C,P,T ANALYSIS ON 1-PARAMETER MODEL (2 × 2) MATRIX IN COMPLEX QUANTUM SYSTEMS

Biswanath Rath

Department of Physics, North Orissa University, Takatpur, Baripada -757003,

Odisha, INDIA; E.mail: biswanathrath10@gmail.com.

We present an one -parameter model (2×2) matrix model PT-symmetric operator and study its matrix form of C,P,T.

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1. Introduction

Nearly two decades ago Bender and Boettecher [1] proposed a thought provoking idea on study of real spectra in complex quantum systems. According to Bender and Boettecher [1], quantum operators satisfying the commutation relation

$$[H, PT] = 0 \Leftrightarrow T^{-1}(P^{-1}HP)T = H \quad (1)$$

can yield real spectra. Here P stands for parity operator and T stands for time reversal operator. The proposed model was well explained by considering the operator

$$H = p^2 + ix^3 \quad (2)$$

where x is the coordinate and p is the corresponding momentum . The commutation relation between x and is

$$[x, p] = i \quad (3)$$

Further we have

$$P^{-1}xP = -x \quad (4)$$

$$T^{-1}xT = x \quad (5)$$

$$P^{-1}pP = -p \quad (6)$$

$$T^{-1}pT = -p \quad (7)$$

In order to give some more understanding on complex quantum systems, Bender Brody and Jones [2] have proposed a (2×2) matrix model analysis and foundout the symmetry operator in matrix form. In fact after this model, Ahmed [3] also presented a matrix model on PT symmetry systems. Unfortunately in a recent analysis Rath [4] reported that C-symmetry and P-parity form of matrix in Ahmed paper were incorrect. However aim of this paper is to analyse matrix form of C, P, T considering an one-parameter model matrix.

2. Model 1-parameter matrix in complex space

Here we consider a model matrix

$$H = \begin{bmatrix} \sin\theta + 1 & i\sin\theta \\ i\sin\theta & \sin\theta - 1 \end{bmatrix} \tag{8}$$

The spectra of this model is always real and are the following

$$E_{\pm} = \sin\theta \pm \cos\theta \tag{9}$$

The corresponding wave functions are the following

$$|\psi_{+}> = \frac{1}{\sqrt{\cos\theta}} \begin{bmatrix} \cos(\theta/2) \\ i\sin(\theta/2) \end{bmatrix} \tag{10}$$

and

$$|\psi_{-}> = \frac{1}{\sqrt{\cos\theta}} \begin{bmatrix} -i\sin(\theta/2) \\ \cos(\theta/2) \end{bmatrix} \tag{11}$$

Further one can verify that in complex space the wave functions satisfy the relations

$$<\psi_{\pm}|\psi_{\pm}> = 1 \tag{12}$$

and

$$<\psi_{\pm}|\psi_{\mp}> = 0 \tag{13}$$

One will also notice that using above eigenfunctions one can verify the eigenvalue relation as

$$H|\psi_{\pm}> = E_{\pm}|\psi_{\pm}> \tag{14}$$

3. C, P, T in matrix form

Following the work Bender, Brody and Jones [2], we find

$$C = \begin{bmatrix} \sec\theta & i\tan\theta \\ i\tan\theta & -\sec\theta \end{bmatrix} \quad (15)$$

Now following Wang, Chia and Zhang [5] model analysis,

$$C\left[\sum_i |\psi_i><\psi_i|\right]^{-1} = P \quad (16)$$

we find

$$P = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad (17)$$

Interested readers will notice that the eigenvalues of both C and P are the following

$$\lambda_p = \pm 1 \quad (18)$$

and

$$\lambda_c = \pm 1 \quad (19)$$

Now using antilinear property of T, we express it as [6]

$$T = UK \quad (20)$$

where U is a (2×2) matrix and K is a complex conjugation operation . Let us consider that U is an unknown matrix having the form

$$U = \begin{bmatrix} u_1 & u_2 \\ u_3 & u_4 \end{bmatrix} \quad (21)$$

Considering the relation

$$K^2 = 1 \quad (22)$$

and

$$T^{-1}(-i)T = KU^{-1}(-i)UK = K(i)U^{-1}UK = iK^2 \quad (23)$$

we find

$$T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} K \quad (24)$$

Hence it is easy to verify the relations

$$[H, C] = 0 \quad (25)$$

$$[H, P] \neq 0 \quad (26)$$

$$[H, PT] = 0 \quad (27)$$

and

$$[H, CPT] = 0 \quad (28)$$

4. Discussion and Conclusion

In this model we have derived matrix form of C, P. However matrix form of T is still incomplete due to unknown form of K. Hence the term PT and CPT ant not be considered as symmetry operators. However they can be considered as invariances of H. From the above matrix form of PT one can not get eigenfunctions of H. The same argument is also valid for CPT. Hence the only symmetry operator of the above analysis is C. One can also find that F(C) also commute with H where

$$[H, F(C)] = 0 \quad (29)$$

where F(C) is given by.

$$F(C) = \cfrac{C}{\lambda + \cfrac{C}{\lambda + \cfrac{C}{\lambda \cfrac{C}{\lambda + C}.....}}} \quad (30)$$

In fact analytical form of F(C) is very difficult, however one can test its validity considering a numerical model matrix easily.

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