RESEARCH ON THE CULTIVATION STRATEGIES OF HIGH SCHOOL STUDENTS' MATHEMATICAL ABSTRACT LITERACY

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ABSTRACT

The abstract of mathematical knowledge is obvious to all. Improving students' mathematical abstract literacy is a very important teaching task for high school mathematics teachers, which is the basis for improving students' academic performance and the key to promoting students' comprehensive literacy. Based on this, through the reflection of high school mathematics teaching time, this paper puts forward the strategy of high school students' mathematics core literacy training for reference.

Keywords: high school mathematics; abstract core literacy; training strategy;

THE QUESTION RAISED

The Mathematics Curriculum Standard for Senior High School (2017 edition) (hereinafter referred to as the curriculum standard) defines mathematical abstraction as one of the core qualities of mathematics. Teachers agree on the importance of mathematics abstraction, but there are two problems in the actual teaching: the traditional teaching method is simple, only focus on the result, lack of experience to get a complete concept of teaching abstract process; The understanding of mathematical abstraction is not in place, and we do not know how to train students' mathematical abstraction. So, how to understand mathematical abstraction? In the course of teaching, from what aspects can we concretely implement the cultivation of mathematical abstraction?

MATHEMATICAL ABSTRACT LITERACY

What is the connotation of mathematical abstract literacy?

The modern Chinese dictionary explains abstraction as follows: ① Abandoning individual and non-essential attributes and extracting common and essential attributes from many things is called abstraction, which is a necessary means to form concepts; ② can not be specific experience, general, empty. The meaning of the former layer can be understood as: extracting the common essential characteristics from concrete things, and the meaning of the latter layer is the extraction and generalization at a higher level, with imagination components, which can be symbolic expression. The modern Chinese dictionary defines accomplishment as: daily accomplishment. Based on this, abstract literacy can be understood as: the ability level of refining and

summarizing concrete things formed in daily learning and practice, eliminating the rough and the essence, eliminating the false and preserving the true, eliminating differences and seeking common ground, and seeking the basic appearance. Then what is mathematical abstract literacy? It is the cultivation level of mathematical research objects abstracted from quantitative relations, spatial forms and concrete things in daily learning and practice, mainly the level of abstract generalization ability. The curriculum standard gives a more precise description of mathematical abstract literacy: "Mathematical abstraction refers to the attainment of mathematical research objects through the abstraction of quantitative relations and spatial forms." It mainly includes: abstracting mathematical concepts and the relations between concepts from the relationship between quantity and quantity and the relationship between figures and figures, abstracting general laws and structures from the specific background of things, and using mathematical language to represent them." According to such requirements, the educational goal of cultivating core mathematical literacy in senior high school mathematics teaching can be gradually realized in the teaching process of mathematical concepts, geometric algebra, and innovative problems [1].

The basic characteristics of mathematical abstract literacy

Symbolic Characteristics

Mathematics abstractly studies the quantitative relations and spatial forms of things or phenomena, and the result is to use mathematical symbols as the carrier of mathematical thinking activities to exchange and spread mathematical ideas, so that the expression of things or phenomena becomes unified, concise and orderly. For example, the study of the law of change of an object with the change of another object is abstracting as the monotonicity of a function, and the concrete symbolic language is described as "Let the domain of the function f(x) be I, the interval D□I: if □ $\forall x_1, x_2 \in D$,when $x_1 < x_2$ there is $f(x_1) < f(x_2)\text{Or}(f(x_1) > f(x_2))$." Then the function f(x) is said to increase monotonically (decrease) in the interval D ". In mathematics teaching, the symbolic characteristics of mathematical abstraction can cultivate students' ability of abstract thinking. In the specific learning environment, students gradually feel and understand the symbolic characteristics of abstraction, feel the ability of mathematical abstraction to overcome complex and highly generalized, and can reasonably use mathematical symbols in the process of problem solving, and carry out mathematical reasoning logically.

Modeling Features

The object of mathematical research is things in the real world, which is changeable and diverse. Only by modeling the object of study through mathematical abstraction, can we study the problem through general modeling.

Such as the cyclical phenomenon of sunrise and sunset in nature, we can idealize this phenomenon as a cyclic change, and then model it into the circular motion model of the points on the circle of the unit circle to study the problem. This is to simplify the research object through idealization, abstract the general model from the direct model of things, and establish mathematical abstract relations on the basis of the research of the general model. In teaching, mathematical abstract modeling features can improve students' thinking level and promote the development of students' wisdom.

Quantitative characteristic

Mathematical abstraction is a research method to reveal the nature and law of objective things from the quantitative relationship, and it is constantly developing layer by layer abstract. For example, the abstraction of the concept of function has undergone the process of quantification from the correspondence between things in the objective world to the correspondence between quantities, and then from the correspondence between variables in junior high school to the correspondence between real numbers on the non-empty number set in senior high school. Quantification of concrete things into variables is the result of quantification. The abstraction of variables to real numbers and the transition from variable theory to real number correspondence theory are the result of hierarchical abstraction of function concept. The quantitative characteristics of mathematical abstraction in mathematics teaching can promote the generation of students' higher order thinking and the development of innovative thinking.

THE ROLE OF MATHEMATICAL ABSTRACTION

As the core of mathematical ability, mathematical abstraction occupies the most important position in mathematical core literacy. It can even be said that mathematical abstraction is the foundation of mathematical core literacy. On the one hand, the learning process of mathematics requires learners to have a certain ability of abstract generalization. Because mathematics is abstract and generalized, learners can realize a qualitative leap in the cognition of mathematical objects only if they have a certain degree of abstract generalization ability. On the other hand, the application process of mathematics also has certain requirements for learners' mathematical abstract cultivation. In the application activities of mathematics, learners are required to mathematize and abstract the situations they encounter, connect them with the mathematics knowledge they have learned, and finally use the mathematics knowledge to solve the problem and return the problem to the actual situation. This series of activities are built on the basis of mathematical abstraction, so teachers should pay special attention to the cultivation of mathematical abstract ability in teaching activities.So that students can grasp the essence quickly in the learning process of mathematics, improve the learning efficiency.

INFLUENCING FACTORS

Teacher level

According to the survey results, most teachers are aware of the purpose of the curriculum reform, especially the importance of cultivating the core mathematics literacy. However, some teachers do not have a thorough grasp of the essence of the core mathematics literacy, and some teachers rarely refer to the curriculum standards in the process of education and teaching, which leads to a lack of in-depth understanding of the core mathematics literacy. Relatively speaking, the recognition is slightly low. The traditional teaching ideas and models have lasted for a long time, so some teachers have not broken through the traditional ideas in time, lack of innovative thinking, and have not accepted the new ideas of training methods. Considering the actual situation, the current training mode is mainly in school, which must rely on mathematics classroom. As the "general director" of mathematics classroom, if the teacher's understanding of mathematics core literacy does not reach a certain depth, it will become a stumbling block on the road of reform.

Student-level

Education is a kind of social activity to train people, and the object of training is students. How to cultivate the core literacy of mathematics should also depend on the students' final mastery. Think deeply about the problem with mathematical thinking, and look at the problem rigorously with mathematical perspective; Using mathematical methods to solve practical problems in People's Daily lives is the ultimate goal of cultivating students' ability in this aspect. According to the survey, it can be found that some students are not interested in math learning, think that math is difficult, learning math is the result of the pressure of college, and some students do not have the perseverance and patience to learn math well. In fact, in mathematics learning, it is the subjective factors of students that play an important role, while the objective factors do not have the same effect, but exist in a collaborative way. At present, teachers should attach importance to students' previous learning level, so as to use appropriate teaching methods and teaching means to mobilize students' enthusiasm and initiative in learning. Based on the investigation and analysis, it can be seen that there is no difference between male and female students in senior high school in mastering the current content, and there is a significant difference between students with excellent learning and students with poor learning.

TRANING STRATEGIES FOR HIGH SCHOOL STUDENTS' MATHEMATICAL ABSTRACT LITERACY

Pay attention to concept teaching

Mathematical concept is the foundation of mathematical knowledge and the basic form of mathematical thinking. However, in teaching, teachers often neglect to tell the concept, but use a lot of exercises to make students deepen

their understanding of the concept. According to such teaching strategies, students' understanding of concepts is often preliminary and one-sided, which is not conducive to the cultivation of students' ability of abstract generalization. The process of students' independent discovery is one of the most important ways to cultivate students' ability of abstract thinking. Therefore, after students have a preliminary understanding of concepts through students' independent discovery, teachers will guide students to think about concepts in various teaching forms and form a relatively complete knowledge structure independently. Of course, necessary exercises can also deepen students' understanding of concepts. It is more practical and effective to cultivate students' ability of abstract generalization in the real student-centered learning process. Teachers should encourage students to learn independently, and after a preliminary understanding of the concept,Then, through the form of teacher's questions and group discussion, students are guided to make certain speculations on the key points in the concept.The teacher can encourage students to repeat the concept in their own understanding and language, and then the teacher can consolidate the learning results through certain exercises and group presentations. It is often more scientific and effective to cultivate students' ability of abstract generalization from such a learning process similar to flipped classroom. For example, when learning the definition of ellipses, students can prepare several strings, and find two students in each group to demonstrate on the blackboard. One student in each group held down both ends of the string with two fingers, and another student used chalk to tighten the string and move the chalk, but some of the drawings made were irregular. At this time, students could explore: Why did the drawings drawn in this way not meet the standards in the book? Through discussion, students can fully understand the two fixed points and fixed length in the definition of ellipse. On this basis, continue to explore the drawn ellipses, why some are flat and some are round? How does the distance between the two points relate to the length of the rope? Through students' hands-on operation, raising and solving problems in specific situations, students can fully understand the concept and stimulate their enthusiasm and interest in learning.

Pay attention to the internal connection of knowledge

Since the new curriculum reform, although each module of mathematics is relatively independent, but there is a certain connection between each other. On the one hand, in the process of mathematics teaching, teachers should be good at grasping the connection between knowledge points of a chapter and different chapters and modules, so that students can perceive the essential characteristics between them, and improve mathematical abstract ability by finding the connection between knowledge. On the other hand, teachers should also cultivate students' ability to summarize independently, and exercise their mathematical abstract ability in the process of summarizing rules. First of all, after the completion of each lesson or chapter, teachers can guide students to

summarize and summarize what they have learned. We can draw a mind map to find out the development and connection of the knowledge in this chapter, or we can summarize the knowledge in this chapter by summarizing the knowledge points and thinking methods. This kind of generalization is not only the review and consolidation of the learned knowledge, but also can urge students to refine the learned knowledge, in order to exercise students' abstract generalization ability. Secondly, when teaching concepts, teachers can also guide students to make analogical reasoning in connection with the relevant content learned before, which exercises students' mathematical abstract ability. For example, it is possible to make an analogy between the properties of plane geometry and that of solid geometry, an analogy between arithmetic series and arithmetic series, and an analogy between classification and step in the counting principle. This way of learning can not only improve students' interest in learning. Improve the quality of students' mathematics learning, and also provide students with ways to exercise the ability of mathematical abstraction and generalization. Finally, teachers can guide students to dig and summarize the rules through some examples, so as to reach a conclusion. For example, when learning the induction formula of trigonometric functions, teachers can first demonstrate the relationship between the trigonometric function value of the Angle with the final edge falling in the first quadrant and the trigonometric function value corresponding to the Angle with the final edge symmetric about the longitudinal axis to get "Formula 2", then ask students to find out several other symmetric relations and get several other formulas, and finally put the formulas together to guide students to make comparison. Inspire students to conclude from "odd change even unchanged, symbol to see the quadrant" conclusion.

Construction of practical situation

Practical situation is to create a learning situation for students to operate in class.To enable students to connect theory with practice and apply it to practice while learning indirect experience.Timely practice in the teaching process can not only effectively solve the contradiction between the abstract of mathematical knowledge and the figurative thinking of students, but also have a positive significance in stimulating students' interest in learning, improving students' hands-on ability and improving students' mathematical thinking. Teachers should make use of students' "fun, active and curious" psychology, guide students to seriously practice, transform abstract mathematical knowledge into a vivid living situation, so as to abstract mathematical concepts and conclusions, and penetrate mathematical abstract literacy into students' minds. For example, in the teaching of the concept of ellipses, the teacher first asks the question: in junior high school, we already know that the trajectory of a point whose distance to a fixed point is a constant value is a circle, then what is the trajectory of a point whose distance sum to two fixed points is a constant value? Then send a prepared piece of rope to each table, let a student at the

same table fix the two ends of the rope at two points on the paper, another student straighten the rope with the pen tip, make the pen tip slowly turn on the paper, and observe what kind of figure can be drawn in the end. We found that some of you can draw ovals very quickly, and some of you can't. At this time, the teacher can guide them appropriately and let them think, discuss and analyze the reasons why they can't draw ovals. Finally, the students agreed that an ellipse can only be drawn if the length of the string is greater than the distance between the two points. Next, ask students to summarize the characteristics of the points on the ellipse. How is it formed? What quantities are constant during this formation? On the basis of practical operations, students can easily abstract the definition of an ellipse: that is, the trajectory of a point whose distance sum from the plane to two fixed points F1 and F2 is equal to a constant (greater than I F1, F2l) is called an ellipse. Students' ability to abstract the essence of mathematics in hands-on practice is enhanced [7].

Master the method of mathematical abstraction

Concept formation is a basic mathematical abstraction method under the guidance of teachers. Derivative, as a basic concept, is taught in the form of concept formation. Much knowledge in the field of function, such as the concept of function, the concept of monotonicity of function, etc., is abstracted by the way of concept formation. It uses more practical context. Students undergo multiple concept-forming learning activities, and teachers intentionally guide students to compare and reflect on the learning process of these knowledge. Students can gradually internalize this learning method and learn a guided mathematical abstraction method. The continuation of the original abstract idea of growing point knowledge is another basic method of mathematical abstraction. Many contents of high school mathematics are the further extension of existing mathematical concepts. The idea of mathematical abstraction is to continue the original abstract idea of the growing point knowledge, and to expand the new knowledge through the discussion of new concrete examples. This kind of thinking, with initiative, is another basic method of mathematical abstraction in high school. For example, how to develop the concept of fractional exponents? Review existing knowledge to activate ideas effectively. Reviewing the derivation method of zero exponential power, it is found that it is obtained by extending the scope of the division algorithm of the same base power (the division algorithm of the same base power is obviously asymmetric with the multiplication algorithm, and the division algorithm has the restriction that the dividend exponent is greater than the divisor exponent. Lifting this restriction, by examining some individual cases, one develops zero exponents and negative integer exponents). Apply this idea, examine the power of power rule, explore some examples, naturally can be derived fractional exponential power. Students have successfully applied the same method many times, and in the process of application, in the process of reflection and review afterwards, they gradually strengthen this idea, and

gradually learn the mathematical abstract method contained in it.Another example is the definition of an ellipse.Teachers can guide students to explore the direction of generalization by using the definition of a circle - the trajectory of a point whose distance to a fixed point is equal to its length - so as to develop students' basic methods of mathematical abstraction. The definition of a circle has two elements, one is a fixed point and the other is a fixed length. Generalizing in the direction of a fixed point, a circle is a fixed point, so naturally what is the trajectory of a point that is equal to the distance between two fixed points? This naturally leads to the definition and drawing of ellipses. It also laid the direction and method for the re-creation of the following knowledge -- hyperbola and parabola. Intensive use of the same idea over a period of time is conducive to the formation of mathematical abstract habits. The key to continuing the knowledge growth point idea is to identify the correct knowledge growth point. This requires teachers to have the awareness and ability of holistic vision and thematic teaching [6].

Group discussion and communication

Group discussion is to divide the students in the class into several groups (preferably the group of neighboring students), and hand over the problems that are difficult to complete independently in class to the group for communication and discussion, so as to form a relatively complete and unified conclusion. Group discussion can not only promote the improvement of students' learning ability and teamwork ability, but also gradually improve mathematical abstraction and achieve the purpose of enhancing literacy. Therefore, teachers should actively carry out group discussions to make classroom teaching more vivid and effective.For example, in teaching the image of the function $y = A\sin(wx+\phi)$ （A ＞ 0，w ＞ 0）, the teacher can give the problem set:

Question 1: What is the relationship between $y = \sin(x+1)$ and the image of $y = \sin x$? Question 2: How does the graph $y = \sin 3x$ relate to $y = \sin x$? Question 3: How does $y = \sin 2x$ relate to the graph of $y = \sin x$? Question 4: How does $y = \sin(2x+1)$ relate to the graph of $y = \sin 2x$?

The teacher asked the students to discuss according to the original group, and then each group reported on the stage, and the teacher gave appropriate comments and encouragement. Then, the teacher uses the geometric drawing board for dynamic demonstration to verify the conclusion, so as to abstract the various transformation rules of the function. Group discussion enables students to fully exchange different ideas and views, reduce individual thinking limitations, form scientific thinking literacy, and gradually improve mathematical abstraction.

CONCLUDING REMARKS

Abstract is the most important thinking in learning mathematics, which makes mathematics a system of "high generalization, accurate expression, general conclusion, order and multilevel". Abstract thinking runs through the whole process of mathematics learning and helps to acquire new concepts. Accurate use of mathematical language, the formation of mathematical structure system. In teaching, teachers should make full use of the carrier of concept teaching, intentionally develop students' abstract thinking, and promote the improvement of students' thinking quality.

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高中生数学抽象素养培养策略探究

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摘要：数学知识的抽象性有目共睹，提高学生的数学抽象素养是高中数学教师极为重要的教学任务，是提高学生学习成绩的基础，是推动学生综合素养提升的关键。基于此，通过对高中数学教学时间进行反思，针对性提出高中学生数学核心素养培养策略，以供参考。

关键词：高中数学;抽象核心素养;培养策略;

一、问题的提出

《普通高中数学课程标准(2017年版)》(以下简称课程标准)将数学抽象确定为数学学科的核心素养之一。教师对数学抽象的重要性是认同的，但在实际教学中存在以下两个问题:教师教学方法传统单一，只注重结果，缺乏经历得到完整概念的教学抽象过程;对数学抽象的认识不到位，不知道可以从哪些方面入手培养学生的数学抽象。那么，如何认识数学抽象?在教学过程中，可以从哪些方面具体地落实数学抽象的培养呢?

二、数学抽象素养

1、数学抽象素养的内涵

何为抽象?现代汉语词典对抽象的解释为:①从许多事物中，舍弃个别的、非本质的属性，抽出共同的、本质的属性，叫抽象，是形成概念的必要手段;②不能具体经验到的，笼统的，空洞的。前一层的意思可以理解为:从具体事物中提炼出共同的本质特征，后一层的意思是更高层面的提炼与概括，带有想象的成分，可以是符号化的表达。现代汉语词典将素养解释为:平日的修养。基于此对抽象素养可以理解为:在平日的学习与实践中所形成的对具体事物的去粗取精、去伪存真、去异求同、去表求本的提炼与概括能力水平，那么何为数学抽象素养呢?

就是在平日的学习与实践中所形成的在数量关系、空间形式，具体事物中抽象出数学研究对象的修养水平，主要是抽象概括能力水平。课程标准对数学抽象素养做了更精准的描述:“数学抽象是指通过对数量关系与空间形式的抽象，得到数学研究对象的素养。主要包括:从数量与数量关系，图形与图形关系中抽象出数学概念及概念之间的关系，从事物的具体背景中抽象出一般规律和结构，并用数学语言予以表征。”按照这样的要求，在高中数学教学中，实现培养数学核心素养的教育目标，可以在数学概念、几何代数、创新问题等教学过程中逐步实现[1]。

2、数学抽象素养的基本特征

（1）符号化特征

数学抽象研究事物或现象的量的关系和空间形式，其结果就是用数学符号作为数学思维活动的载体，进行数学思想交流与传播，使得事物或现象关系表述变得统一、简洁、有序。例如，研究一个对象随着另一个对象的变化而变化的变化规律，就抽象为函数的单调性，具体的符号化语言就描述为“设函 $f(x)$ 的定义域为 I，区间 $D \in I$：如果 $\forall x_1, x_2 \in D$，当 $x_1 < x_2$ 时都有 $f(x_1) < f(x_2)$或$(f(x_1) > f(x_2))$。那么称函数 $f(x)$ 在区间 D 单调递增（减）”。在数学教学中，数学抽象的符号化特征可以培养学生抽象思维能力。学生在具体的学习环境中逐步感受和理解抽象的符号化特征，感受数学抽象以简驭繁高度概括的能力，并能在问题解决过程中合理地使用数学符号，合乎逻辑地进行数学推理。

（2）模型化特征

数学研究对象是现实世界中的事物，它是多变而又是多样的。只有把研究对象通过数学抽象进行模型化，才能通过一般模型化进行问题研究。如自然界中的日出日落，寒来暑往等周而复始现象，我们可以把这种现象理想化为周而复始的变化，进而模型化为单位圆圆周上的点的圆周运动模型来进行问题研究。这是对研究对象通过理想化进行简化，在事物的直接模型上抽象出一般模型，在一般模型的研究基础之上，建立数学抽象关系。在教学中，数学抽象的模型化特征能够提高学生思维水平，促进学生智慧的发展。

（3）量化特征

数学抽象是从数量关系上揭示客观事物的本质及规律的一种研究方法，是逐层抽象不断发展的。如函数概念的抽象，就经历了从客观世界中的事物对应关系量化为数量的对应关系，再从初中变量对应关系转变到高中的非空数集上的实数对应关系的过程。具体事物量化为变量是量化的结果。变量抽象为实数，变量说过渡到实数对应说，是函数概念分层抽象的结果。数学教学中数

学抽象的量化特征能够促进学生高阶思维的产生，促进创新思维的发展。

三、数学抽象的作用

作为数学能力的核心，数学抽象在数学核心素养中占据最重要地位。甚至可以说，数学抽象是数学核心素养的基础。一方面，数学学习过程中需要学习者具备一定的抽象概括能力。由于数学具有抽象性和概括性的特点，学习者只有具备一定程度的抽象概括能力，才能对数学对象的认知实现质的飞跃"。另一方面，数学的应用过程也对学习者的数学抽象素养有一定的要求。在数学的应用活动中，要求学习者能将所遇情境进行数学化和抽象化，并与已学数学知识相联系，最终利用数学知识解决问题并将问题还原回实际情况。这一系列的活动都是建立在数学抽象基础上的，所以，教师在教学活动中要特别注重对数学抽象能力的培养，从而使学生在数学的学习过程中能迅速抓住本质，提升学习效率。

四、影响因素

1、教师层面

根据调查结果可知，多数教师意识到了课改的目的性，尤其是数学核心素养培养的重要性，但是，有一部分教师对于数学核心素养的精髓把握不够透彻，还有一部分教师，在教育教学过程中很少参照课程标准，这导致了教师对数学核心素养的理解不够深入，相对而言认知度略低。传统的教学思路和模式持续时间较久，所以有些教师还没有及时突破传统理念，缺乏创新思维，没有接受培养方式的新理念。考虑实际情况来说，现在培养方式以在校为主，那就必须依赖于数学课堂。而教师作为一个数学课堂的“总导演”，若对数学核心素养理解未达到一定深度，这必将成为改革道路上的绊脚石。

2、学生层面

教育是培养人的一种社会活动，而培养对象就是学生。关于数学核心素养的培养情况如何，还应看学生最后的掌握情况。用数学的思维深入的思考问题，用数学的角度严谨的看待问题;用数学的方法解决人们日常所要面对的生活中的实际问题，这是培养学生这一方面能力的终极目标。根据调查可以发现，有部分学生对数学学习不感兴趣、认为数学难、学数学是升学压力使然，有的学生没有学好数学的毅力与耐性。其实，在数学学习中，起着重要作用的是学生的主观因素，而客观因素不会有同样的作用，只是以协同的方式存在。当前，教师应重视学生此前的一个学习水平，从而运用恰当的教学方法以及教学手段调动学生学习的积极性、主动性。基于调查分析，可看出高一男女生对于当前所学内容掌握情况并无差异性，而学优生与学困生之间存在显著差异。

五、高中生数学抽象素养的培养策略

1、注重概念教学

数学概念是数学知识的基础，是数学思维的基本形式。但是在教学中，教师往往忽略对概念讲述，而是利用大量的练习使学生加深对概念的理解。按照这样的教学策略，学生对概念的理解往往是初步的和片面的，不利于对学生抽象概括能力方面的培养。而学生自主发现的过程才是培养学生抽象思维能力的最主要方式之一，所以通过学生的自主发现，让学生对概念有初步了解后，教师采用多种教学形式引导学生对概念进行思考，自主的形成较为完整的知识结构，当然必要的习题练习也可以加深学生的概念的理解。这种真正以学生为主体的学习过程中培养学生抽象概括能力的方法往往更加实际、有效。教师要鼓励学生自主学习，在对概念有初步了解后，再通过教师提问和小组讨论的形式引导学生对概念中的要点做一定的思辨。教师可以鼓励学生用自己的理解和语言复述概念，然后教师可以通过一定的习题和小组展示巩固学习成果。从这样类似于翻转课堂的学习过程中培养学生抽象概括能力的做法往往更加科学而有效。如在学习椭圆定义时，可以让学生准备几根细绳，每组找两名学生到黑板上演示。每组的一名学生用两指按住细绳的两端，另一名学生用粉笔拉紧细绳移动粉笔，但作出的图有的不规范，此时可以让学生探究:为什么这样画出的图没有书上的标准?经过探究讨论可以让学生充分认识椭圆定义中的两定点和定长。在此基础之上，继续共同探究画出来的椭圆，为什么有的扁一些有的圆一些呢?两定点间的距离与绳长有什么关系呢?通过学生的动手操作，在具体情境下提出问题、解决问题，可以让学生充分的理解概念，也激发了学生学习的积极性和兴趣。

2、关注知识内在联系

新课改以来，虽然数学学科的各个模块相对独立，但相互之间还是有一定联系的。一方面，教师在数学教学过程中要善于抓住一个章节以及不同章节、不同模块的知识点间的联系，让学生感知它们之间的本质特征，通过寻找知识间的联系来提升数学抽象能力;另一方面，教师也应该培

养学生的自主总结能力，从总结规律的过程中锻炼其数学抽象能力。

首先，在每节课或者一个章节学习完毕后，教师可以引导学生对所学内容进行总结和概括。可以用画思维导图寻找本章知识的发展与联系的方法，也可以通过知识点和思想方法的归纳对本章知识进行总结的方法。这种概括不但是对所学知识的复习和巩固，而且从中能督促学生对所学知识进行提炼，以锻炼学生的抽象概括能力。

其次，教师在讲授概念时也可以联系以前所学的相关内容引导学生进行类比推理，从中锻炼学生的数学抽象能力。比如，可以进行平面几何的性质与立体几何的性质之间的类比、等差数列与等比数列之间的类比、计数原理中分类和分步的类比等。这样的学习方式不仅能提高学生的学习兴趣。提升学生的数学学习质量，还能给学生提供锻炼数学抽象概括能力的途径。

最后，教师可以引导学生通过一些实例挖掘和总结其中的规律，从而得到结论。比如，在学习三角函数的诱导公式时，教师可以先示范终边落在第一象限的角的三角函数值和与之关于纵轴对称的终边所对应角的三角函数值之间的关系得出“公式二”，然后让学生找出其他几种对称关系并得出其他几个公式，最后将公式放在一起引导学生进行对比，启发学生从中归纳出“奇变偶不变，符号看象限”的结论。

3、构建实际情境

实践情境是在课堂上营造出可供学生动手操作的学习情境,让学生在学习间接经验时能够将理论联系实践并应用于实践。在教学过程中适时让学生实践操作，不仅能有效地解决数学知识的抽象性与学生思维形象性的矛盾，也能对激发学生的学习兴趣、提高学生的动手能力、完善学生的数学思维有着积极的意义。教师要巧妙利用学生“好玩、好动、好奇”的心理，引导学生认真进行实践操作，把抽象的数学知识变成活灵活现的生活情境，从而抽象出数学概念和结论，将数学抽象素养渗透到学生的头脑里。

例如，在椭圆的概念教学中，教师先提出问题:初中我们已经知道到一个定点的距离为定值的点的轨迹是圆，那么到两个定点的距离之和为定值的点的轨迹又是什么呢?然后把准备好的一段绳子发到每一桌，让同桌中一个同学将绳子的两端固定在纸上的两个定点处，另一个同学用笔尖将绳子拉直，使笔尖在纸上慢慢转动，观察最后能画出什么样的图形。我们发现有的同学能迅速画出椭圆，还有部分同学不会画。教师此时可适当引导，让他们思考、讨论、分析画不出椭圆的原因。最后同学们一致认为只有当绳子的长度大于两定点间的距离的时候才能画出椭圆。接着，让学生总结椭圆上的点有什么特征?它是如何形成的?在此形成过程中哪些量是不变的?学生在实践操作的基础上容易抽象概括出椭圆的定义:即平面内到两定点F1、 F2，的距离之和等于常数(大于ⅠF1，F21)的点的轨迹叫作椭圆。学生在动手实践中抽象数学本质的能力得到增强[7]。

4、掌握数学抽象的方法

概念形成是一种教师指导下的基本数学抽象方法。导数作为一种基本概念，其教学采用的是概念形成的方式。函数领域内的很多知识，像函数的概念，函数单调性的概念等，都是采用概念形成的方式抽象得到的。它用到较多的实际背景。学生经历多次的概念形成学习活动，教师有意指导学生比较，反思这些知识的学习过程，学生能够逐渐内化这种学习方法，学会一种有指导的数学抽象方法。延续生长点知识原有的抽象思路，是数学抽象的另一种基本方法。高中数学的许多内容，是对已有数学概念的进一步推广。其数学抽象的思路，是延续生长点知识原有的抽象思路，通过对新的具体例证的探讨，扩展得到新知识。这样的思路，具有主动性，是高中阶段数学抽象的另一种基本方法。例如，怎样发展分数指数幂的概念?审视已有相关知识，有效激活思路。回顾零指数幂的推导方法，发现是通过对同底数幂的除法运算法则使用范围的扩展得到的(同底数幂的除法运算法则明显与乘法运算法则不对称，除法运算法则多了被除数指数大于除数指数的限制。解除这个限制，通过考察一些个例，会发展出零指数幂与负整数指数幂)。套用这个思路，考察幂的乘方法则，探讨一些例证，自然可推出分数指数幂。学生多次成功应用同一方法，在应用过程中，在事后的反思审视过程中，逐渐强化了这个思路，就逐渐学会了蕴藏在内的数学抽象方法。 又如，探究椭圆的定义。教师可以指引学生利用圆的定义--到定点的距离等于定长的点的轨迹————探讨推广的方向，从而培养学生的数学抽象基本方法。圆的定义中有两个要素，一是定点，二是定长。沿着定点的方向推广，圆是一个定点，那么自然提出，到两个定点的距离和等于定长的点的轨迹是什么呢?这样自然导出椭圆的定义与画法。并且为接下来的后续知识————双曲线和抛物线————的再创造，奠定了方向与方法。在一个时间段密集运用同一思路，有利于形成数学抽象的习惯。延续知识生长点思路的关键，是确认正确的知识生长点。这需要教师具有整体视野与主题教学的意识与能力[6]。

5、构建小组讨论交流

小组讨论是把班级学生划分为若干个小组（最好是相邻同学组合），将课堂上难以独立完成的问题交给小组进行交流、讨论，形成较为完整统一的结论。小组讨论不仅能推动学生学习力和团队合作力的提升，而且能逐步完善数学抽象，达到增强素养的目的。因此，教师要积极开展小组讨论，使课堂教学变得更加鲜活、有效。

例如，在函数 $y = A\sin(wx+\phi)$（A ＞ 0，（w ＞ 0）的图象教学中，教师可给出问题组 ：

问题 1 ： $y = \sin(x+1)$ 与 $y = \sin x$ 的图象有什么关系？

问题 2 ： $y = \sin 3x$ 与 $y = \sin x$ 的图象有什么关系？

问题 3 ： $y = \sin 2x$ 与 $y = \sin x$ 的图象有什么关系？

问题 4 ： $y = \sin(2x+1)$ 与 $y = \sin 2x$ 的图象有什么关系？

教师要求学生按照原定的小组进行讨论，再由各小组上台汇报，教师适当地进行点评鼓励。接着，教师用几何画板进行动态演示，验证结论，从而抽象出函数的各种变换法则。小组讨论能让学生充分交流不同的思路与看法，减少个体存在的思维局限，形成科学的思维素养，从而逐步完善数学抽象。

六、结束语

抽象是学习数学最重要的思维，抽象使得数学成为“高度概括，表达准确、结论一般、有序多级”的系统″。抽象思维贯穿数学学习的整个过程，抽象思维有助于获取新概念。，准确运用数学语言、形成数学结构体系。教学中，教师要充分利用概念教学这一载体，有意发展学生的抽象思维，促进学生思维品质的提升。

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