

# THE STUDY ON THE APPLICATION OF MATHEMATICAL THINKING METHODS IN TEACHING MATHEMATICS IN MIDDLE SCHOOL

Yang Lin Yanzhi He\*

Department of Mathematics, College of Science, Yanbian University, Yanji 133002, CHINA

## ABSTRACT

With the implementation and development of the new curriculum reform, it is required to pay attention to the quality education of students and promote the individualized development and comprehensive development of students. In the process of teaching junior high school mathematics, it is necessary to strengthen the cultivation of students' mathematical thinking ability and practical ability, mainly focusing on the cultivation of students' mathematical thinking methods, guiding students to master and use mathematical thinking methods to learn mathematical knowledge and solve mathematical problems. Mathematical thinking methods are the essence of mathematical knowledge, which is an important tool for students to transform mathematical knowledge into their own ability. This paper discusses the specific use of the four important mathematical methods of thought, namely, regression, combination of numbers and shapes, classification discussion, and equations and functions, in the teaching practice of junior high school, in order to provide ideas for the cultivation of students' mathematical methods of thought.

**Keywords:** Mathematical ideas; mathematical methods; teaching.

## THE QUESTION RAISED

Mathematical ideological method is the essential understanding of mathematical knowledge and its use of mathematical methods, which is embedded in the specific mathematical content and mathematical methods, and after a certain amount of refinement and generalization, to become a rational understanding. Mathematical thinking method is an important part of mathematical knowledge, which is implied in the specific mathematical knowledge. In the past junior high school mathematics classroom teaching, teachers often use “reading from the book”, “indoctrination”, “sea of problems” mode to carry out teaching, resulting in a lack of students to participate in the math. As a result, students lack the autonomy and enthusiasm to participate in mathematics classroom teaching, and they cannot have a thorough understanding and cognition of the mathematical knowledge they have learned. The use of mathematical thinking in junior high school mathematics classroom teaching, can let students in the learning, problem solving process to master the rules and techniques, reduce the learning difficulty of students, students in the learning of mathematical knowledge at the same time to cultivate good mathematical thinking ability, so as to be able to flexibly cope with all kinds of mathematical problems, abstract, complex mathematical problems simple, intuitive, and maximize the quality of mathematical learning. In the learning process of junior high school mathematics, students can only have a thorough understanding of the knowledge they have learned by accurately grasping the mathematical thinking methods and constructing a complete system of mathematical knowledge, so that they can use what they have learned. Whether in the learning process or in the process of solving problems in the application of mathematical thinking methods, can reduce the difficulty of learning, help students clear thinking, subtle enhancement of their mathematical thinking ability, so teachers in junior high school mathematics teaching to strengthen the use of mathematical thinking methods, so that students get more

comprehensive development.

### COMBINING NUMBER AND SHAPE IDEAS

The idea of combining numbers and shapes is one of the most commonly used mathematical ideas that combines abstract numbers with intuitive shapes to analyze and study, and then solve problems. <sup>[1]</sup> Through the combination of numbers and shapes, students to a new way of thinking to understand the problem, so that the abstract problem is concrete, complex problems are simplified, reduce the difficulty of the problem, so that students are more likely to solve the problem, but also help to improve the ability of students to use mathematical methods to solve practical problems. The specific application of the idea of combining numbers and shapes can be roughly divided into two categories: “solving shapes by numbers” and “using shapes to help numbers”. In daily teaching, teachers should flexibly use the idea of combining numbers and shapes. <sup>[2]</sup> For example, when teaching “rational numbers and their operations”, teachers can make full use of the number axis, so that students have a deep understanding of “the opposite of the amount of” what is expressed, which is a great help for students to grasp the opposite number, absolute value and other concepts. Teachers in teaching “life in the data” chapter, you can graphically represent the data, but also let the students intuitively understand. When teaching the chapter of “plane figures and their positional relationships”, teachers can make students have a very intuitive understanding of the length of a line segment and the size of an angle through the change in quantity, which helps students to better compare line segments and angles. Teachers in the teaching of the “plane coordinate system”, can be targeted to guide students to think, try to treat the classroom as a plane, to build the reality of the plane coordinate system, so that the specific location of each student and the number of rows and columns (how many rows and columns) will be combined. Through the active participation of students, the idea of combining numbers and shapes can easily penetrate into the thinking of students, so that students are clear about the significance of each point on the plane coordinate system, which also lays a more solid foundation for the subsequent study of the primary function, the quadratic function and the inverse proportional function.

Example: In the plane right-angle coordinate system, the line  $y=kx+3$  passes through the point  $(-1,1)$ , find the solution set of the inequality  $y=kx+3<0$ .

Solution: Substituting the point  $(-1,1)$  into  $y=kx+3$  gives  $1=-k+3$ , which solves for  $k=2$ . So the equation of the line is  $y=2x+3$ . When  $y=0, x=3/2$ . So the coordinates of the intersection of the line with the x-axis are  $(-3/2,0)$ . Combined with the image of the primary function  $y = 2x + 3$  (figure omitted), the solution set of the inequality  $kx + 3 < 0$  is  $x < -3 / 2$ . Combine the familiar function image with the inequality, so that the abstract problem becomes concrete, intuitive, which is the use of the combination of ideas of the number and shape of the problem solving to achieve the effect.

### REGRESSION IDEAS

Regression is transformation and induction, that is, when solving a specific mathematical problem, the existing problem is transformed in a certain way, to transform it into a simpler, more familiar to students to answer a problem of a kind of problem solving ideas. <sup>[3]</sup> The idea of reductio ad absurdum is the idea of making one kind of mathematical object transformed into another kind of research object under certain conditions. The idea of recursion is one of the basic ideas of problem solving in middle school mathematics. In terms of specific transformation methods, it can be carried out by a variety of specific means, such as the transformation between addition, subtraction, multiplication and division, the transformation between multiplication and prescribing, adding auxiliary lines as well as adding auxiliary elements. In daily teaching, the teacher should subconsciously make students realize that many common mathematical methods are essentially a method of transformation. Teachers can then

provide targeted training to students in conjunction with the actual teaching content, so that students can naturally master the ideological method of reduction. Of course, in the specific practice of teaching, you can first put forward specific problems, and then guide students to think independently, so that students to explore the idea of transformation. In the usual teaching process, teachers should consciously penetrate the idea of regression. For example, in solving the fractional equation, let the students recall the method of solving the integral equation, and then use the regression idea to transform the fractional equation into the integral equation familiar to the students, so that the students can successfully find the solution to the fractional equation; for example, in solving the system of binary equations, the “elimination of the method” is also a specific application of regression idea, through appropriate constant transformations, first of all the constant transformations, then the constant transformations. Through appropriate constant transformation, the first elimination of an unknown, so that the system of quadratic equations into the students are familiar with the one-variable equations, so that students can easily master the solution of the system of quadratic equations. Similarly, there is the solution of quadratic equations used in the “descending method”, divided into whole forms, etc., these are the specific embodiment of the idea of naturalization.

Example: a clothing store purchased a number of A, B two models of fashion T-shirts, A models with a total of 7,800 yuan, B models with a total of 6,400 yuan, the number of A models is the number of B models 1.5 times the number of pieces, A models of the price of each piece of the price of each piece of B models less than 30 yuan.

(1) How many T-shirts are purchased for each of the two models?

(2) store to raise the price of 60% markup sales, sales for a period of time, A models all sold out, B models of the remaining half, the store decided to B models at 50% off the price of the marked price reduction in sales, and soon sold out, sold out of this batch of T-shirts, the store a total of how many dollars in profit?

Solution: (1) set the store purchased B type T-shirts  $x$ , then purchased A type T-shirts  $1.5x$ , according to the meaning of the question  $\frac{7800}{1.5x} + 30 = \frac{6400}{x}$ , Solve  $x = 40$

After testing  $x = 40$  is the solution of the original equation, and in line with the meaning of the question,  $1.5x = 1.5 \times 40 = 60$ .

Therefore, A, B two types of T-shirts were purchased 40 and 60 pieces.

(2) According to the question, the store profits.

$7800 \times 60\% + 1/2 \times 6400 \times 60\% + 1/2 \times 6400 \times [(1 + 60)\% \times 50\% - 1] = 5960$  (yuan).

Therefore, after selling the T-shirts, the store made a total profit of 5960 yuan.

Based on the meaning of the problem list of fractional equations, in the process of solving fractional equations, it involves the idea of regression, the fractional equations into the students are familiar with the integer equations, and then find the solution to the equation.

## CLASSIFICATION AND DISCUSSION IDEAS

Classification and discussion is an important method of mathematical thinking, which runs through the whole process of teaching mathematics. First of all, junior high school mathematics is divided into two categories, algebra and geometry, which is the embodiment of classification ideas; secondly, the classification of real numbers, the classification of angles, the classification of equations and functions in junior high school mathematics, etc., all reflect the classification ideas. Therefore, when teaching, teachers should instill in students the idea of categorization and discussion, that is, different criteria for categorization lead to different results. [4] In the specific teaching process, teachers should inspire students to think about the same object in different situations and classify it rigorously and methodically according to the classification criteria. Generally speaking, under the precondition that the answer or conclusion of a problem is not unique, we can consider using the idea of classification and discussion to analyze all the possible scenarios that exist in the problem one by one, and come up with answers or

conclusions under different conditions.

Example: The lengths of the two sides of an isosceles triangle are 4cm and 6cm respectively, what is its perimeter?

Solution: Since the question does not specify the specific lengths of the base and the waist, resulting in the perimeter of the triangle appears to be not unique, then the idea of classification and discussion should be used. Thus, there are two cases: 4 cm, 4 cm, 6 cm and 6 cm, 6 cm, 4 cm. In the first case, the perimeter of the triangle with three side lengths of 4 cm, 4 cm, 6 cm is 14 cm and in the second case, the perimeter of the triangle with three side lengths of 6 cm, 6 cm, 4 cm is 16 cm.

In this type of question, there is also an implied condition that the three-side relationship of the triangle must satisfy that the sum of the two sides is greater than the third side and the difference between the two sides is less than the third side, and if the classification carried out fails to satisfy this requirement, it is not necessary to discuss it. It is worth noting that, although classification discussion is a very effective mathematical method, but in the specific use of classification discussion to solve practical problems, according to the known conditions of rigorous and organized classification, so as to avoid omissions and repetitions in the classification, that is, focus on the completeness of the classification. Of course, do not blindly and mechanically rush to classify, should be to solve the problem in-depth study, fully explore the relationship between known and unknown quantities, to arrive at the best solution.

## EQUATIONS AND FUNCTIONAL IDEAS

Function thinking is the idea of discovering relationships between variables. Equation thinking is the idea of establishing a relationship between an unknown variable under study and a known variable, or transforming it into an equation. They are some of the more widely used mathematical ideas at the middle school level. Equation thinking is involved in both quadratic equations, systems of quadratic equations, and quadratic equations. <sup>[5]</sup> And the function idea by analyzing the correspondence existing between different variables can make mathematical problems more intuitive and logical. Therefore, teachers should guide students to explore the ideas contained in equations and functions in junior high school mathematics and flexibly use them to think and solve problems in the daily teaching process. In specific teaching practice, teachers can start from the actual problem, using known conditions or familiar formulas to grasp the quantitative relationship between the known variables and unknown variables in the problem, so that it is transformed into equations (or systems of equations), inequalities, and other mathematical models, and then through the solution of mathematical models, so that the problem can be solved. In the “binary system of equations” chapter, the more classic chicken and rabbit in the same cage is the specific application of the idea of equations.

Example: a supermarket sales A, B two kinds of goods, A goods each piece of 10 yuan, selling price of 15 yuan; B goods each piece of 30 yuan, selling price of 40 yuan.

(1) If the supermarket to buy a total of 80 pieces of two kinds of goods, and exactly with 1600 yuan, how many pieces of A, B, two kinds of goods each?

(2) If the supermarket to make a total of 80 pieces of two kinds of goods purchased cost no more than 1640 yuan, and the total profit (profit = selling price - purchase price) is not less than 600 yuan, please help the supermarket to design the corresponding stocking program, and pointed out that the supermarket profit the largest program.

Analysis (1) is obviously a specific application of the chicken and rabbit problem, you can use the equation idea to solve the problem. Choose either a quadratic equation or a system of quadratic equations. (2) is a specific application of the set of inequalities, you can construct a set of inequalities to solve the problem according to the known conditions in the topic.

Solution: (1) let the supermarket purchased x pieces of A goods, then purchased B goods (80-x) pieces, by the meaning of the question  $10x + 30(80-x) = 1600$ , solve  $x = 40$ , then  $80-x = 40$ .

Therefore, the supermarket purchased A, B two kinds of goods each 40 pieces.

(2) set the supermarket purchased A goods  $y$ , the purchase of B goods  $(80-y)$ , from the meaning of the question: 
$$\begin{cases} 10y + 30(80 - y) \leq 1640 \\ (15 - 10)y + (40 - 30)(80 - y) \geq 600 \end{cases}$$

Solution  $38 \leq y \leq 40$

Since  $x$  can only be a non-negative integer, so  $x = 38, 39, 40$ , then  $80-x = 42, 41, 40$ , there are three different options.

The profit of the supermarket is  $5 \times 38 + 10 \times 42 = 610$ (yuan) when option 1 is adopted.

The profit of the supermarket is  $5 \times 39 + 10 \times 41 = 605$ (yuan) when option two is adopted.

The supermarket's profit is  $5 \times 40 + 10 \times 40 = 600$ (yuan) when option 3 is adopted.

Obviously, the supermarket purchased 38 pieces of goods A, B 42 pieces of goods to obtain the maximum profit, for 610 yuan. Similarly, teachers in the teaching process should be conscious, planned, purposeful penetration of function thinking. For example, in teaching "positive and inverse proportion function and primary function", from the beginning to the end of the function of the correspondence between the variables instilled in students.

## APPLICATION STRATEGIES OF MATHEMATICAL THINKING METHODS IN JUNIOR HIGH SCHOOL MATHEMATICS TEACHING

### Penetrate the idea of transformation and naturalization to improve the ability of strain

The idea of transformation and reduction is one of the most basic mathematical ideas, and it is also a thought method that is easy for students to accept in junior high school. [6] It is very important to grasp the idea of transformation and generalization to improve the efficiency of the classroom. Transformation and generalization of the idea of the overall embodiment of the "unfamiliar knowledge" into "familiar knowledge", "complex knowledge" into "simple knowledge", so it is also known as the "knowledge", so it is also known as the "knowledge". Knowledge", so also known as 'transformational thinking'. For example, when students are learning "solving a system of quadratic equations", they can convert a system of quadratic equations into a single equation through the "elimination method", so as to achieve the purpose of transforming a complex system of equations into a single equation. Another example, students in the study of "the solution of fractional equations", for how to break through the problem of the solution of fractional equations, the textbook also penetrates the idea of transformation and naturalization, the fractional equation into a whole equation, students through the solution of whole equations, to achieve the purpose of learning fractional equations. The textbook penetration of transformation and naturalization of ideas there are many, such as in the process of students learning three-dimensional graphics, the textbook will be permeated by plane graphics, so as to achieve the three-dimensional graphics problems into plane graphics problems to achieve the purpose of plane graphics and three-dimensional graphics into each other; students in the study of irrational equations can be converted into rational equations to solve; students in the face of the problem of polygon can be converted into a polygonal problem to deal with; and so on. Triangle problem to deal with; and so on.

### Infiltrate the idea of combining numbers and shapes to improve the migration ability

Combination of mathematical thinking is a very important mathematical thinking in junior high school mathematics, is the abstract mathematical language, quantitative relationships and specific intuitive graphic combination, the use of abstract thinking and image of the organic combination of thinking, with the help of graphics to reflect the relationship between the number of graphs, with the help of the number of graphic descriptions of the essence of the connotation. [7] This mathematical thinking is very clever to the abstract mathematical language, quantitative relationships and intuitive images together, played a "shape to help the number" and "to the number of auxiliary shape" effect, can help students effectively solve many

common problems. For example, students in the study of “absolute value and the opposite number” related content, in thinking “two numbers to compare the size of the absolute value of the larger number must be larger? When asking the question, they compare 5 with 3 and -5 with -3, combine the axes, mark the positions of the two numbers, and then compare the absolute values according to their distances from the origin.

Textbook reflects the combination of number and shape ideas are very many, this method in the use of the process can be very clever to make some students think that it is difficult to get started to effectively solve the problem, especially in the relationship between equations and functions, geometric shapes in the process of determining the most important value of the idea of combination of numbers and shapes is particularly important.

### **Experience the idea of classification and discussion, the development of students' thinking rigor**

Every conclusion in mathematics is established under specific conditions, such as two straight lines are perpendicular to the third line, the two lines are parallel and need to be in the same plane. In daily teaching, the conditions of some problems are not clear, then the conclusion is not fixed, which requires teachers to study and analyze under different conditions according to the specific requirements. This mathematical idea of dividing all the problems into multiple categories according to the specific context and transforming them into multiple sub-problems to be analyzed and studied is known as categorical discussion idea. [8] Teachers in the process of studying the textbook, can be found in the classification discussion exists everywhere, from the “understanding of rational numbers” on the beginning of the emergence of the obvious classification requirements, that is, positive rational numbers, negative rational numbers and zero three types of forms; students in the process of learning isosceles triangles, if the topic is not clear which side of the waist, which side is the base, it is necessary to study the classifications according to the requirements; students in the study of hook and loop triangle, if the topic is not clear which side of the waist, which side is the base, it is necessary to study the classifications according to the Requirements for classified research; students in the process of learning the collinear theorem, if it is not clear which are right-angled edge which is the hypotenuse, it is necessary to further classify the discussion between teachers and students; students in the study of the function, the value of greater than 0 or less than 0, quadratic coefficients of the quadratic function of the quadratic coefficients of greater than 0 or less than 0, are required to classify the discussion of the problem.

### **Understanding the whole idea and cultivating students' sense of pattern**

The whole idea is that when encountering a problem according to its specific situation from the problem as a whole, highlighting the overall structure of the problem of the analysis and transformation, to find the structural characteristics of the problem, good use of “integrated” vision, some of the algebraic formula or part of it as a whole, and then according to the connection between the problem, to carry out a purpose, The whole is then purposefully and consciously processed according to the connections between the problems. [9] This way of thinking is often not very clear in the textbooks, but teachers should take into account the whole idea when teaching, such as in the simplification of algebraic formulas and value, in solving more complex equations (groups), in determining the characteristics of some geometric shapes will be applied to the whole idea. In the process of using this idea often have to take into account the whole substitution, the whole set of elements, the whole operation, superposition and multiplication processing, geometry in the complementary shapes and other methods, the teacher should pay attention to the teaching process to guide the students to observe carefully, to understand the overall idea of a pattern consciousness.

## CONCLUSION

The ideological methods contained in junior high school mathematics are not limited to the ones discussed above, but there are also such ideological methods as observation and experiment, analysis and synthesis, and induction and analogy, etc. How to infiltrate them in the teaching process requires teachers to dig them out continuously in the teaching process. In the specific teaching process, teachers should be based on the requirements of the “Compulsory Education Mathematics Curriculum Standards”, fully extract the specific mathematical methods of thought in the textbook, carefully design each class, deliberate each teaching link, conscious, targeted penetration, so that students in the subtle understanding of mathematical methods of thought. In addition, teachers in teaching and teaching at the same time, but also good at guiding students to think independently, and gradually clarify the ins and outs of mathematical knowledge and master some of the basic mathematical ideas, so that students in solving practical problems, can actively use these ideas. As the saying goes, “teaching a fish is not as good as teaching a fisherman”, the traditional transfer of knowledge can no longer meet the teaching requirements, students need to master the skills and means of learning knowledge, teachers should be able to actively guide students to explore the key materials in the textbook, with appropriate exercises and summarization, so that students will be familiar with these mathematical ideas and methods, skilled in hand, so that their mathematical solutions to problems. In this way, their mathematical problem solving ability will be improved and their core mathematical literacy will be enhanced.

**Remark:**\*Corresponding author:Yanzhi He E-mail:yzhe@ybu.edu.cn.

## REFERENCE

- [1] Xiong Chengju. Exploration of the application of the idea of combining numbers and shapes in the teaching practice of junior high school mathematics[J]. Xueyuan Education,2024,(21):61-63.
- [2] Wei Anzhong. The application and investigation of the idea of combining numbers and shapes in junior high school mathematics teaching[J]. Mathematics Learning and Research,2024,(15):56-58.
- [3] Zhao Ruqin. Discussion on the Application of Regression Ideas in Middle School Mathematics Teaching[J]. Intelligence,2021,(25):59-60.
- [4]Sun Dagui. Analysis of the application of classification and discussion ideas in junior high school mathematics teaching[J]. Mathematics Learning and Research,2021,(14):16-17.
- [5]Chancheng. The application of equation thinking in junior high school mathematics teaching[J]. Middle school math teaching and learning,2021,(15):25-27.
- [6] LI Xinyue,CHEN Zhaoying. Research on the application of function thought method in junior high school mathematics teaching[J]. Research on Middle School Mathematics(South China Normal University Edition),2022,(24):14-16.
- [7] Chen Tongling. The Integration and Penetration of Mathematical Thinking Methods in Middle School Mathematics Teaching[J]. Middle School Curriculum Counseling,2022,(27):117-119.
- [8] Chen Wenyao. How to penetrate mathematical thinking methods in junior high school mathematics teaching[J]. Test and Research,2023,(25):28-30.
- [9] Li Lu. Strategies for infiltrating the ideological method of classification and discussion in junior high school mathematics teaching[J]. The second classroom(D),2022,(04):34-35.