ON TWO STEP SIMULATION MODELS FOR THE NUMERICAL SOLUTION OF SOME SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS

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ABSTRACT

This work presents a new set of two step finite difference schemes for the numerical solution of some second order ordinary differential equations using self-compensating exponential interpolation functions. The differential equations were subjected to some non-standard transformations and were combined with some interpolating functions. The technique produced new simulation models that can be used for the simulation of the dynamics of physical phenomena whose state equations can be represented by second order ordinary differential equations. The resulting schemes have been applied to some second order initial value problems and the results have been found to be suitable.

Keywords: Self-compensating exponential function, Non-standard method, Hybrid method, Interpolation function, Non-standard modeling rules, Standard finite difference method.

INTRODUCTION

The mathematical model for most physical phenomena results in non-linear second order ordinary differential equations which a lot of researchers have attempted to solve using linearization method. This generally approximates the original equations in such a way that it produces solutions that are close enough to the dynamics of the behaviour of the original phenomena, Lambert (1991).

However, such linearization is not always feasible. This being the case, researchers now results into various forms of approximations that are used to arrive at some desired results. In general, numerical models have been in the fore front of such approximations. Given enough information on the dynamics of an equation, a non-standard technique can be used to develop schemes that replicate the dynamics of the modeled equation. Such technique seek to replace the conventional denominator function in the standard numerical scheme by a generic function of the step size (h) that satisfy certain conditions and also renormalize the discretization function

$f(x, y, h)$ according to some rules proposed by Mickens (1994), Anguelov and Lubuma (2003), and Mickens (2000).
Standard finite difference methods have been found to be more valuable in finding solutions at close ranges and around special grid points. However looking holistically at the nature of the solution curves and behavioral patterns of the schemes, studies have shown that most of this standard algorithms produce solution curves that does not carry along the qualitative properties of the original dynamic equations, Mickens (2000), Obayomi and Oke (2015), Obayomi and Oke (2016).

This work will introduced an interpolant and use non-standard techniques to construct finite difference schemes that will be suitable for the numerical solution of some second order ordinary differential equations using self-compensating exponential interpolation functions. The research work will follow a mix of some standard techniques and non-standard method represented in the works of Mickens (1994).

DERIVATION OF THE SCHEMES

Let us assume an initial value problem of the form

$$y'' = f(x, y, y') \;,\; y(x_0) = \theta \qquad (1)$$

A non-standard model for the second derivative of a central difference scheme may be written as:

$$y'' \equiv \frac{y_{k+1} - 2y_k + y_{k-1}}{\varphi} \qquad (2)$$

where $\varphi(h) \rightarrow h^2 + 0(h^4)\; as\; h \rightarrow 0$

We assume a solution of equation (1) that can be represented by a combination of two reciprocating exponential components in the form:

$$y(x) = a_o + a_1 e^{\alpha x} + a_2 e^{-\beta x} \qquad (3)$$

Then, at points $= x_{n-1}$, $x = x_n$ and $x = x_{n+1}$ we have

$$y(x_{n-1}) = a_o + a_1 e^{\propto x_{n-1}} + a_2 e^{-\beta x_{n-1}} \qquad (4)$$

$$y(x_n) = a_o + a_1 e^{\propto x_n} + a_2 e^{-\beta x_n} \qquad (5)$$

$$y(x_{n+1}) = a_o + a_1 e^{\propto x_{n+1}} + a_2 e^{-\beta x_{n+1}} \qquad (6)$$

it therefore follows that :

$$y(x_{n+1}) - 2y(x_n) + y(x_{n+1}) =$$

$$= a_1(e^{\propto x_{n+1}} - 2e^{\propto x_n} + e^{\propto x_{n-1}}) + a_2(e^{-\beta x_{n+1}} - 2e^{-\beta x_n} + e^{-\beta x_{n-1}})$$

$x_{n-1} = a + (n-1)h, x_n = a + nh$ and $x_{n+1} = a + (n+1)h.$

Then $y(x_{n+1}) - 2y(x_n) + y(x_{n+1})$

$$= a_1.\left(e^{\alpha(a+nh)}(e^{\alpha h} - 2 + e^{\alpha h})\right) + a_2.e^{-\beta(a+nh)}(e^{-\beta h} - 2 + e^{-\beta h}) \qquad (7)$$

Let $y'(x) = f_n$, $y''(x) = f'_n$

it therefore follows that :

$y(x_{n+1}) - y(x_n) = y_{n+1} - y_n$

$$= a_1(e^{\alpha x_{n+1}} - e^{\alpha x_n}) + a_2(e^{-\beta x_{n+1}} - e^{-\beta x_n}) \qquad (8)$$

$$y(x_{n+1}) = y(x_n) + a_1(e^{\alpha x_{n+1}} - e^{\alpha x_n}) + a_2(e^{-\beta x_{n+1}} - e^{-\beta x_n}) \qquad (9)$$

$$y_{n+1} = y_n + a_1 e^{\alpha x_{n+1}} + a_2 e^{-\beta x_{n+1}} - a_1 e^{\alpha x_n} - a_2 e^{-\beta x_n}$$

$$y_{n+1} = y_n + a_1(e^{\alpha x_{n+1}} - e^{\alpha x_n}) + a_2(e^{-\beta x_{n+1}} - e^{-\beta x_n}) \qquad (10)$$

$$y(x) = a_0 + a_1 e^{\alpha x} + a_2 e^{-\beta x} \qquad (11)$$

$$y'(x) = \alpha a_1 e^{\alpha x} - \beta a_2 e^{-\beta x} \qquad (12)$$

$$y''(x) = \alpha^2 a_1 e^{\alpha x} + \beta^2 a_2 e^{-\beta x} \qquad (13)$$

From (1), we have:

$$a_0 = y(x) - a_1 e^{\alpha x} - a_2 e^{-\beta x} \qquad (14)$$

and from (2), we have:

$$a_1 = \frac{y'(x) + \beta a_2 e^{-\beta x}}{\alpha e^{\alpha x}} \qquad (15)$$

Putting (15) in (13), we have:

$$y''(x) = \alpha^2 e^{\alpha x}\left(\frac{y'(x) + \beta a_2 e^{-\beta x}}{\alpha e^{\alpha x}}\right) + \beta^2 a_2 e^{-\beta x}$$

$$y''(x) = \alpha(y'(x) + \beta a_2 e^{-\beta x} + \beta^2 a_2 e^{-\beta x}$$

$$y''(x) = \alpha y'(x) + a_2(\alpha\beta e^{-\beta x} + \beta^2 e^{-\beta x})$$

$$\therefore a_2 = \frac{y''(x) - \alpha y'(x)}{\alpha\beta e^{-\beta x} + \beta^2 e^{-\beta x}} \quad (16)$$

Putting (15) and (16) in (14), we have:

$$a_0 = y(x) - e^{\alpha x}\left(\frac{y'(x) + \beta a_2 e^{-\beta x}}{\alpha e^{\alpha x}}\right) - e^{-\beta x}\left(\frac{y''(x) - \alpha y'(x)}{\alpha\beta e^{-\beta x} + \beta^2 e^{-\beta x}}\right)$$

$$= y(x) - \frac{y'(x)}{\alpha} - \frac{\beta a_2 e^{-\beta x}}{\alpha} - \left(\frac{y''(x) - \alpha y'(x)}{\alpha\beta + \beta^2}\right)$$

$$a_0 = y(x) - y'(x)\alpha^{-1} - \beta e^{-\beta x}\alpha^{-1}\left(\frac{y''(x) - \alpha y'(x)}{\alpha\beta e^{-\beta x} + \beta^2 e^{-\beta x}}\right) - \left(\frac{y''(x) - \alpha y'(x)}{\alpha\beta + \beta^2}\right) \quad (17)$$

$$y(x_{n+1}) - 2y(x_n) + y(x_{n+1}) =$$

$$= a_1.\left(e^{\alpha(a+nh)}\left(e^{\alpha h} - 2 + e^{-\alpha h}\right)\right) + a_2.e^{-\beta(a+nh)}\left(e^{\beta h} - 2 + e^{-\beta h}\right)$$

$$= \left(\frac{y'(x) + \beta a_2 e^{-\beta x}}{\alpha}\right)\left(\left(e^{\alpha h} - 2 + e^{-\alpha h}\right)\right) + \left(\frac{y''(x) - \alpha y'(x)}{\alpha\beta + \beta^2}\right)\left(e^{\beta h} - 2 + e^{-\beta h}\right) \quad (18)$$

The Interpolating function must coincide with the theoretical solution at $x = x_{n-1}$, $x = x_n$ and $x = x_{n+1}$ such that

at $y(x_{n-1}) = y_{n-1}$, $y(x_n) = y_n$, $y(x_{n+1}) = y_{n+1}$

$$y(x_{n+1}) - 2y(x_n) + y(x_{n+1}) = y_{n+1} - 2y_n + y_{n-1}$$

$$y_{n+1} - 2y_n + y_{n-1} =$$

$$\left(\frac{y'(x) + \beta a_2 e^{-\beta x}}{\alpha}\right)\left(\left(e^{\alpha h} - 2 + e^{-\alpha h}\right)\right) + \left(\frac{y''(x) - \alpha y'(x)}{\alpha\beta + \beta^2}\right)\left(e^{\beta h} - 2 + e^{-\beta h}\right)$$

$$= \left(\frac{y'(x) + \beta\left(\frac{y'' - \alpha y'}{(\alpha\beta + \beta^2)e^{-\beta x}}\right)e^{-\beta x}}{\alpha}\right)\left(e^{\alpha h} - 2 + e^{-\alpha h}\right) + \left(\frac{y'' - \alpha y'}{\alpha\beta + \beta^2}\right)\left(e^{\beta h} - 2 + e^{-\beta h}\right) \quad (19)$$

$$y_{n+1} - 2y_n + y_{n-1} =$$

$$\left(\frac{y'}{\alpha} + \frac{y'' - \alpha y'}{\alpha(\alpha+\beta)}\right)\left(\left(e^{\alpha h} - 2 + e^{-\alpha h}\right)\right) + \left(\frac{y'' - \alpha y'}{\alpha\beta + \beta^2}\right)\left(e^{\beta h} - 2 + e^{-\beta h}\right) \quad (20)$$

The above relationship is true for any second order ordinary differential equation

whose solution can be approximated by the derived interpolating function.
The renormalized scheme can therefore be written in line with non-standard modeling rules as:

$$y_{n+1} - 2y_n + y_{n-1} = \varphi.\left(\frac{y'}{\alpha} + \frac{y'' - \alpha y'}{\alpha(\alpha+\beta)}\right)\left((e^{-\alpha h} - 2 + e^{-\alpha h})\right) +$$

$$\left(\frac{y'' - \alpha y'}{\alpha\beta + \beta^2}\right)(e^{-\beta h} - 2 + e^{-\beta h}) \qquad (21)$$

For equations without velocity components, we may assume $y' = 0$. Without loss of generality,

$$y_{n+1} = 2y_n - y_{n-1} + \varphi\left(\frac{y''}{\alpha(\alpha+\beta)}\right)\left((e^{-\alpha h} - 2 + e^{-\alpha h})\right) +$$

$$\left(\frac{y''}{\alpha\beta + \beta^2}\right)(e^{-\beta h} - 2 + e^{-\beta h}) \qquad (22)$$

Hence, we have a class of schemes that can be used for approximating second order ordinary differential equations. For the purpose of testing, we may choose $\varphi \in [0,1]$. This will be renormalized. Applying rule 2 of the non-standard modeling rules, we will obtain two new schemes by replacing h with a dynamic function of h as follows:

$\psi(h) \to h + 0(h^2)\ as\ h \to 0.$

$$\psi = \sin(h), \quad \psi = \frac{(e^{\lambda h} - 1)}{\lambda}, \psi = \sin(\propto h), \psi = h \quad \propto, \lambda \in \mathbb{R}$$

The Standard scheme developed in (20) will be named NEW h

The hybrid scheme obtained by substituting h for $\psi = \sin(h)$ and $\psi = \frac{(e^{\lambda h} - 1)}{\lambda}$

which will be named NEW SIN, NEW EXP respectively

CONVERGENCE, CONSISTENCY AND STABILITY OF THE NEW SCHEME

Convergence

$$y_{n+1} = 2y_n - y_{n-1} +$$

$$\varphi\left(\frac{y'}{\alpha} + \frac{y'' - \alpha y'}{\alpha(\alpha+\beta)}\right)\left((e^{\alpha h} - 2 + e^{-\alpha h})\right) + \left(\frac{y'' - \alpha y'}{\alpha\beta + \beta^2}\right)(e^{\beta h} - 2 + e^{-\beta h})$$

$$y_{n+1} = 2y_n - y_{n-1} + \varphi\left(\frac{f_n}{\alpha} + \frac{f_n' - \alpha f_n}{\alpha(\alpha+\beta)}\right)\left((e^{\alpha h} - 2 + e^{-\alpha h})\right) +$$

$$\left(\frac{f_n' - \alpha f_n}{\alpha\beta + \beta^2}\right)(e^{\beta h} - 2 + e^{-\beta h}) \qquad (23)$$

Let $A = \frac{\varphi\beta(e^{\alpha h}-2+e^{-\alpha h})}{\alpha(\alpha+\beta)}$, $B = \frac{\varphi(e^{\alpha h}-2+e^{-\alpha h})}{\alpha(\alpha+\beta)}$, $C = \frac{\varphi(e^{\beta h}-2+e^{-\beta h})}{\beta(\alpha+\beta)}$ and

$D = \frac{\varphi\alpha(e^{\beta h}-2+e^{-\beta h})}{\beta(\alpha+\beta)}$

$$y_{n+1} = 2y_n - y_{n-1} + Af_n + Bf_n' + Cf_n + Df_n'$$

$$y_{n+1} = 2y_n - y_{n-1} + (A+C)f_n + (B-D)f_n' \qquad (24)$$

For small h, $2y_n - y_{n-1} \cong y_n$

Simplifying, we obtain

$$y_{n+1} = y_n + (A+C)f_n + (B-D)f_n'$$

$$y_{n+1} = y_n + \mathrm{M}f_n + Nf_n'$$

The incremental function can be written as

$\phi(x_n, y_n, h) = \mathrm{M}f_n + Nf_n'$, Fatunla (1988) (25)

$$\phi(x_n, y_n, h) - \phi(x_n, y_n^*, h) = \mathrm{M}[f(x_n, y_n, h) - f(x_n, y_n^*, h)] + N[f'(x_n, y_n, h) - f'(x_n, y_n^*, h)]$$

$$= \mathrm{M}[f(x_n, y_n) - f(x_n, y_n^*)] + N[f'(x_n, y_n) - f'(x_n, y_n^*)]$$

$$= \mathrm{M}\left[\frac{\partial f(x_n, \bar{y})}{\partial y_n}(y_n - y_n^*)\right] + N\left[\frac{\partial f'(x_n, \bar{y})}{\partial y_n}(y_n - y_n^*)\right]$$

L1 = $\mathrm{SUP}_{(x_n, y_n)\in D} \frac{\partial f(x_n, \bar{y})}{\partial y_n}$ and L2 = $\mathrm{SUP}_{(x_n, y_n)\in D} \frac{\partial f'(x_n, \bar{y})}{\partial y_n}$

then

$$\phi(x_n, y_n, h) - \phi(x_n, y_n^*, h) = \mathrm{M}[L1(y_n - y_n^*)] + N[L2(y_n - y_n^*)] \qquad (26)$$

Let L = |M.L1+N.L2|

$\phi(x_n, y_n, h) - \phi(x_n, y_n^*, h) \le L|y_n - y_n^*|$.

This is the condition for convergence.

Consistency

$$y_{n+1} = y_n + Mf_n + Nf_n' \tag{27}$$

Then

$y_{n+1} = y_n + h\,\phi(x_n, y_n, h)$

when $h = 0$, $(e^{\beta h} - 2 + e^{-\beta h}) = 0, (e^{\propto h} - 2 + e^{-\propto h}) = 0$

$\Rightarrow y_{n+1} = y_n$ and the incremental function is identically zero when $h = 0$

$\Rightarrow \phi(x_n, y_n, 0) \equiv 0$

Stability

Consider the equation

$$y_{n+1} = y_n + \{M\}f_n(x_n, y_n) + \{N\}f_n'(x_n, y_n) \tag{28}$$

Let $p_{n+1} = p_n + \{M\}f_n(x_n, P_n) + \{N\}f_n'(x_n, P_n)$

$y_{n+1} - p_{n+1} = y_n - p_n + \{M\}[f_n(x_n, y_n) - f_n(x_n, P_n)] + \{N\}[f_n'(x_n, y_n) - f_n'(x_n, P_n)]$

$= y_n - p_n + M[\frac{\partial f(x_n, p_n)}{\partial p_n}(y_n - p_n)] + N[\frac{\partial f'(x_n, p_n)}{\partial p_n}(y_n - p_n)]$

L1 = $\mathrm{SUP}_{(x_n, y_n) \in D} \frac{\partial f(x_n, p_n)}{\partial p_n}$ and L2 = $\mathrm{SUP}_{(x_n, y_n) \in D} \frac{\partial f'(x_n, p_n)}{\partial p_n}$

$$y_{n+1} - p_{n+1} = y_n - p_n + M.L1(y_n - p_n) + N.L2(y_n - p_n) \tag{29}$$

$|y_{n+1} - p_{n+1}| = |y_n - p_n| + [M.L1 + N.L2]|(y_n - p_n)|$

Let L = $|1 + [M.L1 + N.L2]|$

$$|y_{n+1} - p_{n+1}| \leq L\,|y_n - p_n| \tag{30}$$

Let $y_0 = y(x_0) = \xi$ and $p_0 = p(x_0) = \xi^*$ then

$$|y_{n+1} - p_{n+1}| \leq K\,|\xi - \xi^*| \tag{31}$$

APPLICATION TO SOME SECOND ORDER INITIAL VALUE PROBLEMS

For the purpose of testing, we have selected a class of second order initial value problems for which the velocity components are assumed to be zero. We now applied equation (21) above to the problem.

Problem 1

$$y'' = 16 + 64y, \; y(0) = 1 \; y(0) = 1, y'(0) = 0 \quad (32)$$

Using

$$y_{n+1} = 2y_n - y_{n-1} +$$

$$\varphi\left(\frac{y''}{\alpha(\alpha+\beta)}\right)\left((e^{-\alpha h} - 2 + e^{-\alpha h})\right) + \left(\frac{y''}{\alpha\beta+\beta^2}\right)(e^{-\beta h} - 2 + e^{-\beta h})$$

The standard scheme is

$$y_{n+1} = 2y_n - y_{n-1} + \varphi\left(\frac{(16+64y)}{\alpha(\alpha+\beta)}\right)\left((e^{-\alpha h} - 2 + e^{-\alpha h})\right) +$$

$$\left(\frac{(16+64y)}{\alpha\beta+\beta^2}\right)(e^{-\beta h} - 2 + e^{-\beta h}) \quad (33)$$

The Analytic solution is $y = \frac{5}{8}e^{8x} + \frac{5}{8}e^{-8x} - \frac{1}{4}$, Zill and Cullen (2005)

The Nonstandard scheme that does not involve interpolating function can be obtained using non-standard modeling rules 2 and 3.

Thus

$$y'' = 16 + 64y$$

$$\frac{y_{k+1} - 2y_k + y_{k-1}}{\varphi} = 16 + 64y_k$$

where $\varphi(h) \rightarrow h^2 + 0(h^4) \; as \; h \rightarrow 0$ (34)

$$y_{k+1} = 2y_k + y_{k-1} + \varphi(h)(16 + 64y_k) \quad (35)$$

Problem 2

$$y'' = y, \; y(0) = 1, y'(0) = 1 \quad (36)$$

Using

$$y_{n+1} = 2y_n - y_{n-1} +$$

$$\varphi\left(\frac{y''}{\alpha(\alpha+\beta)}\right)\left((e^{-\propto h}-2+e^{-\propto h})\right)+(\frac{y''}{\alpha\beta+\beta^2})(e^{-\beta h}-2+e^{-\beta h})$$

The standard scheme is

$$y_{n+1}=2y_n-\ y_{n-1}\ +\varphi\left(\frac{y_n}{\alpha(\alpha+\beta)}\right)\left((e^{-\propto h}-2+e^{-\propto h})\right)+$$

$$(\frac{y_n}{\alpha\beta+\ \beta^2})(e^{-\beta h}-2+e^{-\beta h}) \qquad (37)$$

The Analytic solution is $y=\ e^x,\ \varphi\in[0,1]$, Zill and Cullen (2005)

The non-standard scheme that does not involve interpolating function can be obtained using rules 2 and 3 thus

$$y''=\ y \qquad (38)$$

$$\frac{y_{k+1}-2y_k+y_{k-1}}{\varphi}=y_k$$

where $\varphi(h)\ \ \rightarrow h^2+0(h^4)\ as\ h\rightarrow 0$

$$\mathrm{y}_{k+1}=2y_k+\ y_{k-1}+\varphi(h)(y_k) \qquad (39)$$

Problem 3

$$y''=\ -2-4y,\ \ y\left(\frac{\pi}{8}\right)=\frac{1}{2} \qquad (40)$$

Using $y_{n+1}=2y_n-\ y_{n-1}$ +

$$\varphi\left(\frac{y''}{\alpha(\alpha+\beta)}\right)\left((e^{-\propto h}-2+e^{-\propto h})\right)+(\frac{y''}{\alpha\beta+\beta^2})(e^{-\beta h}-2+e^{-\beta h})$$

The standard scheme is

$$y_{n+1}=2y_n-\ y_{n-1}\ +\varphi\left(\frac{-2-4y_n}{\alpha(\alpha+\beta)}\right)\left((e^{-\propto h}-2+e^{-\propto h})\right)+$$

$$(\frac{-2-\ 4y_n}{\alpha\beta+\beta^2})(e^{-\beta h}-2+e^{-\beta h}) \qquad (41)$$

The Analytic solution is $y=\ \sqrt{2}\,sin2x-\frac{1}{2},\ \varphi\in[0,1]$, Zill and Cullen (2005)

The non-standard scheme that does not involve interpolating function can be obtained using rules 2 and 3 thus

$$y'' = -2 - 4y$$

$$\frac{y_{k+1} - 2y_k + y_{k-1}}{\varphi} = -2 - 4y_k \quad (42)$$

where $\varphi(h) \rightarrow h^2 + 0(h^4)\ as\ h \rightarrow 0$

$$y_{k+1} = 2y_k + y_{k-1} + \varphi(h)(-2 - 4y_k) \quad (43)$$

TESTING AND EXPERIMENTATION

The schemes have been tested using various step sizes and the behaviours of the curves were consistent. We present below the 3D graphs for the schemes using step size $h = 0.01$

Problem 1: Schemes of $y'' = 16 + 64y, y(0) = 1, y'(0) = 0$

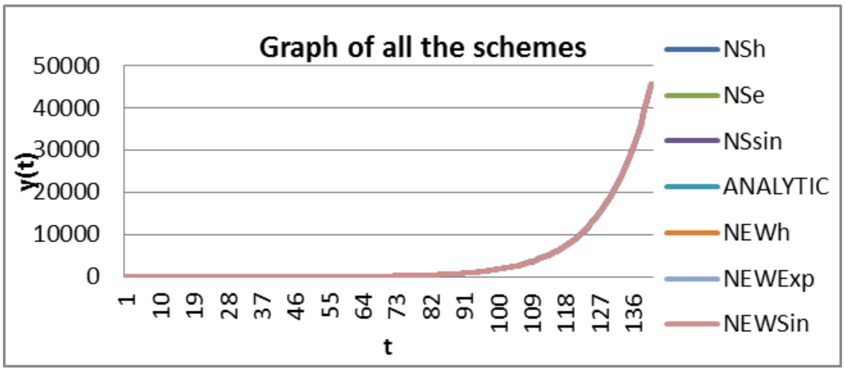

Fig 1:Solution curves for the standard, hybrid and non-standard schemes of Problem 1

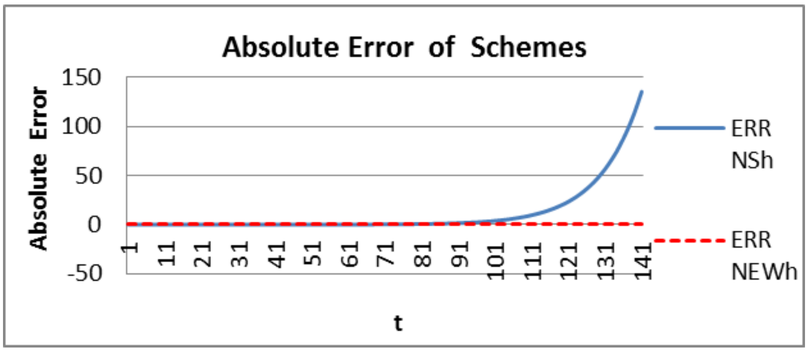

Fig 2a: Graph of Absolute Error for the hybrid and non-standard schemes of Problem 1 for NSh and NEWh.

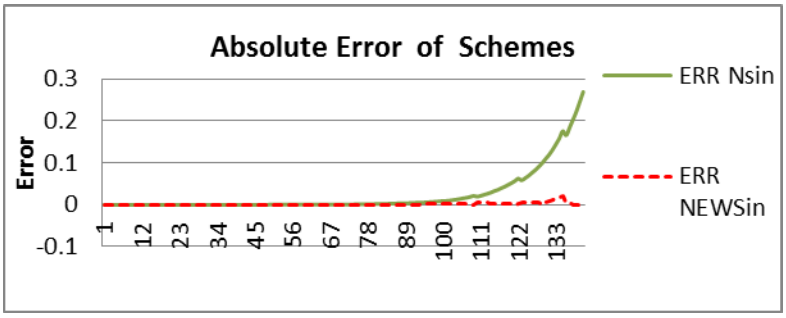

Fig 2b: Graph of Absolute Error for the hybrid and non-standard schemes of Problem 1 for NSin and NEWSin

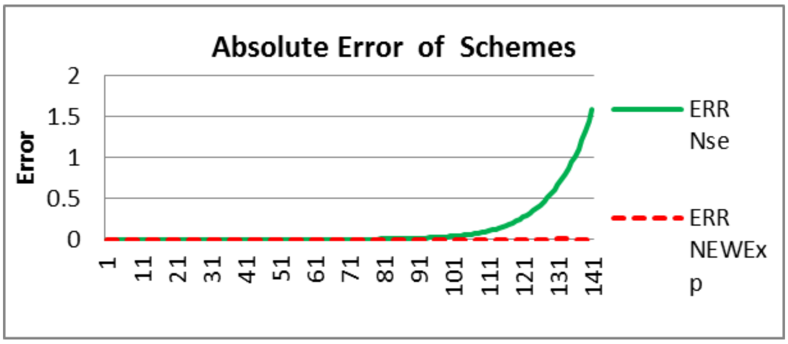

Fig 2c: Graph of Absolute Error for the hybrid and non-standard schemes of Problem 1 for NSe and NEWExp

Problem 2 Schemes of $y'' = y,\ y(0) = 1, y'(0) = 1$

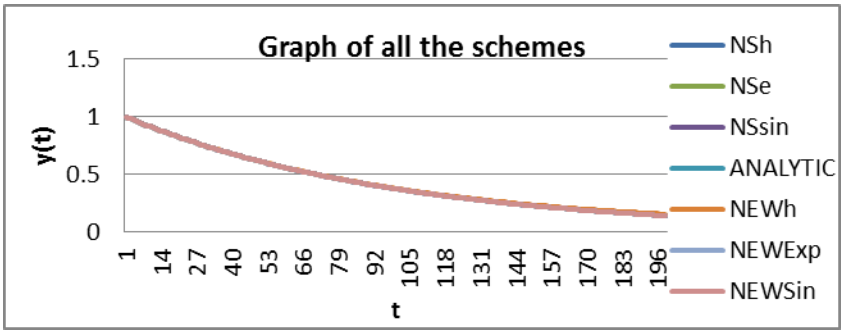

Fig 3:Solution curves for the standard, hybrid and non-standard schemes of Problem 2

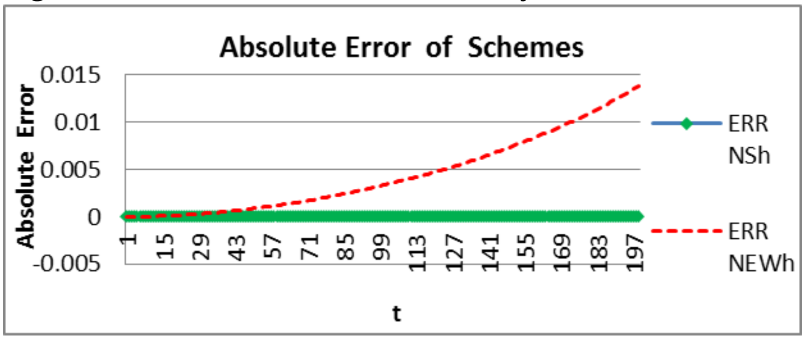

Fig 4a: Graph of Absolute Error for the hybrid and non-standard schemes of Problem 2 for NSh and NEWh

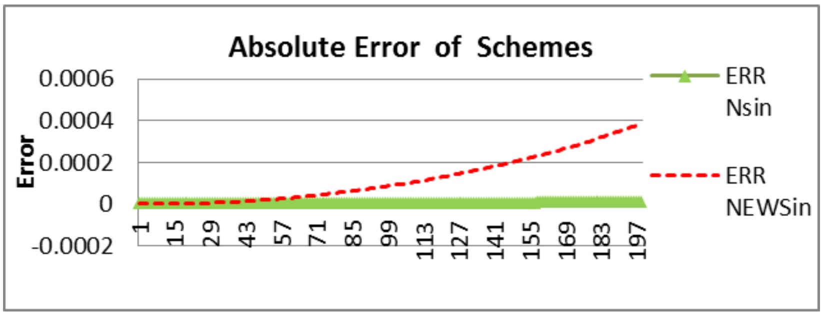

Fig 4b: Graph of Absolute Error for the hybrid and non-standard schemes of Problem 2 for NSin and NEWSin

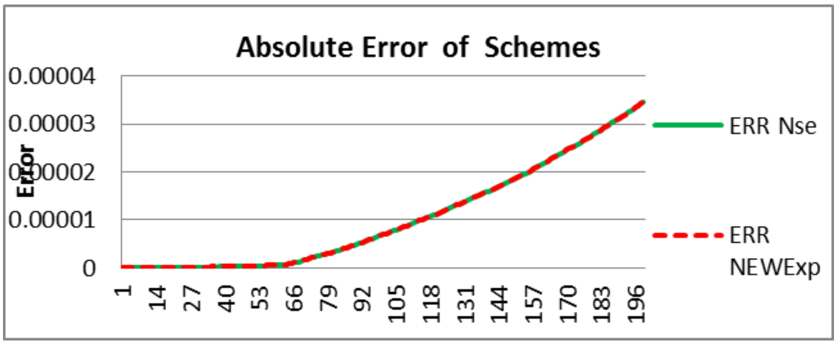

Fig 4c: Graph of Absolute Error for the hybrid and non-standard schemes of Problem 2 for NSe and NEWExp

Problem 3 schemes of $y'' = -2 - 4y, \; y\left(\frac{\pi}{8}\right) = \frac{1}{2}$

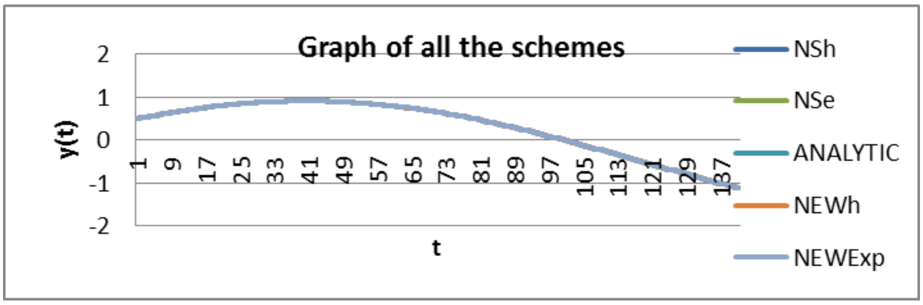

Fig 5:Solution curves for the standard, hybrid and non-standard schemes of Problem 3

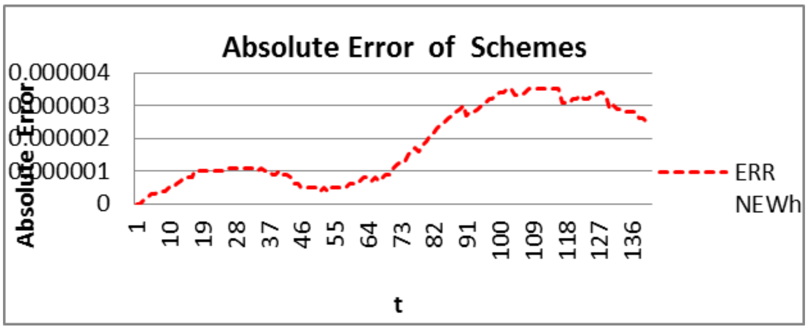

Fig 6a: Graph of Absolute Error for the hybrid and non-standard schemes of Problem 3 for NEWh

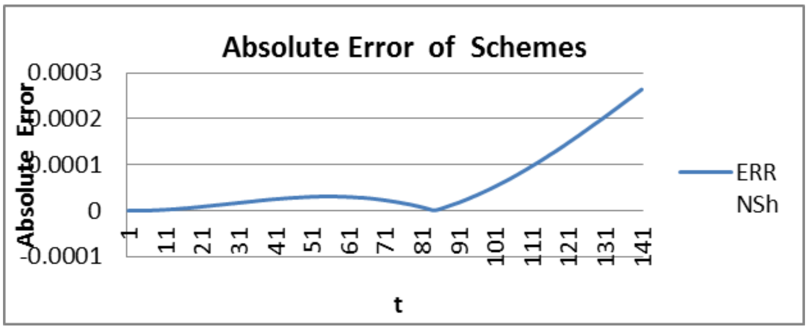

Fig 6b: Graph of Absolute Error for the hybrid and non-standard schemes of Problem 3 for NSh

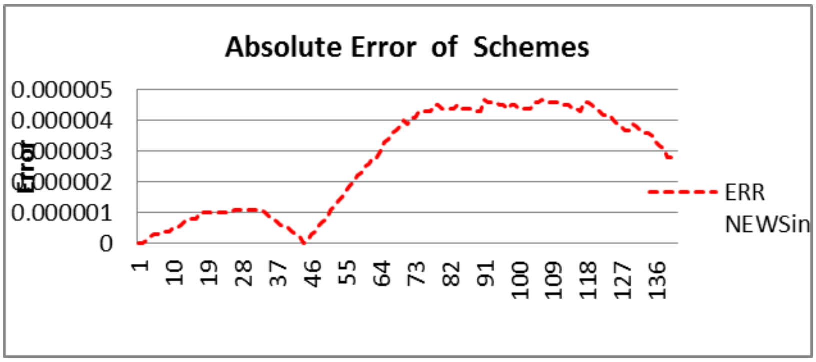

Fig 6c: Graph of Abs. Error for the hybrid and non-standard schemes of Problem 3 for NEWSin

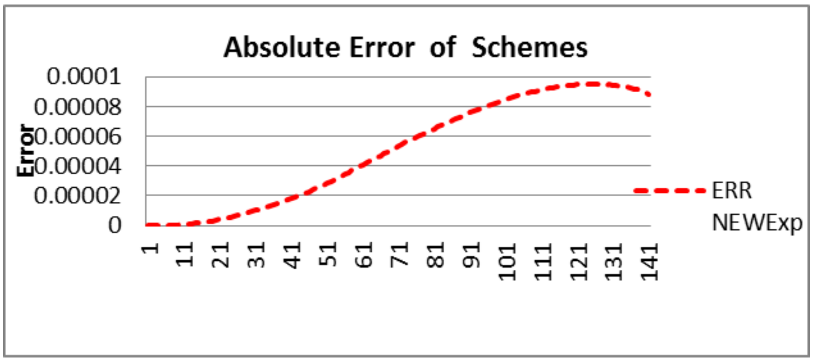

Fig 6d: Graph of Abs Error for the hybrid and non-standard schemes of Problem 3 for NEWExp

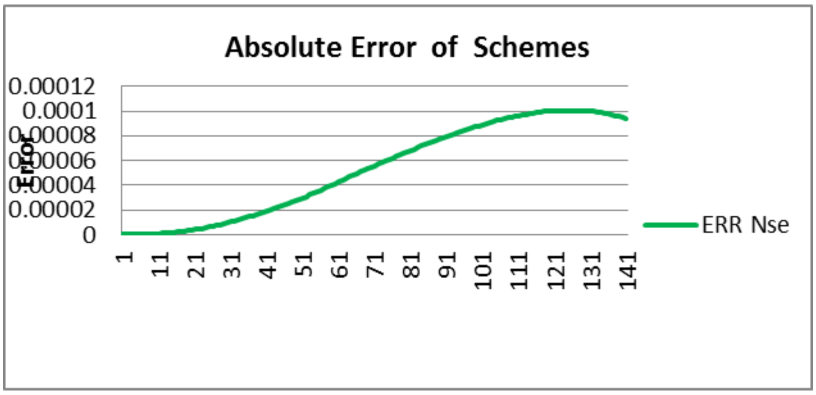

Fig 6e: Graph of Absolute Error for the hybrid and non-standard schemes of Problem 3 for NSe

CONCLUSION

All the graphs of the schemes followed the analytical solutions monotonically as shown in Figures 1, 3 and 5. The absolute error of deviation from actual solution is relatively small for the tested equations and intervals. It can be observed from the third problem the nonstandard scheme (NSh) possesses the highest absolute error of deviation from the Analytic solution as we can see in figure 6b. NSh was found to be very unsuitable for second order equation, because the Nonstandard model with step function Sin(h) fails. The choice of parameters for the nonstandard step functions does have impact on the error of deviation but this does not affect the monotonicity of solutions and dependence on initial values. Appropriate values for each parameter may be determined using the technique proposed by Angueluv and Lubuma in (2003)

This experiment has been performed on a very small class of differential equations. All the schemes have been found to be consistent with literature and compared favourably in all cases.

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