DECISION DEVELOPMENT OF MANAGEMENT PROBLEMS OF BIOTECHNOLOGICAL SYSTEMS AT AN UNCERTAINTY OF ENVIRONMENTAL STATES USING THE MATHEMATICAL STATISTICS METHODS

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ABSTRACT

The methods for formalizing management problems based on the uncertainty of initial information and situation are developed in the paper. An assumption is made and accepted that the theory of decision-making under conditions of uncertainty be considered as a theory of rationalization of decision making taking into account the elements of uncertainty of environmental conditions, and its methods - as the methods of constructing a rational choice function under conditions of uncertainty. For an optimal strategy, control is calculated first, starting from the last level, and then the control process is carried out, starting from the first level. With this in view, any decision-making procedure, except for the one introduced by the levels, gives a lower score, partially proper optimal one, but not lower than locally optimal. It means that any decision-making procedure, except for the one introduced by the levels, gives a lower rating, partially proper, but not lower than locally optimal. The management processes of a biotechnological system (BTS) almost always occur in conditions of uncertainty due to the lack of information necessary for management. Therefore, it is advisable to consider the management process as a decision-making process in conditions of uncertainty.

Keywords: Formalization methods, conditions of uncertainty of initial information, management problems, rationalization of decision making, construction of a rational decision choice function.

INTRODUCTION

Let biotechnological systems (BTS), denoted by S, consist of objects $S = \{1,...S_m\}$ and management units $U^0, U^1...., U^m$. Each object S_n is characterized by a state vector (H_n, a_n) H_n - is the number of the management unit and S_n - its proper state, where $H_n \in H = \{1,...m\}$, $a_n \in A_n = \{a_n^1,...a_n^m\}$.

Each management unit U^m is characterized by a set of states (decisions) $X^k = \prod_{n=1}^{l} x_n^k$.

The transition of an object S_n from state (H_n, a_n) to state (H_n, a_n^1) is carried out under conditional probability $P(a_n a_n^1 IX^k)$, and the transition from state (H_n, a_n) to state (H_n, a_n)- under conditional probability $P_2(H_n H_n^1 IX_l^0)$.

Denote by 2^s the set of all subsets S. Consider a single-digit image $V^-: S \rightarrow 2^s$. Define $V^-: S \rightarrow 2^s$ in such a way that $V^-(S_n) = \{(S_n) = (Sr \in S) S_n \in V^+(Sr)\}$ are some subsets S compared to the object S_n.

Statement of the problem

Introduce generalized state vectors

$$a_l^+ \in A_l^+ = \prod_{S_r \in V\varphi(S_n)} A_r \quad a_l^- \in A_l^- = \prod_{S_r \in \varphi^-(S_n)} A_r \tag{1}$$

Assume that $S_A \bar{\in} V^+(S_n) V\varphi(S_n)$ for each object S_n; an assessment of its state $fH(a_n, a_n^+)$ is introduced, where S_n, a_n^+ is the state of the object $\varphi^+(S_n)$ is a part of the state vector of the objects.

If the estimate depends on some random parameter, for example, the state of the medium $fH(a_n, a_n^+, \theta)\theta \in \theta$ (θ is a finite set of values, and $P(\theta)$ is the probability that the parameter takes the corresponding values), then we will assume that

$$f^H(a_n, a_n^+) = \sum_{\theta \in \theta}^{l} P(O) f^H(a_n a_n^+ \theta) \tag{2}$$

Define an estimate for the entire system as the sum of the estimates for the objects:

$$F(x,a) = \sum_{n=1}^{l} fH(a_n a_n^+) \text{ где } a \in A = \prod_{n=1}^{l} A_n, x \in H^2 \tag{3}$$

By management we mean epy decision-making by management units

$$U \hat{U}^0, \ldots, \hat{U}^m \tag{4}$$

As a management estimate, consider the mathematical expectation of the system estimate $F(x,x,a) = \underset{a \in A}{\mathrm{M}} \underset{x \in H}{\mathrm{M}} F(x^1, a^1)$ where $x \in X = \prod_{k=o}^{m} X^k(x,a)$ - is the vector of the initial states of the objects $x = \prod_{n=1}^{l} H_n$, $a = \prod_{n=1}^{l} a_n$.

The management process is divided into two levels. A management unit $\hat{U}^0$ is assigned to the first level with an appropriate set of decisions, the management units $\hat{U}^0 : \ldots \hat{U}^m$ and the set of decisions $X^1, \ldots, X^m$ - to the second level.

The management units can make decisions sequentially one after another and in any order according to the vector of their state.

Objects respond to a change in management unit states, and information sources respond to a change in object states $H^n \in A$.

A management strategy is an image of the set of object states on the set of decisions X, where $H^n - n -$ is the multiple direct product {1, ...m}.

We divide the set of objects S by a subset of two levels. First, consider the decision making process at the second level.

Here a management strategy is an image of a set of states on the set of decisions $X = \prod_{k=1}^{m} X^k$

We denote the strategy $x = \underset{\alpha \in A}{U} x(a'), x(a') = \bar{x}$ where a - is the vector of the initial states of system objects.

The strategy $\bar{x}$ is called a proper optimal one if

$$x_n^k\left(a_n, a_n^*\right) \overset{=}{\max_{x_{k_n} a_n \in A_n}} \mathrm{M} f^{k*}\left(a_n, a_n^*\right) \tag{4}$$

where $a_n^* = a_n^+ x a_n^- x \prod_{S_x \in \varphi^-(S_n)} a_n^* \quad f^{k*}\left(a_n, a_n^* = f^k\left(a_n, a_n^+\right)\right) + \sum_{S_2 \in \varphi^-(S_n)} \mathrm{M}_{a_2 \in A} f^k\left(a_k, a_n^*\right)$

For an optimal strategy, the management is preliminarily calculated starting from the last level, and then the management process is carried out starting from the first level, in this case $\bar{x}_n^k\left(a_n, x_h x_R^*\right)$.

Let a partition of the set S into subsets ($i_1 \neq i_2$) be given

$\Sigma=\{g_1,\ldots\ldots gi\}$; $g_i \epsilon \sigma$; $g_i=\sigma$, $g_{i1} n\ g_{i2}=\varnothing$ (5)

We call $\Sigma \bar{X}$ a partial optimal strategy if $\bar{X}_r{}^n$- is the optimal strategy for the set of objects $\sigma_j U$ and $\bar{x}\Sigma = \underset{I \le j \le i}{U} \sigma_j \bar{X}$

With this in view, any decision-making procedure, except for the one introduced by the levels, gives a lower estimate, partially proper optimal one, but not lower than locally optimal.

Next, consider the management process at the first level.

Assume that the objects $S_n \in S$ are in states $\left(H_n a_n\right)$, respectively.

The strategy at the first management level is taken as an image of the set A on X and denoted as $\bar{x}^0 = \underset{a \in A}{U} \bar{X}^o(a)$

The strategy $\bar{x}^o$ is called a conditionally optimal one, when for all $n \in \{I, \ldots, m\}$

$$\bar{x}_{n\left(a_n^+, H_n\right)} \max_{x_n^o \in X_n^o} f_n\left(x_n^o, a_n^+\right) \tag{6}$$

$$f_h\left(x_n^0, a_n^=\right) = \max_{x_n \Xi X_n} \quad \mathrm{M}_{a_n \Xi \Xi A_n} \mathrm{M}_{1 \le k \le m} f^k\left(a_{n,} a_n^+\right)$$

Similar to the second level of management, the strategy, in addition to the set states, is characterized by the order of decision-making, determined by the optimality conditions introduced in the previous paragraph.

We can say that management $f_h\left(x_n^0, a_n^=\right)$ is optimal if

$$P\left(x, a_g, x, a_g l x_t\right) = \max_{x \Xi X} P\left(x, a, x_g, a_g l x\right),$$

$$P\left(x, a_g x, a_g\right) x_t = \prod_{1 \le n \le n} P\left(a_n, a_n^g x\right) P\left(H_n, H_n^g x\right)$$

Denote $g\left(x, \underset{x}{a}\right) = \Pi\left(x, a_g x_g, a_g x_g\right) F$ and $\overset{=}{x}$ as a system management strategy with the adopted decision-making procedure; it depends on the initial state of the system objects $\overset{=}{x} = \bar{x}(x, a)$, the management strategy $\overset{=}{x}(x, a)$ will be acceptable if

$$F\left(\overset{=}{x}\right) = \underset{a \in A}{M}\left[F\left(x^1, a^1\right) / x, a, \overset{=}{x}\right] \ge g(x, a)$$

All this allows for the repeated (and sometimes single) operation of the system to adopt relatively simple management strategies that differ little from the optimal assessment.

BTS management processes almost always occur in conditions of uncertainty due to the lack of information necessary for the management. Therefore, it is advisable to consider the management process as a decision-making process in the conditions of uncertainty.

Method and algorithm for solving problems

In the study of decision-making processes in conditions of uncertainty, the following scheme of decision choice [1] is used.

1. The management object (MO) operates in the environment (C). The behavior of C in relation to the goals and targets of the MO functioning can be characterized as passive and active behavior, or an intermediate one between these extreme cases.

2. The management unit (MU) has a set of (F) decisions on turning the MU to one of the possible states: $\Phi = \{u_1, ..., u_\kappa\} \kappa \geq 2$.

At each stage of the management process, MU can and must make one and only one decision from F.

3. At each stage of the management process, C can be in one and only one state out of many possible:

$\Theta = \{\Theta, ...\Theta_м\}, m \geq 2$, here neither decision maker is known nor in what state C is or will be at the time of decision implementation.

$\Theta = \{\Theta, ...\Theta_м\}, m \geq 2$ moreover, decision maker? it is not known in what condition C is or will be at the time of implementation of the decision.

4. On the straight line of the product of H x F, an estimated functional $F = \{f_{ij}\}, f_{il} \in R^1, i = 1, \bar{m}, j = 1, \bar{k}$ is defined whose elements $f_{ij} = f(\Theta_i, U_i)$ are a quantitative assessment of solution $U_i \in \Phi$, under condition that C is in state $\Theta_i \in \Theta$ at the time of its implementation. The triplet $\overset{л}{T} = \{\Phi, \Theta, F\}$ is called a decision-making situation.

5. At each stage of the management process, the information situation $I \in \{I_i\} \overset{6}{i} = 1$ in which decision maker is located is known. The information situation is a certain degree of uncertainty gradation of the choice of C states, known to decision maker at the time of decision making [1].

The information situation is understood as a certain degree of gradation of the uncertainty of the choice of C states from, the presence of which has decision maker at the time of the decision [1].

6. A set K_I of decision-making criteria: $K_I = \{X_n\} n = \overline{1,6}$ corresponds to each information situation $I \in \{I_i\} \overset{6}{i} = 1$.

The choice of one or another criterion (or a set of criteria) from the set K_I is carried out by the decision maker. At given $\hat{T}$ a decision-making problem is characterized by a triplet $\{I_j, K_I, A\}$, where: A is a system of axioms of decision-making criteria analysis.

At fixed $\Phi\Theta, F, I$, a set of values of decision-making criteria $\{x_u(\varphi_s), ...x_t(\varphi_s)\}$ in an information situation I_j can be compared with each decision $\varphi_2 \in \Phi$.

In the general case, the decision-making criterion is a preference operation on the set Φ taking into account the uncertainty element of possible states C. It orders the totality of decisions $\{\varphi_1, ...\varphi_k\}$ according to the degree of their preference for the decision makers [1]. In other words, on any pair of decisions $\varphi_x \varphi_y \in \Phi$ based on the analysis of estimates $\{x_1(\varphi_x), ...x_{tj}(\varphi_x)$ and $\{x_{tj}(\varphi_y), ..., x_{tj}(\varphi_g)\}$ a binary relation of preference - indifference R can be defined, which has the properties of reflectivity (1) and transitivity (2):

$$(V\varphi_x \in \Phi)[\varphi_x R \varphi_x]$$

$$(\forall \varphi_x, \varphi_y, \varphi_t \in \Phi)[(\varphi_x R \varphi_y)\mathrm{E}(\varphi_y R \varphi_t) \rightarrow \varphi_x R \varphi_y] \quad (7)$$

where: $\varphi_x R \varphi_y$ means that the decision $\varphi_x \in \Phi$ is "no less preferable" than the decision $\varphi_y \in \Phi$. There exists the following interdependence between the binary preference-indifference ratio R and the Neumann-Morgenstern real-valued utility function $\varphi(\varphi)$ [2]:

$$(\forall \varphi_x, \varphi_y, \varphi_t \in \Phi)[(\varphi_x R \varphi_y) \leftrightarrow (\varphi(\varphi_{x_x}) \geq U(\varphi_y))] \quad (8)$$

The preferences decision maker on Φ can be represented in the form of defined on Φ x Φ preference matrix, $P(\Phi, R) = // S_{st} //_k^k$, the elements of which $S_{st}, S, t = \bar{I}\, K$ are defined as

$$S_{st} = \begin{cases} 1 & если \varphi_s R \varphi_t \\ 0 & если (\varphi_s R \varphi_t) \end{cases}$$

where $(\varphi_s R \varphi_t)$ means denying the statement $\varphi_s R \varphi_t$; note that from (7) and (8) it follows that

$$(\forall X \in (I,...k)\ )[S_{xx} = 1] \quad (9)$$

$$\forall x, u, t \in (1,....k\ ))[S_{xy} = 1][(S_{yt} = 1) \rightarrow (S_{xt} = 1)\ ] \quad (10)$$

Another convenient way of representing preferences on Φ is a preference graph $\in (\Phi, R)$ in which to each decision from Φ corresponds a vertex, and the presence of an arc $r(x, y)$ oriented from the vertex $x \in (I,....k)$ to the vertex $y \in (I,...k)$ means that $\varphi_x R \varphi_y$ or, $r(x, y) \leftrightarrow S_{xy} x, y \in (I,..., K)$, which is the same.

Since condition (4) is always satisfied, loops $r(\delta, \delta)$ on preference graphs, as a rule, are not indicated. Using the ratio R (preference-indifference), the ratio p and P^{-1} (strict preference) and the ratio I (equivalence) are defined as

Another convenient way of presenting preferences on Φ is a graph of preferences $\in (\Phi, R)$, in which each decision from Φ there corresponds a vertex, and the presence oriented from the top $x \in (I,....k)$ to the top $y \in (I,...k)$ arcs $r(x, y)$ means that $\varphi_x R \varphi_y$ or, what is the same, $r(x, y) \leftrightarrow S_{xy} x, y \in (I,..., K)$

$$\forall \varphi_x, \varphi_y \in \phi, \varphi_x P \varphi_y = (\varphi_x R \varphi_y) \varepsilon^I (\varphi_y R \varphi_x) \quad (11)$$

$$\forall \varphi_x, \varphi_y \in \phi, \varphi_x P^{-1} \varphi_y = (\varphi_x R \varphi_y) \varepsilon (\varphi_y R \varphi_x) \quad (12)$$

$$\forall \varphi_x, \varphi_y \in \phi : \varphi_x I \varphi_y = (\varphi_y R \varphi_y) \varepsilon (\varphi_y R \varphi_x) \quad (13)$$

where: P - means "more preferable than"; P^{-1} - "less preferable than"; I- "equivalent" (if $\varphi_x I \varphi_y$, then the choice between φ_x and φ_y is indifferent for the decision-maker) [9].

If the set of decisions $\varphi = \{\varphi_1 ..., \varphi_k\ \}$ is defined as the set of decisions admissible for the decision-maker, the functioning of the latter is manifested in the choice and adoption of a certain decision $\varphi^0 \in \Phi$ (or the choice of a certain set of decisions $\varphi^0 \in \Phi$ and the adoption of one of them; and the decision-maker is indifferent which decision from F1 will be adopted). The function C (Φ), which compares a certain set of "selected" elements to the set Φ is called the choice function (CF). Obviously, here the following condition must be met

$$\Phi < C(\Phi) \leq \Phi \quad (14)$$

As was shown above, the theory of decision making under conditions of uncertainty allows determining the binary preference-indifference R ratio on Φ and to order (taking into account the element of uncertainty in choosing one's states from environment C) the decision from Φ in accordance with the decision maker's preferences.

The theory of decision making under conditions of uncertainty allows, as was shown above, to determine the binary relationship of preference - indifference R on Φ and to order (taking into account the element of uncertainty in choosing one's states from environment C) of a decision from Φ in accordance with the decision maker's preferences.

(CF) will be called the rational choice function [9] (RCF) and denoted by C (Φ, R) if

$$C(\Phi, R) \geq \varepsilon \{\varphi \in \Phi : (\forall \varphi \in \Phi)[\varphi^o R \varphi]\} \quad (15)$$

If <Φ,R> is given as a preference matrix, then the set of values of C (Φ, R) can be determined from the expression

$$C(\Phi.R) = \varepsilon \left\{ \varphi_i^o \in \Phi I \prod_{j=1}^{\kappa} S_{ij} = 1 \right\} \quad (16)$$

On the preference graph G<Φ,R>, the set of rational decisions corresponds to the set of decisions from which the arcs go to all vertices G <Φ, R>:

$$C(\Phi.R) = \varepsilon \left\{ \varphi_i^o \in \Phi I \prod_{j=1}^{\kappa} r(ij) = 1 \right\} \quad (17)$$

Using expression (9), we represent definition (17) in the form

$$C(\Phi.R) = \varepsilon \{\varphi^o \in \Phi(\varphi \in R)\ ^]\![\varphi P U^o]\}$$

CONCLUSION

Thus, the theory of decision making under conditions of uncertainty can be considered as a theory of decision making rationalization taking into account the elements of environmental states uncertainty, and its methods - as the methods of RCF construction under conditions of uncertainty.

It is important to emphasize that the decision-making is carried out by the person (or a group of persons) responsible for implementing the decision. Decision theory allows choosing (specifying) just a subset of decisions that, according to its concept, are considered as rational ones.

Therefore, to study the management processes, it is necessary to introduce some essential characteristic of the decision maker behavior. Such a characteristic is the rationality (or irrationality) of his behavior. Since the decision maker functioning in the management process is manifested in decision-making, the rationality (or irrationality) of his behavior should be judged by the stages of decision-making. We assume that decision maker behavior is rational if the decisions he makes are rational, i.e. belong to the RCF. The proposal of rational behavior of the subsystem forming the management unit, decision maker since otherwise the study of the models of decision-making processes is devoid of practical meaning.

REFERENCES

1. Averkin A.N. et al. Fuzzy sets in artificial intelligence control models. -M.: The science, 1986
2. Khalin.V.G. Decision Making Theory. M.: The science 2018
3. Akhmetov K.A., Ismailov M.A. Mathematical modeling and process control of biochemical production. Tashkent: Science.1988. p. 96.
4. Rutanov.V.G. Filatov. G. Intelligent system automation systems. Ucheb, posobie. Tutorial. BGTU. name V.G.Shuxova 2010 y.
5. Kafarov V.V., Dorokhov I.N., Markov E.P. System analysis of chemical technology processes. Application of the fuzzy set method. M.: The science,1986. 360 p.
6. Kafarov V.V., Glebov M.B. Mathematical modeling of the main processes of chemical production. M.: graduate School, 1991, 400 p.

7. Fuzzy sets in control and artificial intelligence models. Edited by D.A. Pospelov. M.: The science,1986. 312 p.
8. Construction of mathematical models of chemical and technological objects. E.G. Dudnikov, V.S. Balakirev, V.N. Krivsunov, A.M. Tsirlin. M.: Chemistry,1970. 312 p.
9. Fuzzy sets and the theory of possibilities. Recent Achievements. M., Radio and communication. 1986, 408 p.