# ON THE CULTIVATION OF HIGH SCHOOL STUDENTS' 

# ABILITY OF CALCULATING AND SOLVING 

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#### Abstract

The ability of calculating and solving is one of the most basic and important abilities in the mathematics ability under the new curriculum standard. It is also the most important one in the college entrance examination. According to the investigation and analysis of the author, $80 \%$ of the real questions in the college entrance examination of mathematics are related to calculation every year, and students are easy to lose points here every year, so the level of calculation ability directly affects the scores of candidates. Therefore, in the teaching, we should vigorously cultivate the students' ability of calculating and solving, and find the problem. This paper analyzes the real problems of college entrance examination over the years, and puts forward scientific methods to cultivate students' ability of calculation and solution.


Keywords: Computing ability, Culture method, True questions of college entrance examination.

## INTRODUCTION

## Research Background

In middle school, the ability of operation and solution mainly refers to: students can use laws, formulas, theorems, etc. to carry out an accurate operation, that is to say, they can carry out a series of data deformation and data processing through their own properties, find a reasonable solution through the known conditions of the problem itself, and use axioms to design a simple path, at the same time, logarithm According to a valuation and approximate calculation. The ability of operation and solution is a combination of abstract thinking ability and mathematical operation ability. Operation includes arithmetic operation, relation operation (traditional set operation, special relation operation), logic operation, etc. ${ }^{[1]}$ the ability of operation and solution is formed through constant use of mathematical concepts, formulas, theorems and certain quantification. ${ }^{[2]}$ For example, some formulas, rules and properties can only be obtained by simple reasoning. At this time, we must develop students' logical thinking ability and abstract thinking ability to deal with problems correctly, flexibly, quickly and reasonably. Under the new curriculum standard, the ability of calculation and solution is endowed with richer connotation, and the requirements of estimation ability, the ability of using calculator and computer, and the ability of seeking approximate solution are also given. ${ }^{[3]}$ In addition, we pay more attention to whether we can make clear the calculation ability and the calculation method in the process of calculation and solution, which are
also the three focuses of the debate in the compulsory curriculum reform. Fundamentally speaking, operation solving is a process of reasoning, and it is also one of the main abilities of high school mathematics core literacy.

## THE LEVEL OF COMPUTING ABILITY

Students' ability of operation and solution is closely related to their ability of observation, analysis, association, comparison, understanding, highly abstract generalization, language expression, and the most important ability of logical reasoning, which is a comprehensive mathematical ability of interaction and mutual penetration. For example, in the 2018 college entrance examination mathematics (Theory), there is a question: the tangent equation of curve $y=2 \ln (x+1)$ at point $(0,0)$. For questions like this, students should first observe the tangent equation with known conditions at $(0,0)$, so students should first consider whether this point is on the curve, After verification, $(0,0)$ is substituted into the curve, and this point is on the curve; Secondly, if we want to find the tangent equation, we must have a slope, so we can derive the slope at $(0,0)$ from the curve. After calculation, the derivative of the curve at this point is 2 . Finally, we can use the point slope formula to find the tangent equation $y=2 x$. We can see from the above questions that the improvement of the students' ability of calculating and solving cannot exist in isolation. If the students can solve better and more accurately, they must promote each other and develop together with various basic abilities. The ability of operation and solution can be summarized into three levels: 1. Directly substituting the known conditions given in the problem into the formula, rule, theorem or directly solving through the concept, which is the most basic ability of operation and solution. 2. Students can flexibly and accurately apply the formulas and rules to the questions. For the example of the above question, we can find the tangent line equation through the calculated slope and then use the point oblique equation; or we need a large number of induction formulas to simplify the trigonometric function in compulsory 4 , which is also a level expression of the operation and solution ability. 3. Students can also use a variety of mathematical ideas, such as induction, analogy, association, classification, transformation and our commonly used functional ideas to find the fastest and most effective solution, which is also a higher level expression of the ability to solve problems.

## AN ANALYSIS OF THE REAL PROBLEMS IN COLLEGE ENTRANCE EXAMINATION

## Familiar with formula, etc

Students should fully understand, master concepts, theorems, formulas and rules, and deeply understand the significance of each quantity in known conditions. A question in Science (2) of 2017 national college entrance examination: There are the following problems in the ancient Chinese mathematics masterpiece "suanzongzong": "looking at the towering tower with seven floors in the distance, the red light is multiplied by a little bit, and the total number of lights is three hundred and eighty-one. How many pointed lights are there?" A 7 -storey tower has 381 lights in total, and the number of lights on the next two adjacent floors is twice the number of lights on the previous floor,
so the top floor of the tower has lights ().
Analysis: the number of lights in the next two adjacent floors is twice the number of lights in the upper floor. We also know that there are seven floors in the tower, with a total of 381 lights. Then we can directly substitute $\mathrm{Sn}=381, \mathrm{n}=7, \mathrm{q}=2$ into the first n terms and formulas of the equal ratio sequence, and finally we can find the number of lights in the top floor of the tower. Finally, the number of top towers is 3 . For example, the above-mentioned real problem belongs to one of the levels related to our ability to calculate and solve. Such a problem is the easiest to score. It can be directly substituted into the formula. As long as students do not make mistakes in calculation, it is easy to solve. However, some students do not pay attention to the additional conditions and implicit conditions in the questions, so they easily substitute them into the formula, resulting in the final error. For example, if the function $f(x)=x^{\wedge} 2+b x+c$ has $f(2+$ $t)=f(2-t)$ for any real number $t, s o()$.
(A) $\mathrm{f}(2)<\mathrm{f}(1)<\mathrm{f}(4)$ (B) $\mathrm{f}(1)<\mathrm{f}(2)<\mathrm{f}(4)(\mathrm{C}) \mathrm{f}(2)<\mathrm{f}(4)<\mathrm{f}(1)$ (D) $\mathrm{f}(4)<\mathrm{f}(2)<\mathrm{f}(1)$

Analysis: the function $f(x)$ in the problem is $f(2+t)=f(2-t)$ for any real number $t$, implying that the image of the function is symmetric with respect to the line $\mathrm{x}=2$, which is known from $f(x)=x^{\wedge} 2+b x+c$. The image of the function is a parabola with an opening upward, so it is $\mathrm{f}(2)<\mathrm{f}(1)<\mathrm{f}(4)$. So choose (A). For example, the discovery of the test questions with implicit conditions is based on the solid grasp of the double base, so the teaching of the double base should be strengthened, and the important mathematical concepts, formulas, rules, properties, etc. should be mastered accurately, as well as the cultivation of mathematical thinking ability.

## Simplification solution

For example, exchange method, valuation method, inverse formula method, integral substitution, exclusion method, etc. At the same time, we should remember some basic formulas or valuable specific data. We can use them directly in the calculation, which can improve the efficiency of problem solving. In the 2017 national college entrance examination of Science (2), a question: the internal angles A, B and C of the triangle ABC correspond to the sides of $\mathrm{A}, \mathrm{B}$ and C respectively, and the known $\sin (\mathrm{A}+\mathrm{C})=$ $2(\sin (B / 2))^{\wedge} 2$. Find $\cos B$.
Analysis: we can know $\mathrm{A}+\mathrm{C}=\Pi-\mathrm{B}$ by using the inner angle and theorem of triangle, then simplify $\sin (A+C)$ by using the induction formula, simplify $\sin (B / 2)^{\wedge} 2$ by using the power-down formula, and combine with $(\sin B)^{\wedge} 2+(\cos B)^{\wedge} 2=1$, so that students must simplify and solve the final solution, so as to find $\cos B$. There is another problem: let $A+B=\Pi / 3$, and find the value of $\tan A+\tan B+3 \wedge(1 / 2) \tan A \tan B$ $(\mathrm{A}>0, \mathrm{~B}>0)$.
If we cut the chord of the formula directly, because it is the common solution to solve the trigonometric function problem, but if we use this method, we will go directly into the maze, because this method is not the only way to solve the problem, we can think of the tangent formula $\tan (A+B)=(\tan A+\tan B) /(1-\tan A \tan B)$ of the sum of two corners, can the formula be simplified in reverse We can solve it as follows: from tan $(\mathrm{A}+\mathrm{B})=(\tan \mathrm{A}+\tan \mathrm{B}) /(1-\tan \mathrm{A} \tan \mathrm{B})$, because $\mathrm{A}+\mathrm{B}=\Pi / 3$, so $(\tan \mathrm{A}+\tan \mathrm{B}) /(1-$ $\tan A \tan B)=3^{\wedge}(1 / 2)$, we can get $\tan A+\tan B+3 \wedge(1 / 2) \tan A \tan B=3 \wedge(1 / 2)$.

Among them, the converse formula is an expression of converse thinking and an important expression of directional thinking. Such simplification is more likely to appear in the trigonometric function in the real questions of college entrance examination, which is a good opportunity to exercise the ability of calculation and solution, and also tests the students' mastery and application of formulas, which greatly lays a solid foundation for students' flexible use. At the same time, the simple and quick simplification is very popular for students. High quality and high speed are suitable for our college entrance examination.

## Use mathematical thinking to solve the problem

Generally, it is scientific and ideological to use mathematical thinking method to analyze and solve problems, because using mathematical thinking method can make the thinking of solving problems clear, logical and particularly meticulous. At the same time, we should pay attention not to understand the ability of operation and solution only as the simple operation of algebraic formula. In the college entrance examination, the question type is complex, so the choice of mathematical thinking method is particularly important. For example: classified discussion thought, function and equation thought, transformation and transformation thought, number shape combination and so on. These ideas have corresponding number questions in the college entrance examination questions over the years.
For example: $\mathrm{a}=\log (2) 0.2, \mathrm{~b}=2^{\wedge} 0.2, \mathrm{c}=0.2^{\wedge} 0.3$ in the 2019 college entrance examination of science mathematics test questions compare the size of $\mathrm{a}, \mathrm{b}, \mathrm{c}$. This question examines the comparison of index and logarithm size, infiltrates the intuitive imagination and mathematical operation literacy, adopts the intermediate variable method, uses the transformation and attribution idea to solve the problem; that is, uses the intermediate quantity 0 to compare $\mathrm{a}, \mathrm{c}$, uses the intermediate quantity 1 to compare $\mathrm{b}, \mathrm{c}$. If such a question type does not consider the problem of intermediate variables, it will only waste time and may make mistakes if it only depends on its algebra operation ability in the examination room of the college entrance examination. As a result, it will take a simple multiple-choice question as a big one.

## HOW TO IMPROVE THE ABILITY OF CALCULATION AND SOLUTION FROM COLLEGE ENTRANCE EXAMINATION QUESTIONS

The ability of calculating and solving is one of the main factors that affect students' mathematics achievement. In reality, there are many problems in high school students' mathematical operation ability. For example, the operation speed is slow, the accuracy is low, the formula is mechanically applied, only the operation conclusion is paid attention to, but not the solution process (either simple or complex). Therefore, we should take corresponding measures to solve these problems:

## Develop interest in learning

Students should pay attention to the cultivation of interest in mathematics learning, strengthen the cultivation and promotion of mathematical operation ability, and pay attention to the fun and sense of achievement of solving problems successfully. We
should make friends with mathematics from the heart and understand the application field of mathematics. At the same time, we should pay special attention to the combination of theory and practice, close contact with life, and seek fun from life, so as to abstract the fun of mathematics.

## Cultivate good habit of examination

We must hold a rigorous and scientific attitude, carefully examine the questions, and do not doubt whether the question is wrong. As the most authoritative and fair form of examination in our country, the college entrance examination is scientific. Therefore, when doing the real questions or in the examination room, we must do: (1) see the numbers and symbols given in the questions clearly, because mathematics is a science of quantitative relations and spatial forms, and we must make clear what meanings the numbers and symbols are given. (2) There is also to find the relationship between variables, especially the implied conditions mentioned above. Often, the implied conditions involved in the question will determine whether the question can be done right or faster, which requires our classmates to dig. (3) To find out the quantity required in our questions, we need to know the given conditions in doing the questions, and make clear what you want to do and what quantity you should use.

## Remember some important formulas, rules, properties or conclusions.

Keeping these in mind will greatly increase our problem-solving efficiency. For example, in the expansion formula of Volume 1 of 2016 new curriculum standard: ( $2 x$ $\left.+x^{\wedge} 1 / 2\right)^{\wedge} 5$, what is the coefficient of $x^{\wedge} 3$ ? In this case, we need to use binomial theorem, otherwise it is difficult to solve. At this time, we can use the general term formula of binomial expansion to find the $r+1$ term, make the index of $x^{\wedge} 3$, and find $r$, then we can find the coefficient of $x^{\wedge} 3$ in the expansion. The following is the process of solving this problem: $\left(2 x+x^{\wedge} 1 / 2\right)^{\wedge} 5$ in the expansion, let $5-(r / 2)=3$, the solution is $r=4$, so the coefficient of $x^{\wedge} 3$ is 10 , so the answer is 10 .

## Have a habit of careful calculation.

At ordinary times, students must be patient, not impatient, and think seriously when doing the real questions of college entrance examination, even if it is a very simple question. The calculation should also be written in a neat and standard format. Even on the draft paper, the writing should be clear and logical. This is also to be able to turn around to check. It is customary to be natural. When checking, you should also be patient and careful, check one by one, and remember not to be careless. Mathematics is scientific and meticulous, but also dialectical unity. For example, in the digital space, a 10 represents a lot of meanings, which is a sufficient condition for training students' rigorous thinking and improving the ability of operation and solution.

## Improve the reasoning ability of operation

Because in the process of operational reasoning, we need a reasonable and scientific way of thinking and a series of thinking activities, such as observation, contrast, division and abstraction. In fact, the improvement of computing ability is also an
important way to improve their own logical thinking. First of all, we need to know all kinds of relations among various formulas, laws, theorems, etc., as well as their own meanings. For example, the geometric meaning of derivative is a key and difficult point in our high school elective textbooks. If we don't know its geometric meaning, we will be at a loss in the college entrance examination. We will know how to solve derivative, but we don't know how to do it. Therefore, its concept, definition and physical meaning (volume expansion rate, speed change rate, etc.) should be grasped and understood flexibly. Only in this way, in the process of seeking derivation, can we carry out reasoning step by step and finally get the correct result. It is not a matter of one day and one night, but a matter of long-term accumulation and accumulation to improve students' reasoning and operation ability. I think that the reason why teachers take students as the main body in teaching is to understand the students' knowledge and ability deficiencies by observing the whole process of students' problem-solving. And timely evaluation, and constantly improve students' learning ability and computing ability.

## Be confident in yourself

No matter what problem you are facing, you should have self-confidence. Mentally, you can always use your existing cognitive structure to do it. We should dare to do problems and be good at solving them. In the college entrance examination room, we should treat each one as our own enemy, so this is called "despise the enemy strategically and attach importance to the enemy tactically". When solving specific problems, we must carefully examine the problems, carefully and do not neglect any known conditions and the meaning of each number and symbol. By drawing inferences from one example, we will find that some of them have common characteristics. We can find and grasp the particularity of this problem through the general solutions and ideas of these problems. Seize this problem and this kind of problem different place, because the problem maker will not go to the original problem, always go to change some numbers or change some problem-solving methods, so that the result of this problem has changed, so there is almost no same mathematical problem, there are always one or several conditions are not the same, so the thinking and problem-solving process is not the same. Like some students who are not sensitive to mathematics or numbers, they can only learn mathematics mechanically and meaninglessly, which leads to that some students and teachers can do the questions they have talked about, others can't do them, they can only draw ladles in the same way, and some small changes of the questions will be directly covered. It's absolutely right to grasp its particularity when doing questions. Of course, some teachers will think that mathematics is only based on "the sea of questions tactics", and that there are several advantages to doing more questions: first, practice makes perfect and improves efficiency, which is particularly important in the college entrance examination room with time constraints; second, use problem-solving to consolidate and remember the connection and significance between the definitions, rules, properties, etc., and form a certain virtuous circle. However, in my opinion, mathematics is gifted, and "sea of questions tactics" is mechanical. We need to find out the essence and commonness of this kind of problems,
so as to really draw inferences from one example. Of course, problem-solving requires rich knowledge and self-confidence. This is what we math people have to have. It's like there is no always cloudy day, and we always see bright sunshine. If you don't have confidence, you will be afraid of difficulties and give up. Only with self-confidence can we move forward bravely, never give up lightly, study harder, have the hope to overcome difficulties and usher in our own spring.

## After the operation is solved, we should reflect on it, so as to improve the ability of operation

After we have done everything after careful consideration, we should reflect on it. At the same time, we should improve ourselves and realize our shortcomings. Reflection is not only the combination of learning and thinking, but also a good habit for us. It is a process from receiving knowledge to applying consciousness. It is a process of remembering and recognizing, remembering and thinking. Learning is the basis of thinking, thinking is a deepening of learning. Only through thinking can we turn knowledge into our own. The reflection after solving a problem refers to the review and thinking of the process of examining a problem, the thinking of solving a problem, the principle, thought, method of solving a problem and the knowledge used in solving a problem. Due to the limitation of students' cognitive structure level, they have no idea about knowledge. Under the influence of exam oriented education, a large number of students now think that the problem is right. He is also keen on doing a lot of problems and is not good at reflecting on the problems after solving them, thus losing an important link to reflect on the ability of solving problems. The problem that I did wrong is just skimming the water. I think I only need to know this problem. However, it is possible that the next step of variant training will not. It is lack of mathematical thinking and cognitive structure. Therefore, in order to improve the ability of operation and solution, we must reflect on it. To achieve these points: thinking to examine the question (the nature of the question), thinking rules (the use of methods), thinking to solve (divergent thinking, one question to solve), thinking to change (to prevent thinking stereotype), thinking to category, thinking mistakes. This "six thinking" can gradually develop the habit of students' independent thinking and active exploration, reflection, and enable students to deeply understand mathematical knowledge, and ultimately achieve the purpose of high efficiency, from "learning" to "learning".

## PAY ATTENTION TO THE LEARNING OF MATHEMATICAL CONCEPTS AND CULTIVATE RIGOROUS THINKING

The inherent nature of mathematics itself is formal, strategic and symbolic. In the process of doing mathematics problems, we must hold a rigorous attitude, and require students to pay attention to the learning of mathematical concepts, definitions and algorithms, so as to make sure that they are fluent in words. For example, when doing the true questions about the definition of the function's parity and monotony, you must understand the word "arbitrary", or in doing the true questions about inequality, there will always be the word "constant", so you must carefully examine the questions, or you will be far away. When doing operation problems, students should not only know
"how to operate", but also be clear about "why to operate like this", so that every step has a basis and a good reason. When doing a problem, more consideration should be given to "why". So on the basis of preciseness step by step to reach our highest level of Mathematics - scientificity.

## SUMMARY

Through the exercise of real questions, we need to think, practice and dig out the essence of each question repeatedly. This is a long-term training process, and it can not be achieved overnight. We should not only pay attention to the usual training, but also pay attention to the calculation and solution methods. We should not only master the general method, but also pay attention to the skills and skills. At the same time, we should combine with the training of other basic abilities of mathematics to calculate accurately and quickly, so as to reach the higher level of the ability of calculation and solution. Even the ability of this operation and solution can run through other disciplines and can be used flexibly, which can be harvested and beneficial for a lifetime.

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