APPLICATION OF INEQUALITY

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ABSTRACT

The article shows the faithful inequality as a result of transformations, not violate it justice can be obtained requires inequality. As well as in this article considered applications of inequalities for finding the largest and the least values of the function.

Keywords: Expression, inequality, comparison, average arithmetic, geometric mean, the largest, the smallest.

INTRODUCTION, LITERATURE REVIEW AND DISCUSSION

Application of inequality for comparison of algebraic expressions
To prove justice inequalities of algebraic expressions uses a variety of methods:

1) bust of all possible cases; 2) the use of the laws of action of the algebraic expressions and the use of the properties of inequalities; 3) the use of already known inequalities; 4) the use of the method of the contrary and other.

Example-1. Prove that for any natural number of n inequality

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}<2 \qquad (1)$$

Solution. For any natural numbers $2\le k\le n$ we have $\frac{1}{k^2}<\frac{1}{k(k-1)}$, therefore

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}<1+\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\ldots+\frac{1}{n(n+1)}=1+\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\ldots+$$
$$\left(\frac{1}{n-1}-\frac{1}{n}\right)=2-\frac{1}{n}<2$$

Example-2. Prove that for any real numbers $a_1, a_2, \ldots, a_n$ and $b_1 b_2, \ldots, b_n$ performed inequality Koshi-Bunyakovsky

$$(a_1b_1+a_2b_2+\ldots+a_nb_n)^2\le\left(a_1^2+a_2^2+\ldots+a_n^2\right)\cdot\left(b_1^2+b_2^2+\ldots+b_n^2\right) \qquad (2)$$

Solution. At $a_1 = a_2 = \cdots = a_n = 0$ (2) there is a equality, when at least one of the numbers $a_1, a_2, \ldots, a_n$ nonzero, we have $a_1^2+a_2^2+\ldots+a_n^2>0$

We consider the square expression $ax^2 + bx + c$,

Where $a=a_1^2+a_2^2+\ldots+a_n^2$; $b=a_1b_1+a_2b_2+\ldots+a_nb_n$, $c=b_1^2+b_2^2+\ldots+b_n^2$

Note that $ax^2+2bx+c=(a_1x+b_1)^2+(a_2x+b_2)^2+\ldots+(a_nx+b_n)^2$

Since that $(a_ix + b_i)^2 \ge 0$ then $ax^2 + bx + c \ge 0$ for any actual positive number of x, so the discriminant this square expression of non-positive.

Therefore $b^2 \leq ac$, i.e.

$$(a_1b_1 + a_2b_2 + \cdots +, a_nb_n)^2 \leq (a_1^2 + a_2^2 + \cdots + a_n^2)(b_1^2 + b_2^2 + \cdots + b_n^2)$$

In (2) equality holds if and only if there exist the number of α and β, that the $\alpha^2 + \beta^2 \neq 0$ and for all $k = 1,2,\ldots,n$ performed equality $\alpha a_k + \beta b_k = 0$.

Inequality (2) briefly can be written in the form of

$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 \leq \sum_{i=1}^{n} a_i^2 \sum_{i=1}^{n} b_i^2$$

often employed inequality:

1. For any real numbers $a_1, a_2, \ldots, a_n$

$$\sqrt{a_1^2 + a_2^2 + \cdots + a_n^2} \leq |a_1| + |a_2| + \cdots + |a_n|.$$

2. For any real numbers $a_1, a_2, \ldots, a_n$ and $b_1 b_2, \ldots, b_n$

$$\sqrt{a_1^2 + a_2^2 + \cdots + a_n^2} - \sqrt{(b_1^2 + b_2^2 + \cdots + b_n^2} \leq |a_1 - b_1| + |a_2 - b_2| + \cdots + |a_n - b_n|$$

3. For any real numbers $a_1, a_2, \ldots, a_n$ and $b_1 b_2, \ldots, b_n$

$$\sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_n - b_n)^2} \leq \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2} + \sqrt{b_1^2 + b_2^2 + \cdots + b_n^2}$$

At $n = 2$ this inequality has the form

$$\sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2} \leq \sqrt{a_1^2 + a_2^2} + \sqrt{b_1^2 + b_2^2}$$

and it is possible to give the following geometric interpretation of:

If $A(a_1, a_2)$ and $B(b_1 b_2)$ two point plane, the inequality means that the length of the segment AB not more than the sum of the lengths of segments OA and OB.

4. For any non-negative numbers $a_1, a_2, \ldots, a_n$

$$\sqrt[n]{a_1 \cdot a_2 \cdot a_n} \leq \frac{a_1 + a_2 + \cdots + a_n}{n}$$

(inequality between the average arithmetic and middle geometric *n* numbers).

1. Application of inequality for finding the largest and the least values of the inequality on average arithmetic and middle geometric follows:

1. If the amount of positive numbers $a_1, a_2, \ldots, a_n$ is equal to a, the product value at $a_1 = a_2 = \cdots = a_n = \frac{a}{n}$ and this is the greatest value equal to $\left(\frac{a}{n}\right)^2$.

2. If the amount of positive numbers $a_1, a_2, \ldots, a_n$ is equal to b, the their sum takes the lowest value at $a_1 = a_2 = \cdots = a_n = \sqrt[n]{b}$ and this is the value equal to $\sqrt[n]{b}$

Example-3. Find the most value function

$$f(x) = (1 - x^3)(1 + 3x) \qquad \text{with } -\frac{1}{3} < x < 1$$

Solution. Transform function

$$f(x): \qquad (1 - x^3)(1 + 3x) = (1 - x)(1 - x)(1 - x)(1 + 3x)$$

since the $1 - x > 0 \quad 1 + 3x > 0$ with $-\frac{1}{3} < x < 1$ and

$$(1 - x) + (1 - x) + (1 - x) + (1 + 3x) = 4$$

the greatest value of the product to 1.

Example - 4. Find the most value function

$$y(x) = x^2\sqrt{4 - x} \qquad -2 \leq x \leq 2$$

Solution. Function of $y(x)$ and $\frac{1}{4}y^2(x)$ reach the largest values at the same value of the argument x (since the $y(x) \geq 0$). We represent the expression of $\frac{1}{4}y^2(x) = \frac{x^4}{4}(4-x^2)$ in the form of

$$\frac{x^4}{4}(4-x^2) = \frac{1}{2}x^2 \cdot \frac{1}{2}x^2 \cdot (4-x^2)$$

The amount of $\frac{1}{2}x^2 + \frac{1}{2}x^2 + (4-x^2)$ is equal 4, i.e. takes *a* constant value, therefore function $\frac{1}{4}y^2(x)$, which means, and function $y(x)$ reach the greatest value in

$\frac{1}{2}x^2 = (4-x^2)$, i.e. in $x = \pm\frac{2\sqrt{6}}{3}$

Consequently, $y\left(\frac{2\sqrt{6}}{3}\right) = \frac{16\sqrt{3}}{9}$ the greatest value function.

Example-5. Least squares method.

Find the lowest value of expression

$$A_n = \sum_{i=1}^{n}(y_i - (ax_i + b))^2. \qquad (*)$$

Solution. Least squares method is used in the treatment of measurements for anti-aliasing noise experiment: this method allows you to correct the random errors, inevitable arising in measurements, in the case when nature of according to the measured values from independent variable is set.

We consider the simplest situation when measured value of *y* depends linearly from one variable x. Let made n measurements and for values $x_1, x_2, \dots, x_n$ variable x obtained measurements $y_1, y_2, \dots, y_n$. Task is to conduct *a* direct $y = ax + b$, best adjacent to the points of $P(x_1y_1), P(x_2y_2), \dots, P(x_ny_n)$. The essence of the method least squares is that the best is considered that direct, for which takes the greatest value of the sum of squares deviations that is, the expression of

$$A_n = \sum_{i=1}^{n}(y_i - (ax_i + b))^2 \to min$$

How to find the value of *a* and *b*, minimizing the expression of *(*)*?

Assume that these values are there are, and parameter *a* is already found. To find *b* rewrite A_n in the form of

$$A_n = \sum_{i=1}^{n}(y_i - ax_i)^2 - \sum_{i=1}^{n}2b(y_i - ax_i) + \sum_{i=1}^{n}b^2 = nb^2 - 2b\sum_{i=1}^{n}(y_i - ax_i) + \sum_{i=1}^{n}(y_i - ax_i)^2$$

Let us consider A_n as *a* quadratic function of b. It reaches *a* minimum at

$$b = \frac{1}{n}\sum_{i=1}^{n}(y_i - ax_1) = \frac{1}{n}\sum_{i=1}^{n}y_i - \frac{a}{n}\sum_{i=1}^{n}x_i$$

Putting

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n}x_i, \qquad \bar{y} = \frac{1}{n}\sum_{i=1}^{n}y_i$$

You can write *a* $b = \overline{x} - a\overline{y}$, where to $A_n$we obtain

$$A_n = \sum_{i=1}^{n}((y_i - \overline{y}) - a(x_i - \overline{x}))^2 = a^2\sum_{i=1}^{n}(x_i - \overline{x})^2 - 2a\sum_{i=1}^{n}(y_i - \overline{y})(x_i - \overline{x}) + \sum_{i=1}^{n}(y_i - \overline{y})^2$$

We now consider A_n as a quadratic function from a. it is obvious reaches a minimum at

$$a = \frac{\sum_{i=1}^{n}(y_i - \overline{y})(x_i - \overline{x})}{\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

The best proved to line $y = ax + b$

$$a = \frac{\sum_{i=1}^{n}(y_i - \overline{y})(x_i - \overline{x})}{\sum_{i=1}^{n}(x_i - \overline{x})^2} \qquad b = \overline{y} - \frac{\overline{x}\sum_{i=1}^{n}(y_i - \overline{y})(x_i - \overline{x})}{\sum_{i=1}^{n}(x_i - \overline{x})^2}.$$

Example-6. If $A > 0$, $\alpha > 1$, $x \geq 0$ the expression of $x^\alpha - Ax$ take the lowest value at $x = (\frac{A}{\alpha})^{\frac{1}{1-\alpha}}$ equal to $(1-\alpha)(\frac{A}{\alpha})^{\frac{\alpha}{\alpha-1}}$

As $\alpha > 1$, then $(1+t)^\alpha \geq 1 + \alpha t$, $t \geq -1$, and equality holds only at $t = 0$. Believing here $1 + t = y$, we obtain

$y^\alpha \geq 1 + \alpha(y-1)$, $y^\alpha - \alpha y \geq 1 - \alpha$, $y \geq 0$, and equal sign there is only at $y = 1$.

Multiplying both sides of the inequality by c^α, we obtain

$$(cy)^\alpha - \alpha c^{\alpha-1}(cy) \geq (1-\alpha)c^\alpha, \qquad y \geq 0$$

Assuming

$$x = cy \quad \text{and} \quad ac^{\alpha-1} = A, \quad c = \left(\frac{A}{\alpha}\right)^{\frac{1}{x-1}},$$

We obtain

$$x^2 - Ax \geq (1-\alpha)c^\alpha = (1-\alpha)\left(\frac{A}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}$$

and equal sign there is only at

$$x = c = \left(\frac{A}{\alpha}\right)^{\frac{1}{\alpha-1}}$$

So, the expression of

$$x^\alpha - Ax, \quad \alpha > 1, \quad A > 0, \quad x \geq 0$$

take the lowest value at $x = \left(\frac{A}{\alpha}\right)^{\frac{1}{1-\alpha}}$ equal to $(1-\alpha)\left(\frac{A}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}$. The problem is solved.

Note that the expression of

$$Ax - x^\alpha = -(x^\alpha - Ax)$$

where $\alpha > 1$, $A > 0$, $x \geq 0$ takes the greatest value at the point

$x = \left(\frac{A}{\alpha}\right)^{\frac{1}{\alpha-1}}$, equal $(\alpha-1)\left(\frac{A}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}$

Example-7. Find the most value function.

$$x^{\alpha} - 0{,}5x^{10}$$

Solution. To find the most value function $x^{\alpha} - 0{,}5x^{10}$, we set $y = x^6$. It is clear that $y \geq 0$. Function $y - 0{,}5 \cdot y^{\frac{10}{5}} = 0{,}5(2y - y^2)$ takes the greatest value equal

$$0{,}5(2-1)\left(\frac{2}{2}\right)^{\frac{2}{2-1}} = 0{,}5$$

Example-8. Find the lowest value of expression at any $x > 1,\ y > 1$.

$$\frac{x^2}{y-1} + \frac{y^2}{x-1}$$

Solution. By virtue of inequality between the average and middle geometric have

$$\frac{x^2}{y-1} + \frac{y^2}{x-1} \geq 2\sqrt{\frac{x^2}{y-1} \cdot \frac{y^2}{x-1}} = 2\frac{y}{\sqrt{y-1}} \cdot \frac{x}{\sqrt{x-1}}$$

It remains to note that for *a>1*, we have

$$\frac{a}{\sqrt{a-1}} = \frac{a-1+1}{\sqrt{a-1}} = \frac{a-1}{\sqrt{a-1}} + \frac{a}{\sqrt{a-1}} = \sqrt{a-1} + \frac{a}{\sqrt{a-1}} \geq 2$$

Thus, the

$$min\left\{\frac{x^2}{y-1} + \frac{y^2}{x-1}\right\} = 8.$$

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