USING MAPLE MODULES IN SOLVING TASKS WITH BASIC FORMULAS OF SPHERICAL TRIANGLES

Egamov Mirshohid Kholmurodovich
Senior teacher, Karshi Engineering-Economics Institute
UZBEKISTAN
mirshohid.egamov@mail.ru

ABSTRACT

In this article, solutions of the basic formulas of a spherical triangle, calculations of some problem tasks using Maple modules are highlighted. For students studying in geodesy, cartography, and cadaster, it is difficult to solve problems on the basic formulas of spherical trigonometry. This type of formulas makes it easy to calculate problems on the computer with the MAPLE software. As well as, the accuracy coefficient for solving spherical trigonometry computed by the MAPLE software will considerably be higher. Studying the elements of spherical trigonometry helps students solve some problems of higher geodetic science.

Keywords: Spherical triangle, cosine, sine, theorem.

INTRODUCTION

In spherical geometry, the study of the figure located on the sphere and geometry. The study of this geometry increases the spatial thinking of students. For example, when mapping the world in a globe and a flat map in geography. In the present work, we will conduct calculations of some problems in spherical geometry with the help of Maple modules.

LITERATURE REVIEW AND METHODOLOGY

Basic concept. Consider a sphere with radius $R=1$ (units). Any plane crossing the sphere gives a circle in the section. If the plane passes through the center of the sphere, the cross section is the so-called large circle. Through any two points on the sphere, except diametrically opposite, you can hold a single large circle.

Definition 1. The smaller arc *AB* of a large circle is the shortest of all the lines on the sphere connecting the given points. This arc is called the distance or segment between two points: *A* and *B*.

The length of the segment is conveniently measured by the angle at which it is visible from the center of the sphere. Therefore, the segments are measured both in numbers and radians.

Many concepts of planimetry can be transferred to the sphere, in particular those that can be expressed through distances.

Definition 2. *A* spherical circle is a set of points of a sphere equidistant from a given point. Similarly, the concept of a triangle can be defined.

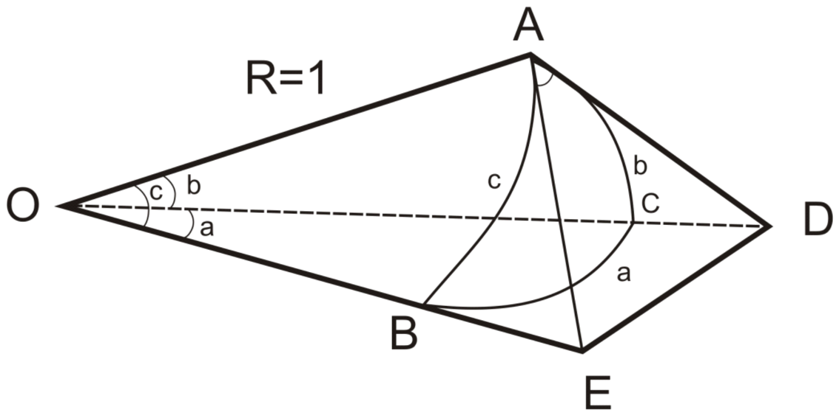
Definition 3. Take three points *A*, *B*, *C* on the sphere. We will connect them in series with each other, in segments. Part of the sphere bounded by these arcs is called a spherical triangle.

The angle between arcs, that is, the internal angles of a spherical triangle, is defined as the angles between tangents at the intersection point.

RESULTS

Consider the *Figure 1*, which shows the triangle *ABC* on a sphere with a radius of one and the center at the point *O*. the Length of the sides of the triangle are, *a*, *b*, *c*. At the top of *A* are tangent *AE* and *AD* to the sides *c* and *b* of the spherical triangle. These tangents intersect at points *E* and *B* with the continuation of the radii of the sphere that pass through the vertices *A* and *C*.
We use the cosine theorem for plane triangles $DE^2 = AE^2 + AD^2 - 2 \cdot AE \cdot AD cosA$, $DE^2 = OD^2 + OE^2 - 2 \cdot OD \cdot OE cosA$ *ADE* and *OED*:

Figure 1. Calculation of a spherical triangle.



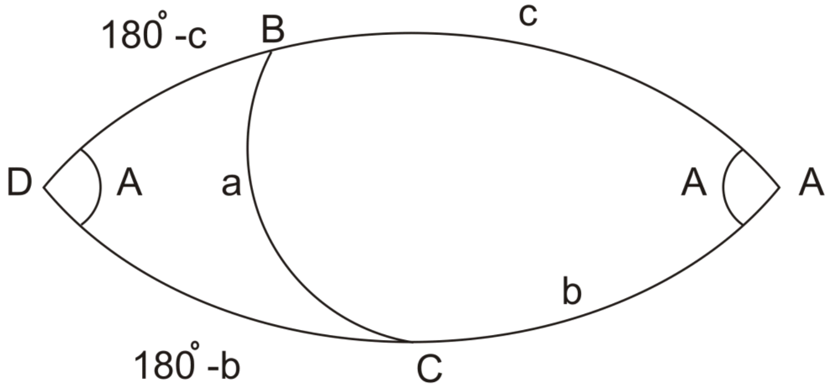
Equating the right-hand sides of the equations, we obtain:
$(OE^2 - AE^2) + (OD^2 - AD^2) = 2 \cdot OD \cdot OE \cdot \cos a + 2 \cdot AE \cdot AD \cos A$
Taking into account that the radius of the sphere is equal to one, from the right-angled triangles of the *OAE* we obtain:

$(OE^2 - AE^2) + (OD^2 - AD^2) = OA^2 + OA^2 = 2 \cdot OA^2$ since $AD = tgb$; $AE = tgc$; $OE = \frac{1}{\cos c}$; $OD = \frac{1}{\cos b}$; we receive: $1 - \frac{\cos a}{\cos b \cdot \cos c} + tgb \cdot tgc \cdot \cos A = 0$.

Multiplying all the terms of the last equation by and $\cos b \cdot \cos c$ we finally get:

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A \quad (1)$$

Construction in *Figure 1* possible if each side of *b* and *c* is less than *90°*. Therefore, the expression (1) should be generalized to the case when the triangle has sides greater than *90°*. To do this, consider the *Figure 2*. It shows *ABC*, which has sides *b>90°* and *c>90°*. If we continue these sides until they intersect at point *D*, we get an adjacent triangle *DBC*, in which each of the sides *180°-b* and *180°-c* is less than *90°*.



Then, for the DBC triangle, the formula (1) takes the form $\cos a = co(180^\circ - b)\cdot\cos(180^\circ - c) + \sin(180^\circ - b)\cdot\sin(180^\circ - c)\cdot\cos A$ or $\cos a = \cos b\cdot\cos c + \sin b\cdot\sin c\cdot\cos A$, which is the same as formula (1).

DISCUSSION

When writing formulas for spherical trigonometry, the method of circular permutation of elements is often used. For example, in order to obtain the second formula, substitute in (1) (substitution is performed according to *Figure 2* in direction of arrow): *Figure 2.* $a\to b, b\to c,\ c\to a, A\to B$

In final we get:

$$\left.\begin{aligned}\cos a &= \cos b\cdot\cos c + \sin b\cdot\sin c\cdot\cos A\\ \cos b &= \cos a\cdot\cos c + \sin a\cdot\sin c\cdot\cos B\\ \cos c &= \cos a\cdot\cos b + \sin a\cdot\sin b\cdot\cos C\end{aligned}\right\}\quad(2)$$

Thus, the cosine of the side of a spherical triangle is the sum of the products of the cosines of the other two sides and the sine of the same sides multiplied by the cosine of the angle between them.

Formulas of cosines of angles of a spherical triangle.

To obtain the formulas of cosines of the angles of a spherical triangle, write down the relations (2) for the sides of the triangle $A_1B_1C_1$ polar with respect to the triangle ABC :

$$\left.\begin{aligned}\cos a_1 &= \cos b_1\cdot\cos c_1 + \sin b_1\cdot\sin c_1\cdot\cos A_1\\ \cos b_1 &= \cos a_1\cdot\cos c_1 + \sin a_1\cdot\sin c_1\cdot\cos B_1\\ \cos c_1 &= \cos a_1\cdot\cos b_1 + \sin a_1\cdot\sin b_1\cdot\cos C_1\end{aligned}\right\}$$

According to the basic property of mutually polar triangles ABC and $A_1B_1C_1$: $a_1 = 180^\circ - A;\ b_1 = 180^\circ - B;\ c_1 = 180^\circ - C \quad A_1 = 180^\circ - a \quad B_1 = 180^\circ - b \quad C_1 = 180^\circ - c$

Substituting these equations into the previous formula gives:

$$\cos(180^\circ - A) = \cos(180^\circ - B)\cdot\cos(180^\circ - C) + \sin(180^\circ - B)\cdot\sin(180^\circ - C)\cdot\cos(180^\circ - a)$$

$$\cos(180^\circ - B) = \cos(180^\circ - A)\cdot\cos(180^\circ - C) + \sin(180^\circ - A)\cdot\sin(180^\circ - C)\cdot\cos(180^\circ - b)$$

$$\cos(180^\circ - C) = \cos(180^\circ - B)\cdot\cos(180^\circ - A) + \sin(180^\circ - B)\cdot\sin(180^\circ - A)\cdot\cos(180^\circ - c)$$

Let's use the formulas of trigonometric functions reduction

$\sin(180^{\circ}-a)=\sin a, \cos(180^{\circ}-A)=-\cos a$, and also multiply by negative one both parts of each relation. As a result, we obtain the formulas of the cosines of the angles of a spherical triangle.

$$\left.\begin{aligned}\cos A=-\cos B\cdot\cos C+\sin B\cdot\sin C\cdot\cos a\\ \cos B=-\cos A\cdot\cos C+\sin A\cdot\sin C\cdot\cos b\\ \cos C=-\cos B\cdot\cos A+\sin B\cdot\sin A\cdot\cos c\end{aligned}\right\}\quad(3)$$

Theorem. The ratio of the sine of the angle of a spherical triangle to the sine of the opposite side, a constant. In other words, the sines of the sides of a spherical triangle are proportional to the sines of the opposite angles, i.e., $\frac{\sin A}{\sin a}=\frac{\sin B}{\sin b}=\frac{\sin C}{\sin c}=K$ where K is a constant (4)

Evidence. Previously, the formula was obtained $\cos a=\cos b\cdot\cos c+\sin b\cdot\sin c\cdot\cos A$, from which it follows:

$$\cos A=\frac{\cos a-\cos b\cdot\cos c}{\sin b\cdot\sin c}$$

Then

$$\sin^2 A=1-\cos^2 A=1-\left(\frac{\cos a-\cos b\cdot\cos c}{\sin b\cdot\sin c}\right)^2=$$

$$=\frac{\sin^2 b\cdot\sin^2 c-\cos^2 a+2\cdot\cos a\cdot\cos b\cdot\cos c-\cos^2 b\cdot\cos^2 c}{\sin b\cdot\sin c}=$$

$$=\frac{(1-\cos^2 b)\cdot(1-\cos^2 c)-\cos^2 a+2\cdot\cos a\cdot\cos b\cdot\cos c-\cos^2 b\cdot\cos^2 c}{\sin^2 b\cdot\sin^2 c}=$$

$$=\frac{1-\cos^2 a-\cos^2 b-\cos^2 c+2\cdot\cos a\cdot\cos b\cdot\cos c}{\sin^2 b\cdot\sin^2 c}:$$

Now we divide both parts of the resulting expression into $\sin^2 a$ which result in:

$$\frac{\sin^2 A}{\sin^2 a}=\frac{1-\cos^2 a-\cos^2 b-\cos^2 c+2\cdot\cos a\cdot\cos b\cdot\cos c}{\sin^2 b\cdot\sin^2 c\cdot\sin^2 c}=K^2$$

The expression standing in the right-hand side, symmetrically relative to the elements and *a*,*b*,*c* does not change its value during circular permutation of the letters. Therefore C can be taken as a constant and we will get $\frac{\sin A}{\sin a}=K$, $\frac{\sin B}{\sin b}=K$; $\frac{\sin C}{\sin c}=K$, which proves the first relation in (4). Similarly we prove the sine theorem (4) establishes a relationship between the sides and the opposite angles of a spherical triangle:

a) equal angles lie against equal sides of a spherical triangle.

b) Against the greatest angle of a spherical triangle is the greatest side and, conversely, against the greatest side of a spherical triangle is the greatest angle.

Task 1. If the sides of the spherical triangle are even $b=72^{\circ}15'$ $c=50^{\circ}30'$ $A=78^{\circ}20'$ find side $a=?$.

Solution. Using the Maple module, we obtain the following results:

$b := evalf\left(\frac{\text{Pi}}{180}\left(72+\frac{15}{60}\right)\right);$ 1.261000385 $c := evalf\left(\frac{\text{Pi}}{180}\left(50+\frac{30}{60}\right)\right);$ 0.8813912725

$A := evalf\left(\frac{\text{Pi}}{180}\left(87+\frac{20}{60}\right)\right);$ 1.524254214 $a := \arccos(\cos(b)\cos(c)+\sin(b)\sin(c)\cos(A));$

1.340661682

$evalf(convert(a, degrees));$ 76.81425613 *degrees*

Answer: $a = 78^{\circ}48'51''$

Task 2. If the following are known in a spherical triangle:
$a = 78^{\circ}48'51''$ and $A = 87^{\circ}20'$, then calculate the modulus of the spherical triangle.
Solution. We use the Maple module. Then the corresponding calculations give the following results. Here we use the sine theorem.

$a := evalf\left(\frac{\mathrm{Pi}}{180}\left(78 + \frac{48}{60} + \frac{51}{3600}\right)\right);$ 1.375566706 ; $A := evalf\left(\frac{\mathrm{Pi}}{180}\left(87 + \frac{20}{60}\right)\right);$ 1.524254214

$M := \frac{\sin(a)}{\sin(A)};$ 0.9820666196

Answer: *M = 0.9820666196*

CONCLUSIONS

The study of these materials makes it possible to understand the spherical triangles and the calculation of some problems with the help of Maple modules helps students easily learn the "Basics of spherical geometry and trigonometry", which is important scientific and practical value for the specialty "Geodesy, cartography, land management". Basic mathematical knowledge obtained after studying this special section will help in the development of such subjects as "Geodesy", "Mathematical methods in geodetic measurements", "Satellite geodesy and spherical astronomy", "Geoinformation technologies".

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