VECTORS HELPS ALGEBRA

Rakhimov, N. N.
Rakhimov Nasriddin Nomozovich
Head of the Department of Mathematics and
Computer Science of the Academic Lyceum
of Samarkand State Foreign Institute,
Uzbekistan, Samarkand

ABSTRACT

The article indicates using vectors the solution of some algebraic problems on the topics of equation and inequality. In each sketch, geometric techniques for solving problems are given. As a rule, they do not have a sign of familiarity for students, but, as experience shows, they are easily perceived by them. Thanks to the integration of the "non-geometric" conditions of the problem and its geometric solution, mathematical knowledge appears to students as a living, dynamic system that can solve problems from other sciences and practice.

Keywords: A vector, numbers, module, equation, inequality, equality, system the equation and trigonometry.

MATERIALS AND METHODS

In this article we will show using vectors not only to solve geometric problems, but it is also possible to solve algebraic problems. Solve problems in this methods useful for preparing students for olympiads in mathematics. Now we solve some algebraic tasks using vector.

1-task. Solve the equation $2\sqrt{x-1}+5x=\sqrt{(x^2+4)(x+24)}$

Decision. We consider two nonzero vectors $\bar{a}(a_1;a_2)$ and $\bar{a}(b_1;b_2)$. We use from the scalar product of vectors and the Cauchy-Bunyakovsky inequality: $a_1a_2+b_1b_2\le\sqrt{a_1^2+b_1^2}\cdot\sqrt{a_2^2+b_2^2}$. Put $\bar{a}(2;x)$ and $\bar{b}(\sqrt{x-1};5)$. Then the Cauchy-Bunyakovsky inequality takes the form $2\sqrt{x-1}+5x\le\sqrt{(x^2+4)(x+24)}$.

By condition, inequalityturns into equality. In this task, this is possible only if

$$\frac{\sqrt{x-1}}{2}=\frac{5}{x}\Leftrightarrow x=5 \qquad [4]$$

2-task. The positive numbers a, b, c are such that abc=1. Prove the inequality $\frac{1}{a^3(b+c)}+\frac{1}{b^3(c+a)}+\frac{1}{c^3(a+b)}\ge\frac{3}{2}$.

Decision. Convenient to move to new variables x=1/a, y=1/b, z=1/c, also a positive and related condition xyz=1. This inequality is equivalent to the following: $S=\frac{x^2}{y+z}+\frac{y^2}{z+x}+\frac{z^2}{x+y}\ge\frac{3}{2}$.

Applying the Cauchy - Bunyakovsky inequality to vectors $\vec{m}\left(\frac{x}{\sqrt{y+z}},\frac{y}{\sqrt{z+x}},\frac{z}{\sqrt{x+y}}\right)$ and $\vec{n}\left(\sqrt{y+z},\sqrt{z+x},\sqrt{x+y}\right)$ we get $(x+y+z)^2\le 2S(x+y+z)$, those. $S\ge(x+y+z)/2$. Using the inequality between the arithmetic mean and geometric mean of three positive numbers, we obtain:

$$S \geq \frac{3}{2} \cdot \frac{x+y+z}{3} \geq \frac{3}{2}\sqrt[3]{xyz} = \frac{3}{2} \qquad [2]$$

3-task. The numbers x, y, z are such that x+y+z=1. Prove the inequality $\sqrt{4x+1}+\sqrt{4y+1}+\sqrt{4z+1}<5$, here $x,y,z \geq -\frac{1}{4}$.

Decision. We consider two vectors $\vec{m}\left(\sqrt{4x+1}; \sqrt{4y+1}; \sqrt{4z+1}\right)$ and $\vec{n}(1;1;1)$. Obviously, the following inequality holds: $\left|\vec{m} \cdot \vec{n}\right| \leq \left|\vec{m}\right| \cdot \left|\vec{n}\right|$ or in coordinate form $\left|x_1 \cdot y_1 + x_2 \cdot y_2 + x_3 \cdot y_3\right| \leq \sqrt{x_1^2+x_2^2+x_3^2} \cdot \sqrt{y_1^2+y_2^2+y_3^2}$. It is this last inequality that is the key to the solution. To mean

$$\vec{m}\cdot\vec{n} = \sqrt{4x+1}\cdot 1 + \sqrt{4y+1}\cdot 1 + \sqrt{4z+1}\cdot 1 \leq \sqrt{4x+1+4y+1+4z+1}\cdot\sqrt{1^2+1^2+1^2} =$$
$$= \sqrt{4(x+y+z)+3}\cdot\sqrt{3} = \sqrt{7\cdot 3} = \sqrt{21} < 5 \qquad [4]$$

4-task. Solve the equation $\sqrt{x-2}+\sqrt{4-x} = x^2-6x+11$.

Decision. Let's evaluate the left and right sides of the equation:

$x^2-6x+11 = (x-3)^2+2 \geq 2$,

$\sqrt{x-2}+\sqrt{4-x} = \sqrt{x-2}\cdot 1+\sqrt{4-x}\cdot 1 \leq \sqrt{x-2+4-x}\cdot\sqrt{1^2+1^2} = 2$. Here the vectors are selected as follows: $\vec{m}\left(\sqrt{x-2}; \sqrt{4-x}\right)$ and $\vec{n}(1;1)$. Now it's clear that the original equation is equivalent to the system $\begin{cases} x^2-6x+11=2 \\ \sqrt{x-2}+\sqrt{4-x}=2 \end{cases}$. We solve this system and find the root. Answer: x = 3. [4]

5-task. The numbers x, y, z are such that $x^2+3y^2+z^2=2$. Find the largest and smallest expression value 2x+y-z.

Decision. It is clear that to evaluate the expression 2x+y-z the coordinates of the vectors must be chosen so that the modulus of one of them is equal to $\sqrt{x^2+3y^2+z^2} = \sqrt{2}$. Consider the vectors $\vec{m}\left(x; y\sqrt{3}; z\right)$ and $\vec{n}\left(2; \frac{1}{\sqrt{3}}; -1\right)$. Then we have

$$|2x+y-z| \leq \sqrt{x^2+3y^2+z^2}\cdot\sqrt{4+\frac{1}{3}+1} = \sqrt{2}\cdot\sqrt{\frac{16}{3}} = 4\sqrt{\frac{2}{3}}.$$

Consequently, $-4\sqrt{\frac{2}{3}} \leq 2x+y-z \leq 4\sqrt{\frac{2}{3}}$. [3]

6-task. Among all system solutions $\begin{cases} x^2+y^2=4 \\ z^2+t^2=9 \\ xt+yz=6 \end{cases}$

Select those for which the value x+z takes the largest value.

Decision. Consider the vectors $\bar{a}=(x,y)$ and $\bar{b}=(t,z)$. By condition $|a|=2$, $|b|=3$, $(a,b)=6=|a||b|$. Therefore, the vectors a and b are directed identically, i.e. $a=2(u,v)$, $b=3(u,v)$.

Put u =cosφ, v =sinφ, тогда $x+z=2\cos\varphi+3\sin\varphi=\sqrt{13}\sin(\varphi+\alpha)\le\sqrt{13}.$ [3]

7-task. Prove that if $a^2+b^2+c^2=1$, $m^2+n^2=3$, then, $|ma+nb+c|\le 2$.

Decision. We introduce vectors in space $\bar{u}(a;b;c)$ and $\bar{v}(m;n;1)$. We apply the Cauchy-Bunyakovsky vector inequality

$$|\bar{u}\cdot\bar{v}|=|am+bn+c|\le|\bar{u}|\cdot|\bar{v}|=\sqrt{a^2+b^2+c^2}\cdot\sqrt{m^2+n^2+1}=2 \qquad [2]$$

8-task. Find the largest value of the function: $y=\sqrt{x+28}+\sqrt{22-x}$.

Decision. The scope of the function is the segment [-28; 22]. Introduce vectors $\bar{u}\left(\sqrt{x+28};\sqrt{22-x}\right)$ and $\bar{v}(1;1)$. We get:
$\bar{u}\cdot\bar{v}=\sqrt{x+28}+\sqrt{22-x}\le|\bar{u}|\cdot|\bar{v}|=\sqrt{x+28+22-x}\cdot\sqrt{1+1}=\sqrt{50\cdot 2}=10.$

Apparently, the greatest value of the function y is 10. It is achieved when the inequality turns into equality, i.e. when the coordinates of the vectors and are proportional:
$\sqrt{x+28}=\sqrt{22-x}\Rightarrow x+28=22-x \Rightarrow x=-3.$
It is significant that the value x = -3 is included in the domain of the function.
Answer: 10 at x = -3. [1]

CONCLUSION

The geometric solution of non-geometric tasks creates the following skills for students: a deep understanding of interdisciplinary communication, the formation of scientific thinking, worldview and creativity.

An important component of human culture is a wide range of ways of its activity. Therefore, it is very important for schoolchildren to establish strong intra-subject relationships in the school course of mathematics. A significant expansion of the methods of their mathematical activity can help visual solutions to problems, because to know mathematics means to be able to solve problems.

Today, in the framework of the current curriculum in mathematics, due to lack of time, it is very difficult for the teacher to carry out the stages of the final repetition. However, the selection of problems for integrated mathematics lessons can partially eliminate this difficulty.

THE TASK FOR INDEPENDENT WORK

№1. Prove the inequality: $\sqrt{a+1}+\sqrt{2a+3}+\sqrt{8-3a}\le 6$

№2. Prove that if $a+b+c=6$, then $\sqrt{4a+1}+\sqrt{4b+1}+\sqrt{4c+1}\le 9$, here $a,b,c\ge-\frac{1}{4}$.

№3. Prove the inequality: $4\sqrt{a}+3\sqrt{16-a}\le 20$ at any $a\in[0;16]$. When is equality achieved?

№4. Prove that if $a-b+c=6$, then $\sqrt{a+1}+\sqrt{2-b}+\sqrt{c+3}\le 6$.

№5. Prove that if $a^2+b^2\le 32$, then $|a+b|\le 8.$

№6. Solve the equation: $x\sqrt{1+x}+\sqrt{3-x}=2\sqrt{1+x^2}$

№7. Find the greatest value of the functions:

a) $y = \sqrt{x-8} + \sqrt{16-x}$ b) $y = \sqrt{\sin^2 x + 1} + \sqrt{\cos^2 x + 1}$

REFERENCES

1. Galkin E. V. Non-standard problems in mathematics. Algebra: Textbook. allowance for students in grades 7-11. Chelyabinsk: “Glance”, 2004. - 448 p.
2. I. Israilov, Z. Pashaev. Geometry 1-part. Academic textbook Lyceum. Tashkent, Teacher Publishing House 2004
3. G.N. Yakovlev, L.P. Kuptsov et al. All-Russian Mathematical olympiads of schoolchildren. Moscow, Publishing House "Education" 1992
4. I.F. Sharygin, V.I. Golubev. Optional course in mathematics. Moscow, Publishing House "Education" 1992