COMMENT ON: OPERATOR P IN 2x2 PT-SYMMETRIC QUANTUM SYSTEM

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ABSTRACT

We find that an incorrect analysis on P-parity operator has been reported by Li et al. in this journal leading to four different forms. Out of four forms only one form is correct and rest three are incorrect. Moreover, three incorrect forms are due to ignoring the relation between P-parity and C-symmetry.

Keywords: Unbroken spectra, parity, PT-symmetry, C-symmetry.

INTRODUCTION

In a recent paper by Li et al. [1] reported that the known (2x2) matrix model operator [2]

$$H = \begin{bmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{bmatrix} \quad (1)$$

can have 4-different forms of P-parity operators as

$$P_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad (2)$$

$$P_2 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \quad (3)$$

$$P_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad (4)$$

and $$P_4 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad (5)$$

In the opinion of present author, P_2, P_3 and P_4 are in-correct. Secondly Li et al. [1] have considered an unphysical case $s = 0$ ignoring the basic physics behind PT-symmetry. Further, we notice that T-time reversal operator in eq. (6) has been incorrectly presented [1]. Below in brief we present our analysis that leads to above remarks.

Present analysis on broken spectra $s = 0$

The above Hamiltonian operator under the constraint $s = 0$ becomes

$$H_{s=o} = h = \begin{bmatrix} re^{i\theta} & 0 \\ 0 & re^{-i\theta} \end{bmatrix} \quad (6)$$

has complex eigenvalues

$$\varepsilon_{\pm} = r\cos\theta \pm ir\sin\theta \qquad (7)$$

Hence PT-symmetry is a broken and does not carry any interesting facts. However, h is still PT-symmetry in nature. In this case parity of the system becomes

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad (8)$$

However, Li et al. [1] suggested an incorrect form

$$P_{incorrect} = \begin{bmatrix} 0 & b \\ 1/b & 0 \end{bmatrix}, b \neq 0,1 \qquad (9)$$

On the other hand above relation for parity can be interpreted as correct provided $b = 1$.

Present analysis on unbroken spectra

The importance of above model PT-symmetry operator is that the spectra of this model are unbroken [2-4]

$$E_{\pm} = r\cos\theta \pm s\sin\alpha \qquad (10)$$

provided $r\sin\theta = s\sin\alpha$ (11)

It is commonly known that parity operator must have eigenvalues ± 1 [2]. Under this condition, we analyse parity operators reflected earlier [1]. It is seen that P_3 and P_4 have the following eigenvalues

$$\lambda_{P_3} = 1 \qquad (12)$$

and $\lambda_{P_4} = -1$ (13)

Hence P_3 and P_4 can not be termed as parity operator. In fact, it is simply positive identity matrix or negative identity matrix. Now we are left with P_1 and P_2. In order to accept the correct one, we use the relation between C-symmetry and Parity as [3] as

$$C = \sum |E_i\rangle\langle E_i| P \qquad (14)$$

where $|E_i\rangle$ is the eigenstates of H having unbroken spectra [2-4]. Further, C is the
symmetry operator, that commutes with H i.e.

$$[H, C] = 0 \qquad (15)$$

In matrix form, it can be represented as

$$C = \begin{bmatrix} i\tan\alpha & \sec\alpha \\ \sec\alpha & -i\tan\alpha \end{bmatrix} \qquad (16)$$

It is seen that the eigenvalues of C are

$$\lambda_C = \pm 1 \qquad (17)$$

Using the wave functions [3,4] we find that

$$P_{correct} = P_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad (18)$$

Hence, we have discarded P_2 as the parity operator .

Meaning of PT-symmetry and T

An operator H is said to be PT-symmetry if satisfies the relation [2-4]

$$[H, PT] = 0 \Leftrightarrow T^{-1}(P^{-1}HP)T = H \qquad (19)$$

or $$[H, PT] = 0 \Leftrightarrow P(THT^{-1})P^{-1} = H \qquad (20)$$

Here T -time reversal operator is an anti-unitary operator even though $T^2 = 1$. It can be expressed as

$$T = UK \qquad (21)$$

where $K^2 = 1$ and U is a unitary operator. Let us focus on the nature of K from the standard literature [5]

$$T^{-1}(i)T = KU^{-1}(i)UK \qquad (22)$$

$$= K(i)U^{-1}UK \qquad (23)$$

$$= (i)K^2 \qquad (24)$$

$$= i \qquad (25)$$

Hence the correct relation between x and T is (incorrect form is presented in Eq(6)[1])

$$T^{-1}(x)T = x \qquad (26)$$

CONCLUSION

In conclusion, we find that out of 4-different forms of parity only P_1 is correct as it is physically and mathematically correct. Secondly discussion on P or T is meaningless if the spectra of an operator is broken. Lastly we focussed some basic features on T, which were not reflected earlier [1].

REFERENCES

[1] X. Li, X. Yong, Y. Han, X. Gong, Y. Tao, Operator P in 2x2 PT-symmetric quantum system, Eur. J. Math and Comp. Sc. (2017),4(2):46-51.
[2] C.M. Bender, D.C. Brody, H.F. Jones, Complex Extension of Quantum Mechanics, Phys. Rev. Lett. (2002), 89(27): 270401
[3] Q. Wang, S. Chia, J. Zhang, PT-Symmetry as a Generalization of Hermiticity, J. Phys. A 43,295301(2010).
[4] B. Rath, Comment on C, PT, CPT invariance of pseudo-Hermitian Hamiltonians [Z. Ahmed, arxiv: quant-ph/0302141 v1], arXiv:1907.05235[quant-ph].
[5] R. Shankar, Principles of Quantum Mechanics, Plenum Press (Chapter -11, section 11.5) (2005).