A NEW COMPLEX NON-HERMITIAN (2 × 2) MATRIX UNDER DIFFERENT NATURE OF REAL EIGENVALUES : INVERSE NATURE

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We propose a new complex non-hermitian matrix and discuss its different nature of real eigenvalues numerically on selective choice of parameters. Interestingly we notice on certain choice of parameters eigenvalues can be made inverse of each other

.MSC nos-

40 C05 -Matrix Methods

15 Bxx-Special Matrices

15 A12-Conditioning of Matrices

57 Q55-Approximations

97 N40-Numerical Analysis

Key words- Complex matrix, Non-hermitian, real eigenvalues, inverse eigenvalues.

Since the development of matrix, it is commonly known to all the readers that Hermitian matrix always yield real eigenvalues [1]. The common form of (2 × 2) Hermitian matrix can be written as

$$h = \begin{bmatrix} c & b \\ b^* & d \end{bmatrix} \tag{1}$$

However in this letter we present a new model complex and study its real nature of eigenvalues as follows. The proposed complex matrix is

$$H = \begin{bmatrix} a_1 + ib & \mp(c_1 + id) \\ \pm(c_2 + id) & a_2 - ib \end{bmatrix} \tag{2}$$

The above matrix model have unbroken eigenvalues

$$E_{\pm} = \frac{\left[(a_1+a_2) \pm \sqrt{(a_1-a_2)^2 + 4\left(d^2 - c_1c_2 - b^2\right)}\right]}{2} \tag{3}$$

provided

$$b(a_1 - a_2) = d(c_1 + c_2) \tag{4}$$

and

$$(a_1 - a_2)^2 + 4(d_2 - b_2 - c_1c_2) \geq 0 \tag{5}$$

In fact the above model is very difficult to handle analytically in view of its typical nature of eigenvalues, which can be visualised considering equivalent numerical model as follows.

Case-1 Un-correlated Eigenvalues

Here the eigenvalues are different from each other and there is no correlation between the two.

$$H_1 = \begin{bmatrix} 7+0.25i & -(1.5+0.25i) \\ (3.5+0.25i) & 2-0.25i \end{bmatrix} \tag{6}$$

The eigenvalues and eigen functions are the following

$$E_{\pm} = 5.5;\ 3.5 \tag{7}$$

$$|\in_{+}>= \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \tag{8}$$

and

$$|\in_{-}>= \begin{bmatrix} \left(\frac{829}{2094} + i\frac{247}{6629}\right) \\ \frac{612}{667} \end{bmatrix} \tag{9}$$

Case-2 Correlated Eigenvalues

Here in addition to earlier condition in Eq(4) , an additional condition

$$a_1a_2 + c_1c_2 + b_2 - d_2 = 0.25 \tag{10}$$

is required to be satisfied . Then the two eigenvalues will be reciprocal to one another. In order to give an example of this we consider the model as

$$H_2 = \begin{bmatrix} \frac{4557}{1894} + \frac{204}{985} & \frac{1239}{5401} + \frac{i}{2} \\ -\left(\frac{258}{443} + \frac{i}{2}\right) & \frac{659}{1477} - i\frac{204}{985} \end{bmatrix} \tag{11}$$

The above matrix model have unbroken eigenvalues

$$E_{\pm} = \frac{1517}{621}; \frac{621}{1517} \tag{12}$$

One will notice that

$$E_{+} = \frac{1}{E_{-}} \tag{13}$$

Interestingly two eigenvalues are inverse of each other. The corresponding eigenfunctions are

$$|\psi_{+}> = \begin{bmatrix} \frac{439}{470} \\ -\left(\frac{91}{310} + i\frac{1096}{5387}\right) \end{bmatrix}$$

(14)

and

$$|\psi_{-}> = \begin{bmatrix} -\left(\frac{548}{4077} + i\frac{2113}{9285}\right) \\ \frac{895}{928} \end{bmatrix}$$

(15)

In this model we notice that if the imaginary part of the diagonal elements are complex conjugate to each other then the matrix under above constraints can yield real spectra that may be correlated of un-correlated to each other.

REFERENCES

[1] E.Kreyszig "Advanced Engineering Mathematics" (John Wiley and Sons, New Delhi, India, 8th edition (2011).