summarized among themselves, and the teacher summarized the following common characteristics:

①both contain two sets of non-empty numbers, denoted by A and B.
②all have a definite correspondence.
③The correspondence of different representations has the following characteristics: for any number x in the number set A, according to the correspondence, there is a uniquely determined number y corresponding to it in the number set B.

Follow-up question 3.1: What are the methods used to represent correspondence in scenarios 1-4?
Teacher and Student Activities: Students responded using analytical, graphic, table, and other methods. The teacher guided the students to understand the inconsistency of these representations, and pointed out that for the convenience of representation, we introduced the symbol f to represent the correspondence in a unified manner, thus introducing the concept of functions. F:A→B is called a function from set A to set B, denoted as

$y=f(x)$，$x\in A$.

where x is called the independent variable, and the value range A of x is called the definition domain of the function. The y-value corresponding to the x value is called the function value, and the set of function values {f(x)|x∈A} is called the range of the function.

Follow-up question 3.2 The functions in Scenario 1 and Scenario 2 have the same correspondence, are they the same function? Why?

Teacher and Student Activities: Students discuss and come up with two answers: "yes" and "no". The teacher guides the students to review the concept of function, and concludes that the definition domain of the function is different in the two contexts, resulting in the difference of the value range, and consolidates the concept of definition domain and value range, and points out that the definition domain, value range and correspondence are the three elements of the function concept.
Design Intent: First, ask students to summarize the common features of different function representations, and then lead to a more general function representation y=f(x).In the teaching process, students interact and explore independently, cultivate students' inductive reasoning ability, and teachers guide and summarize in a timely manner. In the process of moving from concrete to abstract, students learn to think logically, develop students' core literacy of logical reasoning and mathematical abstraction ability.

(4)Interactive exploration and deepening of concepts

Inquiry 1: Use the three elements of the newly learned concept of functions to explain the elementary functions that have been learned in junior high school.

Inquiry 2: Answer the question 1.3 Is y=x and $y = \frac{x^2}{x}$ the same function?

Inquiry 3: Construct a real-world problem situation and be able to express the correspondence analytically.

Teacher and Student Activities: Students use the three elements of the concept of function to explore independently or cooperatively, and the teacher gives guidance and correction.

Design Intent: Through students' interactive inquiry, students will have a deep understanding of the concept of functions, grasp the three elements of the concept of functions, experience the essence of functions, and check whether students can connect mathematics with real life through the answers of inquiry 3.From the actual to the mathematics, and then to the practice, to promote students' understanding of the concept of functions, and cultivate students' deductive reasoning ability. Students understand the "incoming" and "going" of function concepts step by step, grasp the logical relationship between knowledge, and be able to think and express in logical language, and experience the method of logical reasoning, which plays an important role in their formation of logical thinking mode and thinking quality.

(5)Review and summarize, apply improvement
Summary: Ask students to review what they have learned in this section and describe the concept of function and the three elements of function.

Example 1 Which of the following functions is the same as y=x?

A.$y = (\sqrt{x})^2$ B.$u = \sqrt[3]{v^3}$ C.$y = \sqrt{x^2}$ D.$m = \frac{n^2}{n}$

Example 2 If the set A={1,2} and the set B={1,2,3}, write all the functions of the set A to B.
Teacher and Student Activities: Students answer the concept of function and the three elements of function, and the teacher makes a supplementary summary. The teacher prompts the students to determine that the function $u = \sqrt[3]{v^3}$ is the same as y=x based on the defined field of the

function.In Example 2, according to the definition of a function, the two numbers in set A can correspond to the three numbers in set B, and the enumeration method shows that there can be 9 different correspondences, that is, 9 different functions.

Design Intent: It enables students to deepen their understanding of the concept of function and the three elements of function. The design of Example 2 breaks through the conventional expression method of function analytical, which is conducive to students' further understanding of the concept of function from multiple perspectives, and develops students' logical reasoning ability while improving students' expression ability. Through practice, students' deductive reasoning ability can be cultivated and their problem-solving ability can be promoted.

3. Some thoughts on the teaching of function concepts in high school

Functions are not only one of the four main lines of high school mathematics learning, but also run through the whole high school learning as a thought and method, and its importance is self-evident. Mathematical concepts in high school have a higher level of condensation and abstraction, involving more rigorous forms of mathematical expression, and it is often difficult for students to understand and grasp them in connection with real life. In view of the limitations of students' thinking, the following points should be paid attention to when teaching function concepts:

(1) According to the textbook, it is higher than the textbook

From lesson preparation to class attendance, and then to reflection after class, teachers should make teaching arrangements based on the textbooks, on the basis of a deep understanding of the content of the textbooks, combined with their own teaching practice, and according to the subject requirements and students' development needs. In the teaching of concepts, the basic concepts in the textbook are the center and support of teaching, and teachers should be able to stick to the teaching materials, pay attention to generation, and highlight thinking. Through the introduction of the concept of functions in practical situations, and supplemented by GGB and other software to demonstrate function images, guide students to think about problems with mathematical thinking, grasp the logical sequence of teaching content, understand the essence of conceptual content, and cultivate students' logical reasoning (inductive reasoning and deductive reasoning) literacy in this process.

(2)The teaching process should be progressive

In the teaching design, it is necessary to be able to guide students to derive the relevant content of function concepts step by step by setting up a series of practical situations and problem strings according to the arrangement of the textbook, and pay attention to the deformation and application of concepts, so as to cultivate students' inductive reasoning and deductive reasoning ability. Starting from the familiar situation can reduce the abstraction of concepts for students, let students realize that mathematics comes from reality, and is higher than life, and experience the wide use of mathematics, so that students have a desire to explore mathematics, so that they can concentrate on mathematics learning. In this process, teachers should help students build a knowledge system step by step, grasp the connection between concepts, and pay attention to exercising the quality of logical reasoning thinking.

(3)Exercises should be designed to help deepen students' understanding of concepts

The teacher's teaching can only convey to the students the content of the course designed by the teacher based on the characteristics of the students, but cannot represent the content that the students have truly learned and understood. The setting of exercises can not only test students' mastery of knowledge, but also allow students to check and fill in the gaps in knowledge points through exercises. Most of the examples and exercises in the textbook are simple applications of the basic content. After screening and selecting, the teacher can let

students have a deeper understanding of the concept through the explanation of in-depth topics, rather than simply staying on the surface expression. This kind of effective exercise setting can well develop students' reasoning and reasoning ability, and exercise students' ability to use mathematical thinking methods to analyze and solve problems. The correct "brushing questions" is the process of deepening students' understanding of knowledge, feeling the key points of conceptual knowledge in the questions, and cultivating students' logical reasoning ability.

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