RESEARCH ON THE APPLICATION OF MATHEMATICAL THINKING METHODS IN HIGH SCHOOL MATHEMATICS TEACHING

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ABSTRACT

China's college entrance examination has established an index system of "one core", "four layers" and "four wings"[1]. In the new college entrance examination reform, teachers should guide students to explore, explore new methods, actively solve difficulties, and encourage students to solve the limitations of logical thinking, fixed skills and inertial thinking, and have the courage to challenge and innovate independently. Demonstrate the characteristics of comprehensive openness to the outside world in various forms, and then create outstanding talents with strong scalability and continuous learning ability to promote social development. In mathematics teaching, the most important thing for students at this stage is to lay a solid foundation for thinking training, be proficient in using formulas to solve problems, and master the ability to answer questions in mathematics class. This paper analyzes the application of thinking methods such as observation and practice, analogy and conjecture, specialization and generalization, transformation and classification, abstraction and generalization, etc., in solving mathematics problems in ordinary middle schools, which can provide an effective reference for teachers' teaching.

Keywords: High school mathematics; mathematical thinking methods; Mathematics teaching.

INTRODUCTION

Considering the requirements of the new curriculum standards, the course should pay attention to the strict requirements for the development of students' thinking ability, which requires students to be able to effectively rely on mathematical thinking to deal with problems, and to consciously strengthen the course knowledge after the expansion of mathematical thinking, which is the inevitable choice for them to cope with the changes in the form of college entrance examination propositions and course learning requirements. Based on this, high school mathematics teachers should consciously cultivate high school students' mathematical thinking ability in many aspects through mathematical thinking methods. The mathematical thinking methods are more commonly used in observation and experiment, analogy and conjecture, specialization and generalization, transformation and classification, abstraction and generalization.

1. Observation and practice

Mathematics education is one of the main fronts to realize quality education, mathematical observation is the foundation of mathematics learning, mathematical observation ability is the basic ability of understanding, is the channel for students to obtain information from mathematical background materials, and is the starting point of mathematical thinking. Therefore, it is one of the problems that mathematics courses must face to explore how to cultivate students' observation ability in mathematics teaching and use it as a breakthrough to improve students' mathematics performance[2].

Interest is the best teacher. Psychological research believes that children's curiosity is very strong, and as they grow older, their curiosity gradually disappears. However, it is not difficult to find that this is true not only for children, but also for adults. This is especially true of the high school students we are dealing with. Therefore, in order to cultivate observation ability, improving students' observation ability is the key, which can be started from the following three aspects:

(1) Let students have a successful observation experience
Some students have a weak foundation and lack all aspects of mathematics requirements, so it can be said that mathematics is a hardship for them. In the process of learning mathematics for many years, they have never had the experience of learning successfully, that is, in this state, teachers can only design some mathematical problems suitable for them, and can simply find out the relationship, argument, and draw conclusions from the mathematical topics. By allowing them to draw the right conclusions through observation, students get the emotional experience of success, stimulate their inner desire to succeed, and they feel that they can also succeed, and mathematics is not difficult to learn. This greatly enhances students' interest in learning.

(2) Recreate typical events in the history of mathematics to stimulate students' interest in learning
The history of mathematics is the history of the development of mathematics, in which the hesitation, bitterness, difficulties and twists and turns of the development of mathematics are recorded. The history of mathematics is also a record of the struggles of mathematicians to overcome difficulties and overcome crises, and understanding the history of mathematics can help students understand the real process of mathematics development, and gain inspiration and confidence from it. Although the students are more distant from mathematics, they are full of reverence for the creation of mathematics. So I often tell them some stories about mathematicians in the history of mathematics, and they love to hear them, such as: Vedic theorem, Pythagorean theorem, etc... Students will also have fun memorizing mathematical formulas and concepts.

(3) Conduct mathematical observation competitions appropriately
Organize extracurricular activity groups to carry out extracurricular observation activities. Teachers can assign extracurricular observation assignments based on classroom teaching content. You can find questions that are relevant in mathematics, such as the cube in solid geometry, which contains a variety of relationship between points, lines, and planes. Teachers can find a lot of good materials for students to observe repeatedly, which can improve students' interest in observation and cultivate students' mathematical observation ability.
Case From the number of even-strands to Fermat's theorem--- mathematics is not far away.
Teacher: A triangle made of three line segments with lengths of 3, 4, and 5 must be right-angled triangles, so let's study what is the relationship between the three numbers 3, 4, and 5?
Student: (observed) are three consecutive integers.
Teacher: It's three consecutive integers, so when we use three other consecutive integers 4, 3, 2 or 6, 5, 4 as side lengths to form a triangle, is it a right triangle?
Student: No, oh 5, 4, 3 into a series of equal differences, then three equal difference series of integers form a right triangle.

Teacher: 6, 4, 2 or 7, 5, 3 are also equal difference sequences, can they form a right triangle? No, (in fact, it is difficult to see any connection between 5, 4, and 3 through addition, subtraction, multiplication, and division, and students continue to observe and discuss to find patterns) The teacher prompts, what do they get by squareting each of them?
Student: 5^2=25 4^2=16 3^2=9 There is 25=16+9 between them, is5^2=4^2+3^2
Teacher: Can you give any other examples? Please summarize the general pattern
Student: $13^2 = 12^2 + 5^2$...... Through a large number of examples, students make in-depth observations and come to a general conclusion
The inverse theorem of the Pythagorean theorem: when a^2=b^2+c^2, the triangle made with a, b, and c as the sides is a right-angled triangle.
Pythagorean theorem: When the side lengths of a right triangle are a, b, and c respectively, the relation a^2=b^2+c^2 must be satisfied.
Teacher: (Guide) Let's continue to observe, write a^2=b^2+c^2 as x^2=y^2+z^2 from the equation x^2=y^2+z^2,Can we say: The square of an integer can be decomposed into the sum of squares of two integers? Can you give such an example?
Student: 5^2=4^2+3^2 $13^2 = 12^2 + 5^2$ $17^2 = 15^2 + 8^2$ $25^2 = 24 + 7^2$......
Teacher: Obviously, not all integers can be like this, for example, the squares of 6, 7, 8, 9, etc., are written as the sum of the squares of the other two positive integers, which is obviously impossible.
Q: How many integers are there that meet the requirements? How many are there? Is there any way to find them?
In order to answer the above question, we might as well look at it from a different angle, and let the index change $x^3 = y^3 + z^3$ $x^4 = y^4 + z^4$...... Is there a positive integer solution?
More generally: when n is any natural number, is there a positive integer solution to the indefinite equation $x^n = y^n + z^n$? This is the famous Fermat problem.
The observation can go on, and the deeper you go, the more problems you will find, and the many interesting conclusions you can find. With this awareness, maybe one day in the future, you will become a mathematician. Through such examples, students will find that mathematics is not far away from them. This will gradually eliminate their fear of mathematics, they like to observe, and they like mathematics.

2. Analogy and conjecture

Therefore, encouraging students to make bold analogies and conjectures on problems is a way and method to cultivate students' mathematical literacy and ability. The physicist Albert Einstein once said, "Finding a problem is more important than solving a problem, and the discovery of a problem is sometimes due to conjecture and analogy." Polya mentions in the text: "Analogy is a great guide." As far as high school mathematics is concerned, the application of analogy conjecture ability also plays an important role in the teaching of high school mathematics problem solving. In order to cultivate students' ability of analogy and conjecture, it is also affected by factors such as students' mathematical level, cognitive ability development, mathematical cognitive structure and mathematical literacy, so teachers can guide students to try and explore through classroom teaching and problem-solving teaching, which not only solves mathematical knowledge, but also stimulates students' interest in mathematics learning, and effectively cultivates students' learning ability and improves the effectiveness of teaching through teaching under the guidance of correct theory[3].

(1) Reasoning by analogy requires moderation

Only things that are essentially the same or similar can be analogized. It would be erroneous to indiscriminately use analogies about things that are merely formally similar but not fundamentally identical, and to mechanically compare them on the basis of their superficial similarity. Such as not being able to use a·(b· c) = (a·b)·c association analogy to multiplication of multiple vectors a· (b·c)=(a·b)·c.

(2) Analogical reasoning needs to be divergent

From one thing to other things that are similar in nature to it; From one way or method to other ways or methods that are similar to their effects; From a concept or theorem to a string of concepts or theorems that are closely related to it.

(3) Variety, analogical reasoning requires variety

Analogy is often used in conjunction with induction and deduction, and it is also inseparable from analysis. Induction, analogy, and exploratory deductive methods are usually connected by conjecture and mental movements such as association and intuition, so it is necessary to consciously grasp characteristics such as creative thinking and apply them to practical work.

Case During a research study, a student found that the values of the following five formulas are all equal to the same constant.

①$sin^2 13°+cos^2 17°-sin^2 13° cos^2 17°$;

②$sin^2 15°+cos^2 15°-sin^2 15° cos^2 15°$;

③$sin^2 18°+cos^2 18°-sin^2 18° cos^2 18°$;

④$sin^2(-18)°+cos^2 48°-sin^2(-18)° cos^2 48°$;

⑤$sin^2(-25)°+cos^2 55°-sin^2(-25)° cos^2 55°$.

(1) Try to choose one of the above five equations to find this constant;

(2) Based on the results of the calculations in (1), generalize the student's findings as trigonometric identities and prove your conclusions.

Analysis: (1) Select formula (2), $sin^2 15°+cos^2 15°-sin^2 15° cos^2 15°= 1-\frac{1}{2} sin 30°=1-\frac{1}{4}=\frac{3}{4}$

(2) Analogy conjecture: The trigonometric identity is $sincos^2 15°^2 \alpha+cos^2(30°-\alpha)-sin\alpha \cdot \cos(30°-\alpha)=\frac{3}{4}$

The lift side$=sin^2\alpha +(cos 30° cos\alpha + sin 30° sin\alpha)^2-sin\alpha(cos 30° cos\alpha + sin 30° sin\alpha) =sin^2\alpha +\frac{3}{4}cos^2\alpha+\frac{\sqrt{3}}{2}sin\alpha cos\alpha+\frac{1}{4}sin^2\alpha - \frac{\sqrt{3}}{2}sin\alpha cos\alpha-\frac{1}{2}sin^2\alpha=\frac{3}{4}sin^2\alpha+\frac{3}{4}cos^2\alpha =\frac{3}{4}$

For reasoning problems based on analogical reasoning, we usually try to find and probe on the basis of which we can obtain conjectures about relevant problems or solutions, and then use theoretical proof to achieve the goal or disprove the conjecture.

3. Specialization and generalization

The idea of specialization and generalization is one of the important ideas of mathematics, and the learning of mathematical formulas, theorems, and laws is usually to start from the particular, then summarize and summarize, and then prove to obtain a general conclusion, and then use the general conclusion to solve related mathematical problems. This process of repeated understanding from the special to the general and from the general to the special is the concrete embodiment of this mathematical idea[4].

The new textbook emphasizes the cultivation of students' core literacy in mathematics, and the first place is the ability of mathematical abstraction, that is, abstracting general laws and structures from the concrete background of things, and using abstract mathematical language to represent them. In terms of structure, the new textbook will introduce relevant concrete concepts before introducing abstract concepts. The thinking method of specialization and generalization not only conforms to the students' cognitive law of new concepts, but also meets the requirements of improving students' reasoning ability put forward by the new textbook arrangement law and curriculum standards. Therefore, in the teaching process of new knowledge and new concepts, combined with the characteristics of specific teaching content, the special and generalized mathematical ideas are flexibly implemented, and the concepts are formed in the process of analysis, induction and generalization, and after the concepts are formed, specific exercises are used to strengthen the understanding of the new concepts, which can finally improve students' ability to master the abstract mathematical knowledge in the new textbooks.

(1) Concept formation should focus on the process of analysis, induction, and generalization.
Concept formation refers to the process of inducting and summarizing the common essential attributes of a class of things from a large number of specific examples, and embodies the reasoning method of understanding new knowledge from the special to the general. The teaching of concept formation should provide students with familiar concrete examples, guide students to analyze the attributes of each example, and then further abstract the common essential attributes to form preliminary concepts.

(2) Conceptual assimilation should focus on the analysis of the particularity and generality between the new concept and the genus concept.
When students' knowledge is sufficient, some new concepts that are deepened on the basis of genus concepts can also be learned by means of concept assimilation. Concept assimilation refers to the process in which students use the concepts in the original cognitive structure to understand and accept new concepts, that is, to define and recognize new concepts in the way of genus concepts (generality) and species differences (particularity). Embody deductive reasoning methods from the general to the particular.

(3) After the concept is formed, it is necessary to strengthen the understanding of the concept through application.
When students initially learn new concepts, they should set up some specific and simple problems for students to solve, and further strengthen students' understanding of the new concepts through problem solving. Forming or assimilating acquired concepts is a simple application of reinforcement concepts, and this way of learning concepts is prevalent in textbooks.
Case Monotonicity of functions
Teacher: We have learned about the monotonicity of exponential functions and logarithmic functions.
Student: The exponential function $y = a^x (a > 1)$ is a strict increase function in the interval$(-\infty, +\infty)$. The image rises from left to right, and the value of the function y increases with the increase of the independent variable x.
Teacher: How to use symbolic language to describe that the exponential function $y = a^x (a > 1)$is a strictly increasing function in the interval $(-\infty, +\infty)$?
Student: Take$x_1, x_2 \in (-\infty, +\infty)$, when$x_1 < x_2$, there are $a^{x_1} < a^{x_2}$ interval$(-\infty, +\infty)$, then the exponential function is a strict increasing function in the interval$(-\infty, +\infty)$. Teacher: Very good! What about the monotonicity of the logarithmic function $y = \log_a x\ (a > 1)$?

Student: The logarithmic function $y = \log_a x\,(a > 1)$ is a strictly increasing function in the interval $(0, +\infty)$. The image rises from left to right, and the value y increases with the increase of the independent variable x, any $x_1, x_2 \in (0, +\infty)$, and when $x_1 < x_2$, it is a strict increase function in the interval$(0, +\infty)$.
Teacher: Combining exponential functions and logarithmic functions in a symbolic language that is a strict additive function in a given interval, can you give a definition of a strict additive function to the general functions $y = f(x), x \in D$?
Student: Any $x_1, x_2 \in D$, when $x_1 < x_2$, there is always $f(x_1) < f(x_2)$.
Teacher: Do you have any different opinions? Is it true that all functions are strictly incrementally in the definition domain? Can you give a counter-example?
The domain of $y = x^2$ is R, and it is a strict increment function in the interval $[0, +\infty)$.
Teacher: Very good! So we need to revise the interval in the definition, so that interval I is a subset of D. The following is the textbook definition of the monotonicity of a general function: "For the function $y = f(x)$ defined on D, let the interval I be a subset of D. For the value of any given two independent variables on interval I, x_1, x_2, when$x_1 < x_2$, there is always $f(x_1) < f(x_2)$, that is, the function $y = f(x)$ is a strictly increasing function on interval I; If there is always $x_1 < x_2$, $f(x_1) \leq f(x_2)$, then the function $y = f(x)$ is said to be an increasing function in the interval I.
Teacher: By analogy with the definition of a strict subtract function, can you give a definition of a strict subtraction function?
Student: For the function $y = f(x)$ defined on D, let interval I be a subset of D, and for the value of any given two independent variables on interval I, x_1, x_2 when $x_1 < x_2$, there is always $f(x_1) > f(x_2)$, which means that the function $y = f(x)$ is a strictly decreasing function in the interval I; If there is always $f(x_1) \geq f(x_2)$, the function $y = f(x)$ is said to be a subtraction function in the interval I.
Teacher: Very good! The above definitions of "strict increase", "strict decrease", "increase" and "decrease" are collectively referred to as the monotonicity of functions.
Teacher: According to the definition of function monotonicity, is a strictly increasing function an incremental function? Is an increment function a strict increment?
Student: A strict increase function is an increase function, but an increase function is not necessarily a strict increase function.

The new mathematics textbooks have strengthened the cultivation of students' mathematical abstract ability, and put forward higher requirements for the teaching ability and level of front-line teachers. The above case takes the teaching of abstract definition of function monotonicity as an example, and proposes to use the mathematical idea from "special to general" and then from "general to special" to guide students to flexibly switch their thinking between specific cases and general definitions, so as to visualize abstract mathematical definitions and solve students' learning barriers to abstract mathematical knowledge. In actual teaching, teachers should dig deep into the structure of the newly compiled textbooks, understand the knowledge background that students have mastered, and reasonably arrange and select typical teaching cases, so as to promote students' ability to master abstract mathematical concepts in an all-round way.

4. Transformation and naturalization

In order to reduce the difficulty and pressure of students' learning, it is necessary to develop students' independent learning ability, guide them to flexibly change their learning strategies according to the teaching content, and achieve twice the result with half the effort[5]. The idea of transformation and naturalization involves the method of substitution, number transformation, complement transformation, etc., which guides students to use learning methods flexibly in a targeted manner, which is more conducive to achieving the expected learning effect.

(1) Enhance comprehension and cognition

Most high school students have a one-sided understanding of learning methods such as transformation and naturalization, and even regard mathematical thinking methods as a type of mathematical problem or topic, which leads to the unsatisfactory application effect of mathematical thinking and methods. Therefore, teachers need to strengthen and guide students to understand the idea of mathematical transformation and naturalization, so that students can realize that the idea is contained in the basic knowledge and basic skills of mathematics, and form a deep understanding and comprehension of it. Teachers can use certain teaching language to prompt students to analyze and understand problems thoroughly, so that students can clarify the key points in learning, and then realize the successful attribution of mathematical knowledge in familiar similar problems. In the process of high school mathematics teaching, it is necessary to start from the following aspects to improve students' awareness of transformation and naturalization: First, guide students to experience the idea of transformation and naturalization of mathematics in the teaching situation. Teachers design a variety of mathematics activities to attract students to participate in them, and gradually guide students to learn the idea of transformation and naturalization. Through the transformation and mutual transformation of various questions, teachers allow students to experience and return to thought. Second, teachers should strengthen the explanation of the process of knowledge generation in teaching, so that students can understand the mathematical ideas contained in the knowledge, rather than only understanding the final conclusion, which can strengthen students' awareness of transformation and naturalization.

(2) Exploit mathematical ideas within the body of knowledge

When cultivating students' ideas of transformation and naturalization, teachers need to have a thorough understanding of the content of the textbooks, excavate the ideas of transformation and naturalization, and combine the content of mathematical knowledge at different stages to infiltrate the ideas of transformation and naturalization that are in line with them. Teachers should not only pay attention to the teaching of theoretical knowledge, but also combine the teaching with actual cases, so that students can truly feel the application process of transformation and naturalization of ideas, and form mathematical thinking for knowledge transformation. The idea of transformation and naturalization of mathematics is systematically taught, and its connotation is infiltrated into the concepts, theorems, formulas, and problem solving of mathematics, so that students can strengthen their own transformation and naturalization ideas in learning.

(3) Pay attention to the integration of old and new knowledge

Mathematical knowledge is an expansion and expansion of the knowledge and experience learned in the past. Therefore, when cultivating students' ideas of mathematical transformation and naturalization, it is necessary to establish new and old knowledge systems and build a knowledge network system based on the characteristics of the discipline. Cultivate students' ability to transfer learning in the knowledge system, realize the flexible transformation and bypass of various mathematical knowledge, and finally strengthen their own mathematical transformation and naturalization ideas and abilities. For example, when learning the content related to "ternary equations", in order to deepen students' understanding and memory, they are simplified into binary equations and unary equations. In the pre-class teaching preparation stage, teachers systematically analyze the content of the textbooks, sort out the teaching ideas with the ideas of transformation and naturalization, and sort out the transformation process of mathematical knowledge, so as to better carry out teaching in the classroom. In the process of combing the ideas of transformation and naturalization, teachers should fully understand the current situation of students' application of mathematical ideas, and then carry out teaching according to the mastery level of students, so as to help students gradually grasp the in-depth naturalization thoughts. By integrating the idea of transformation and naturalization into every part of mathematical knowledge, students can flexibly transform new knowledge into old knowledge if they master the application of mathematical ideas and methods, and then understand the new knowledge content more deeply.

Case A matter of probability

Teacher: (Question 1) We have learned about mutually exclusive events and opposing events, how to distinguish between these two events?

Student 1: A mutex event is two events that cannot occur at the same time, and the translation into a symbolic language is $P(A \cap B)= P(A)+P(B)$.

Student 2: An opposing event is one of two events that occur in any one trial and only one of them, which translates into symbolic language as $P(A)+P(B)=1$.

Teacher: (Question 2) Students see this question. Only one of the three lottery tickets can only one can win the lottery, and now they are drawn by three students respectively, event A is "the first student did not draw the winning lottery ticket", event B is "the last student to draw the winning lottery ticket", will the occurrence of event A affect the probability of event B occurring?

Student 1: The two events do not affect each other.

Student 2: found $P(A)=\frac{2}{3}$, $P(B)=\frac{1}{3}$, $P(AB)=\frac{2}{9}=P(A)\cdot P(B)$.

Teacher: Students, what are the conditions for the two events to be independent of each other?

Student 3: $P(AB) = P(A)P(B)$.

Teacher: Yes. Therefore, we introduce a general definition of this event relationship and translate it into a symbolic language. For any two events A and B, if $P(AB) = P(A)P(B)$ is true, event A and event B are said to be independent of each other, which is simply called independent.

Teacher: (Question 3) Students think about the example questions in the textbook. There are 4 balls numbered 1, 2, 3, and 4 in a bag, and there is no difference except for the marking, and the ball is touched twice in a non-return manner. If event A = "the first ball is less than 3" and event B = "the first ball is less than 3", are event A and event B independent of each other?

Student 4: Let the sample space be ω, then $\omega=\{(m,n)|m,n\epsilon\{1,2,3,4\},\text{and } m\neq n\}$, $A=\{(1,2), (1,3), (1,4), (2,1), (2,3), (2,4)\}$, $B=\{(1,2), (2,1), (3,1), (3,2), (4,1), (4,2)\}$.

Student 5: According to the question design, it can be concluded that $P(A)=P(B)=\frac{6}{12}=\frac{1}{2}$, $P(AB)=\frac{2}{12}=\frac{1}{6}$, 此时$P(AB) \neq P(A)P(B)$, so event A is not independent of event B.

Transformation and naturalization are the basic methods of mathematics learning, which can be seen everywhere in high school mathematics teaching. Students can learn the ideas of transformation and naturalization, which can help students master other mathematical ideas, thus laying a good foundation for improving the quality and efficiency of learning. Teachers need to pay more attention to cultivating students' ideas of transformation and naturalization, cultivate students' awareness of transformation and naturalization, and improve their ability to solve mathematical problems. This paper excavates the ideas of transformation and naturalization in mathematics textbooks, continuously improves students' knowledge structure in teaching, and improves students' knowledge transformation ability.

5. Abstraction and generalization

Abstract generalization ability, as one of the ten "core keywords" proposed in the new curriculum standards, is the core competency that the new curriculum standards focus on, reflecting the characteristics of mathematics itself, which is not only an important condition for students to learn mathematics well, but also a source of improving problem-solving ability. With the deepening of the new curriculum standards, the cultivation of abstract generalization ability has become the basis of the development of the discipline, and the important position of abstract generalization ability is self-evident[6]. However, due to the abstraction of mathematics itself and the difference in the thinking ability of high school students, it often leads to deviations in some students' mathematical understanding, which requires teachers to guide students to make abstract generalizations in a timely manner in teaching, so as to cultivate students' abstract generalization ability.

(1) Intuitive perception: evokes the ability to abstract generalization

The research of educational psychology shows that high school students are in the transition stage from image concrete to abstract generalization in terms of abstract generalization. With the support of certain perceptual materials and existing experience, the generalization system of students can be further expanded through intuitive perception, which is helpful for the improvement of students' abstract generalization ability. In their daily teaching, teachers should start from the age characteristics of high school students, and lead students to abstract the relationship between numbers and shapes from perceptual materials and summarize them into general relationships and structures, so as to arouse students' ability to abstract and generalize.

(2) Clever questioning: Help students make abstract generalizations

In the teaching of high school mathematics in the classroom, teachers use a variety of teaching sections to stimulate students' active thinking and interest in exploring mathematical problems through clever questions, and then skillfully question them to trigger students' deep thinking. Teachers can ask key questions from the perspective of teaching content and students' psychological needs, which can play a role in inspiring students to abstract generalization, so as to cultivate students' ability to abstract generalization.

(3) Inductive analogy: Sort out abstract generalized thinking

High school students have more than 9 years of mathematics learning experience, and the amount of knowledge has already been sufficient, and a certain knowledge structure has been formed. In the teaching process, teachers should use inductive analogies to play a good role as organizers of students' learning, pay attention to the effective connection of knowledge, outline the framework of knowledge and skills, let students understand the construction of knowledge, and sort out students' abstract and generalized thinking.

(4) Induction: Improve students' generalization habits

Induction and summary is the process of reviewing and summarizing knowledge in a targeted manner, revealing the internal relationship between knowledge, integrating a single knowledge fragment into a complete mathematical knowledge system, integrating it with the knowledge in the original structure, and realizing the improvement of the knowledge structure system. On the one hand, it can improve students' habit of generalization and cultivate students' ability to abstract generalization. On the other hand, it can also deepen students' mathematical understanding.

Case Try to find the minimum value of the function $y = \frac{sinx}{2} + \frac{2}{sinx}(0 < x < \pi)$.

Teacher: This question is the function maximum-value problem, which is an extremely important type of problem in high school mathematics. Let's think about it, how to solve it?

Student 1: According to the conditions, it is not difficult to conclude that $\frac{sinx}{2}$ and $\frac{2}{sinx}$ are both positive numbers, then the mean inequality can be directly used, $y = \frac{sinx}{2} + \frac{2}{sinx} \geq 2\sqrt{\frac{sinx}{2} \cdot \frac{2}{sinx}} = 2$, and the minimum value is 2.
Student 2: Your solution is wrong. The condition for obtaining the minimum value is $\frac{sinx}{2} = \frac{2}{sinx} > 0$, then $sinx = 2$ which is not possible, so the equal sign here does not hold.
Teacher: Student 2 is right, when using the mean inequality to find the maximum, it is necessary to verify whether the equal sign is true, if not, the maximum value cannot be obtained. So how do you solve it here?
Student 3: I think that we can use the commutation method so that $t = sinx \in (0, 1]$, the original function can be deformed into $f(t) = \frac{t}{2} + \frac{2}{t}$, and the function $f(t)$ is a subtraction function on$(0, 1]$, then the minimum value of y should be $f(1) = \frac{5}{2}$.
Teacher: Very good, the commutation method is obviously feasible, but the monotonicity proof process of $f(t) = \frac{t}{2} + \frac{2}{t}$ cannot be omitted here.
Student 4: His problem-solving ideas are a bit cumbersome. Is this the only way to solve?
Teacher: It's really troublesome, which student has a simpler solution?
Student 5: The original function can be converted into $y = \left(\frac{sinx}{2} + \frac{1}{2sinx}\right) + \frac{3}{2sinx}$, according to $0 < x < \pi$, then there is $0 < sinx \leq 1$, so $y \geq 2\sqrt{\frac{sinx}{2} \cdot \frac{1}{2sinx}} + \frac{3}{2} = \frac{5}{2}$, the equal sign is true if and only if $\frac{sinx}{2} = \frac{1}{2sinx}$, that is, $sinx = 1$.
Student 6: Wow! Your problem-solving process is correct and the result is correct. But how did you come up with such an idea?
Student 5: $sinx = 1$ to get the minimum value, so I thought that I could first separate the part of $\frac{1}{2sinx}$ from the larger number $\frac{2}{sinx}$, and make up the equal sign first.
Teacher: Very wonderful problem-solving process. This also fully demonstrates the guiding significance of solving the condition that the middle sign of the mean inequality in the problem is true.
All in all, students' abstract generalization ability is very important for learning mathematical knowledge, teachers should strengthen the cultivation of students' abstract generalization ability, but the improvement of ability can not be achieved overnight, teachers should continue to explore, explore the scientific teaching methods that help students develop their ability, and gradually cultivate students' abstract generalization ability, and students' innovative consciousness and innovation ability will also be developed accordingly.

6. Epilogue

In short, thinking methods are the soul of mathematics, and mathematics has the characteristics of logical rigor and flexible thinking. In order to cultivate students' mathematical ability to analyze problems dialectically and solve problems flexibly, it is necessary to cultivate students' mathematical thinking methods. Therefore, in classroom teaching, teachers strengthen the connection between knowledge, cultivate students' various thinking methods, enable students to form careful thinking habits, deeply understand the connotation and extension of knowledge, and gradually improve their mathematical thinking ability.

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