THE ONE-WAY DEFICIT FOR A CLASS OF TWO-QUBIT STATES UNDER LOCAL NONDISSIPATIVE CHANNELS*

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ABSTRACT

In this paper, we analytically evaluate the one-way information deficit (OWID) for the X states with four parameters under local nondissipative channels, which includes phase flip channel, bit flip channel and bit-phase flip channel. We find that the OWID under the phase flip channel is positive, and greater than the OWID under the bit-phase flip channel, the latter is also greater than the OWID under the bit flip channel.

Keywords: One-way information deficit; phase flip channel; bit flip channel; bit-phase flip channel.

I. INTRODUCTION

Quantum deficit is one nonclassical correlation besides entanglement and quantum discord, which originates in question how to use nonlocal operation to extract work from a correlated system coupled to heat bath. Oppenheim et al.[1] defined the work deficit

$$\Delta \equiv W_t - W_l, \tag{1}$$

where W_t is the information of the whole system and W_l is the localizable information.Recently, Ollivier et al.[2] give the definition of the one-way information deficit (OWID) by Von Neumann measurement on one side,

$$\Delta^{\rightarrow}\left(\rho^{ab}\right)=\min_{\Pi_k} S\left(\sum_k \Pi_k \rho^{ab} \Pi_k\right)-S\left(\rho^{ab}\right). \tag{2}$$

The state of two-qubit system under local environment can be represented with a completely positive trace-preserving map, which can be written in the operator-sum representation[3]:

$$\varepsilon(\rho)=\sum_{i,j}\left(E_i \otimes E_j\right)\rho\left(E_i \otimes E_j\right)^{+},$$

where $E_i^{(k)}(k=A,B)$ is the Kraus operator representing the channel A or B, and $\sum_i E_i^{(k)} E_i^{(k)+}=I$. In this paper, we will discuss three Markovian noise channels: phase flip channel, bit flip channel and bit-phase flip channel, which are represented with two Kraus operators as follows, respectively,

$$E_0^{p-f}=\sqrt{1-\frac{p}{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, E_1^{p-f}=\sqrt{\frac{p}{2}}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \tag{3}$$

$$E_0^{b-f}=\sqrt{1-\frac{p}{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, E_1^{b-f}=\sqrt{\frac{p}{2}}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \tag{4}$$

$$E_0^{bp-f}=\sqrt{1-\frac{p}{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, E_1^{bp-f}=\sqrt{\frac{p}{2}}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}. \tag{5}$$

where p is the probability the noise act on the qubit.

Up to now, there are many achievements about one-way deficit and quantum discord, which can be seen in their literature[4-8]. In 2013, Yao-Kun Wang et al.[8] analytically evaluated the OWID for Bell-diagonal state and X states with four parameters, described the dynamic behavior of the OWID under phase flip channel. In this paper, we intend to evaluate the OWID of X states with four parameters different channels.

This paper is organized as follows. In section II, we will evaluate the dynamics of OWID for X States with four parameters under bit flip channel in detail. In section III, we will calculate the dynamics of OWID for X States with four parameters under phase flip channel and bit-phase flip channel analogously, and make a comparasion. A brief conclusion is given in section IV.

II. The OWID for X States with four parameters under bit flip channel

We consider the following 4-parameter quantum system,

$$\rho^{ab}=\frac{1}{4}\begin{pmatrix} 1+s+c_3 & 0 & 0 & c_1-c_2 \\ 0 & 1-s-c_3 & c_1+c_2 & 0 \\ 0 & c_1+c_2 & 1+s-c_3 & 0 \\ c_1-c_2 & 0 & 0 & 1-s+c_3 \end{pmatrix}. \quad (6)$$

where s, c_1, c_2, c_3 are all real numbers and

$$|c_1|<|c_2|<|c_3|,\ 0<|s|<1-|c_3|, \quad (7)$$

Then under the bit flip channel (4) we have

$$\varepsilon(\rho^{ab})=\frac{1}{4}\left(I\otimes I+(1-p)I\otimes s\cdot\sigma_3+c_1\sigma_1\otimes\sigma_1+(1-p)^2c_2\sigma_2\otimes\sigma_2+(1-p)^2c_3\sigma_3\otimes\sigma_3\right), \quad (8)$$

it is easy to see that $\varepsilon(\rho^{ab})$ is also X state and satisfies conditions in Eqs.(6) and (7).

The eigenvalues of state in Eq.(8) are

$$\lambda_1=\frac{1}{4}\left(1-(1-p)^2c_3-\sqrt{c_2^2+2c_1c_2(1-p)^2+c_1^2(1-p)^4+s(1-p)^2}\right),$$

$$\lambda_2=\frac{1}{4}\left(1-(1-p)^2c_3+\sqrt{c_2^2+2c_1c_2(1-p)^2+c_1^2(1-p)^4+s(1-p)^2}\right),$$

$$\lambda_3=\frac{1}{4}\left(1-(1-p)^2c_3-\sqrt{c_2^2-2c_1c_2(1-p)^2+c_1^2(1-p)^4+s(1-p)^2}\right),$$

$$\lambda_4=\frac{1}{4}\left(1-(1-p)^2c_3+\sqrt{c_2^2-2c_1c_2(1-p)^2+c_1^2(1-p)^4+s(1-p)^2}\right).$$

Then its entropy is given by

$$S(\rho^{ab})=-\sum_{i=1}^{4}\lambda_i\log\lambda_i$$

$$=2-\frac{1}{4}\left[\left(1-(1-p)^2c_3-\sqrt{c_2^2+2c_1c_2(1-p)^2+c_1^2(1-p)^4+s^2(1-p)^2}\right)\right.$$

$$\cdot\log\left(1-(1-p)^2 c_3-\sqrt{c_2^2+2c_1c_2(1-p)^2+c_1^2(1-p)^4+s^2(1-p)^2}\right)$$

$$+\left(1-(1-p)^2 c_3+\sqrt{c_2^2+2c_1c_2(1-p)^2+c_1^2(1-p)^4+s(1-p)^2}\right)$$

$$\cdot\log\left(1-(1-p)^2 c_3+\sqrt{c_2^2+2c_1c_2(1-p)^2+c_1^2(1-p)^4+s(1-p)^2}\right)$$

$$+\left(1-(1-p)^2 c_3-\sqrt{c_2^2-2c_1c_2(1-p)^2+c_1^2(1-p)^4+s(1-p)^2}\right)$$

$$\cdot\log\left(1-(1-p)^2 c_3-\sqrt{c_2^2-2c_1c_2(1-p)^2+c_1^2(1-p)^4+s(1-p)^2}\right)$$

$$+\left(1-(1-p)^2 c_3+\sqrt{c_2^2-2c_1c_2(1-p)^2+c_1^2(1-p)^4+s(1-p)^2}\right)$$

$$\cdot\log\left(1-(1-p)^2 c_3+\sqrt{c_2^2-2c_1c_2(1-p)^2+c_1^2(1-p)^4+s(1-p)^2}\right)$$

. (9)

Next, we discuss the OWID of the X state $\varepsilon(\rho^{ab})$ in Eq.(8). Let $\{\Pi_k=|k\rangle\langle k|, k=0,1\}$ be the local measurement for the particle b along the computational base $\{|k\rangle\}$, then any Von Neumann measurement for the particle b can be written as

$$B_k=V\Pi_k V^+, k=0,1, \tag{10}$$

for some unitary $V\in U(2)$. Since any unitary V can be described as

$$V=tI+i\vec{y}\cdot\vec{\sigma}, \tag{11}$$

with $t\in R, \vec{y}=(y_1,y_2,y_3)$ and $t^2+y_1^2+y_2^2+y_3^2=1$. After the measurement B_k, the state ρ^{ab} will be changed to the ensemble $\{\rho_k, p_k\}$ with

$$\rho_k=(I\otimes B_k)\rho(I\otimes B_k); \qquad p_k=tr(I\otimes B_k)\rho(I\otimes B_k). \tag{12}$$

To evaluate ρ_k and p_k, we write

$$p_k\rho_k=(I\otimes B_k)\rho(I\otimes B_k)$$

$$=\frac{1}{4}(I\otimes V)(I\otimes\Pi_k)(I\otimes V^+)$$

$$\cdot\left(I\otimes I+(1-p)I\otimes s\cdot\sigma_3+c_1\sigma_1\otimes\sigma_1+(1-p)^2c_2\sigma_2\otimes\sigma_2+(1-p)^2c_3\sigma_3\otimes\sigma_3\right)$$

$$\cdot(I\otimes V)(I\otimes\Pi_k)(I\otimes V^+)$$

$$=\frac{1}{4}I\otimes V\Pi_kV^+ +\frac{(1-p)s}{4}I\otimes V\Pi_kV^+\sigma_3V\Pi_kV^+ +\frac{1}{4}\sigma_1\otimes V\Pi_kV^+\sigma_1V\Pi_kV^+$$

$$+\frac{c_2(1-p)^2}{4}\sigma_2\otimes V\Pi_k V^+\sigma_2 V\Pi_k V^+ + \frac{c_3(1-p)^2}{4}\sigma_3\otimes V\Pi_k V^+\sigma_3 V\Pi_k V^+ . \quad (13)$$

By the following relations in [7]

$$V^+\sigma_1 V = (t^2+y_1^2-y_2^2-y_3^2)\sigma_1 + 2(ty_3+y_1y_2)\sigma_2 + 2(-ty_2+y_1y_3)\sigma_3, \quad (14)$$

$$V^+\sigma_2 V = 2(-ty_3+y_1y_2)\sigma_1 + (t^2+y_2^2-y_1^2-y_3^2)\sigma_2 + 2(ty_1+y_2y_3)\sigma_3, \quad (15)$$

$$V^+\sigma_3 V = 2(ty_2+y_1y_3)\sigma_1 + 2(-ty_1+y_2y_3)\sigma_2 + (t^2+y_3^2-y_1^2-y_2^2)\sigma_3, \quad (16)$$

and

$$\Pi_0\sigma_3\Pi_0 = \Pi_0,\ \Pi_1\sigma_3\Pi_1 = -\Pi_1,\ \Pi_j\sigma_3\Pi_j = 0, \quad (17)$$

for $j=0,1, k=1,2$, we obtain that

$$p_0\rho_0 = \frac{1}{4}\left(I + s(1-p)z_3 I + c_1z_1\sigma_1 + c_2(1-p)^2 z_2\sigma_2 + c_3(1-p)^2 z_3\sigma_3\right)\otimes V\Pi_k V^+, \quad (18)$$

$$p_1\rho_1 = \frac{1}{4}\left(I - s(1-p)z_3 I - c_1z_1\sigma_1 - c_2(1-p)^2 z_2\sigma_2 - c_3(1-p)^2 z_3\sigma_3\right)\otimes V\Pi_k V^+, \quad (19)$$

where

$$z_1 = 2(-ty_2+y_1y_3),\ z_3 = 2(ty_1+y_2y_3),\ z_3 = (t^2+y_3^2-y_1^2-y_2^2). \quad (20)$$

It can be directly verified that

$$z_1^2+z_2^2+z_2^3 = 1. \quad (21)$$

Let $M = s(1-p)z_3 I + c_1z_1\sigma_1 + c_2(1-p)^2 z_2\sigma_2 + c_3(1-p)^2 z_3\sigma_3$, then

$$p_0\rho_0 = \frac{1}{4}(I+M)\otimes V\Pi_k V^+,\ p_1\rho_1 = \frac{1}{4}(I-M)\otimes V\Pi_k V^+. \quad (22)$$

By the Eq.(20) in [8], the eigenvalues of $\frac{1}{4}(I+M)$ and $\frac{1}{4}(I-M)$ are $\frac{1}{4}(1+\phi-\theta)$, $\frac{1}{4}(1+\phi+\theta)$, and $\frac{1}{4}(1-\phi-\theta)$, $\frac{1}{4}(1-\phi+\theta)$, where

$$\phi = s(1-p)z_3, \theta = \sqrt{|c_1z_1|^2 + \left|c_2z_2(1-p)^2\right|^2 + \left|c_3z_3(1-p)^2\right|^2}, \quad (23)$$

then the entropy of $\sum_i \Pi_k\rho^{ab}\Pi_k$ is

$$\begin{aligned} S\left(\sum_i \Pi_k\rho^{ab}\Pi_k\right) &= f(\phi,\theta) \\ &= -\sum_{i=5}^{8}\lambda_i\log\lambda_i \\ &= 2-\frac{1}{4}\big[(1+\phi-\theta)\log(1+\phi-\theta)+(1+\phi+\theta)\log(1+\phi+\theta) \\ &\quad +(1-\phi-\theta)\log(1-\phi-\theta)+(1-\phi+\theta)\log(1-\phi+\theta)\big]. \end{aligned} \quad (24)$$

By use of the domain of logarithmic function $f(\phi,\theta)$ in Eq.(7), we obtain the range of θ and ϕ as follows,

$$0 \le |c_1| \le \theta \le |c_3| \le 1, -1 < \phi < 1. \quad (25)$$

We can verify that $f(-\phi,\theta)=f(\phi,\theta)$, the graph of $f(\phi,\theta)$ is symmetrical with respect to the θ-axis; moreover,

$$\frac{\partial f}{\partial \theta}=-\frac{1}{4}\log\left[\frac{(1+\theta)^2-\phi^2}{(1-\theta)^2-\phi^2}\right]<0,\ \ 0<\theta<1,$$

$$\frac{\partial f}{\partial \phi}=-\frac{1}{4}\log\left[\frac{(1+\phi)^2-\theta^2}{(1-\phi)^2-\theta^2}\right]<0,\ \ 0<\phi<1,$$

then $f(\phi,\theta)$ is a monotonic decreasing function. When $\theta=|c_3|$, by Eqs.(7), (22) and (24), we can obtain that $\phi=|s|$.

By Eq.(7), the projection of $f(\phi,\theta)$ on the plane $\phi\circ\theta$ is a symmetrical rectangle with respect to the θ-axis, and by use of the monotonicity of $f(\phi,\theta)$ in the positive directionof ϕ and θ, $f(\phi,\theta)$ can obtain its minimum at the point $(|s|,|c_3|)$, and the minimum of $f(\phi,\theta)$ is given by

$$\min S\left(\sum_i \Pi_k\rho^{ab}\Pi_k\right)=2-\frac{1}{4}[(1+s-c_3)\log(1+s-c_3)+(1+s+c_3)\log(1+s+c_3) \\ +(1-s-c_3)\log(1-s-c_3)+(1-s+c_3)\log(1-s+c_3)]. \quad (26)$$

By Eqs.(9) and (26), the OWID of the state in Eq.(8) is given by

$$\begin{aligned}\Delta^{\rightarrow}\left(\varepsilon\left(\rho^{ab}\right)\right)=\frac{1}{4}\Big[&\left(1-c_3(1-p)^2\right)-\sqrt{c_1^2+c_1c_2(1-p)^2+c_2^2(1-p)^4+s^2(1-p)^2}\\
&\cdot\log\left(1-c_3(1-p)^2\right)-\sqrt{c_1^2+c_1c_2(1-p)^2+c_2^2(1-p)^4+s^2(1-p)^2}\\
&+\left(1-c_3(1-p)^2\right)+\sqrt{c_1^2+c_1c_2(1-p)^2+c_2^2(1-p)^4+s^2(1-p)^2}\\
&\cdot\log\left(1-c_3(1-p)^2\right)+\sqrt{c_1^2+c_1c_2(1-p)^2+c_2^2(1-p)^4+s^2(1-p)^2}\\
&+\left(1+c_3(1-p)^2\right)-\sqrt{c_1^2+c_1c_2(1-p)^2+c_2^2(1-p)^4+s^2(1-p)^2}\\
&\cdot\log\left(1+c_3(1-p)^2\right)-\sqrt{c_1^2+c_1c_2(1-p)^2+c_2^2(1-p)^4+s^2(1-p)^2}\\
&+\left(1+c_3(1-p)^2\right)+\sqrt{c_1^2+c_1c_2(1-p)^2+c_2^2(1-p)^4+s^2(1-p)^2}\\
&\cdot\log\left(1+c_3(1-p)^2\right)+\sqrt{c_1^2+c_1c_2(1-p)^2+c_2^2(1-p)^4+s^2(1-p)^2}\Big]\\
&-\frac{1}{4}\left[(1+s-c_3)\log(1+s-c_3)+(1+s+c_3)\log(1+s+c_3)\right.\\
&\left.+(1-s-c_3)\log(1-s-c_3)+(1-s+c_3)\log(1-s+c_3)\right].\end{aligned} \quad (27)$$

As an example, for $s=0.3, c_1=0.3, c_2=0.4, c_3=0.56$, the dynamic behavior of OWID of the state under the bit flip channel is depicted in Fig.1. We can find that the OWID of the bit flip channel dwindle with increasing probability until $p = 0.4$, then the function of OWID start increasing.

III. The OWIDs under phase flip channel and bit-phase flip channel

In this section, we consider the OWIDs of the state in Eqs.(6) under phase flip channel and bit-phase flip channel.

Firstly, under the phase flip channel (3) the state in Eqs.(6) will change to the following state

$$\varepsilon(\rho^{ab}) = \frac{1}{4}\left(I \otimes I + I \otimes s \cdot \sigma_3 + (1-p)^2 c_1 \sigma_1 \otimes \sigma_1 + (1-p)^2 c_2 \sigma_2 \otimes \sigma_2 + c_3 \sigma_3 \otimes \sigma_3\right), \tag{28}$$

it is easy to see that ε(ρ ab) is also *X* state and satisfies conditions in Eqs.(6) and (7). Its OWID has been calculated by Yao-Kun Wang [8] as follows

$$\begin{aligned}\Delta^{\rightarrow}\left(\varepsilon\left(\rho^{ab}\right)\right) = \frac{1}{4}\Big[&\left(1-c_3+\sqrt{s^2+(1-p)^4(c_1+c_2)^2}\right)\log\left(1-c_3+\sqrt{s^2+(1-p)^4(c_1+c_2)^2}\right)\\ &+\left(1-c_3-\sqrt{s^2+(1-p)^4(c_1+c_2)^2}\right)\log\left(1-c_3+\sqrt{s^2+(1-p)^4(c_1+c_2)^2}\right)\\ &+\left(1+c_3+\sqrt{s^2+(1-p)^4(c_1+c_2)^2}\right)\log\left(1+c_3+\sqrt{s^2+(1-p)^4(c_1+c_2)^2}\right)\\ &+\left(1+c_3-\sqrt{s^2+(1-p)^4(c_1+c_2)^2}\right)\log\left(1+c_3-\sqrt{s^2+(1-p)^4(c_1+c_2)^2}\right)\Big]\\ &-\frac{1}{4}\Big[(1+s-c_3)\log(1+s-c_3)+(1+s+c_3)\log(1+s+c_3)\\ &+(1-s-c_3)\log(1-s-c_3)+(1-s+c_3)\log(1-s+c_3)\Big].\end{aligned} \tag{29}$$

As an example, for $s = 0.3, c_1 = 0.3, c_2 = 0.4, c_3 = 0.56$, the dynamic behavior of OWID of the state under the phase flip channel is depicted in Fig.2. We find that the OWID under the phase flip channel is a monotonic decreasing function, and when $p = 1$, OWID equals zero.

Next the OWID of the state in Eq.(6) under bit-phase flip channel. Under the phase flip channel (5) the state in Eq.(6) will change to the following state

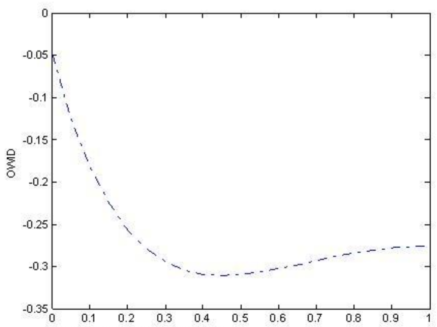
$$\varepsilon(\rho^{ab}) = \frac{1}{4}\left(I \otimes I + I \otimes s \cdot \sigma_3 + (1-p)^2 c_1 \sigma_1 \otimes \sigma_1 + c_2 \sigma_2 \otimes \sigma_2 + (1-p)^2 c_3 \sigma_3 \otimes \sigma_3\right) \cdot \tag{30}$$

Similar to the above calculation, the OWID of the state in Eq.(30) is given by

$$\begin{aligned}\Delta^{\rightarrow}\left(\varepsilon\left(\rho^{ab}\right)\right)=\frac{1}{4}\Big[&\left(1-c_3(1-p)^2\right)-\sqrt{c_1^2+c_1c_2(1-p)^2+c_2^2(1-p)^4+s^2(1-p)^2}\\&\cdot\log\left(1-c_3(1-p)^2\right)-\sqrt{c_1^2+c_1c_2(1-p)^2+c_2^2(1-p)^4+s^2(1-p)^2}\\&+\left(1-c_3(1-p)^2\right)+\sqrt{c_1^2+c_1c_2(1-p)^2+c_2^2(1-p)^4+s^2(1-p)^2}\\&\cdot\log\left(1-c_3(1-p)^2\right)+\sqrt{c_1^2+c_1c_2(1-p)^2+c_2^2(1-p)^4+s^2(1-p)^2}\\&+\left(1+c_3(1-p)^2\right)-\sqrt{c_1^2+c_1c_2(1-p)^2+c_2^2(1-p)^4+s^2(1-p)^2}\\&\cdot\log\left(1+c_3(1-p)^2\right)-\sqrt{c_1^2+c_1c_2(1-p)^2+c_2^2(1-p)^4+s^2(1-p)^2}\\&+\left(1+c_3(1-p)^2\right)+\sqrt{c_1^2+c_1c_2(1-p)^2+c_2^2(1-p)^4+s^2(1-p)^2}\\&\cdot\log\left(1+c_3(1-p)^2\right)+\sqrt{c_1^2+c_1c_2(1-p)^2+c_2^2(1-p)^4+s^2(1-p)^2}\Big]\\&-\frac{1}{4}\big[(1+s-c_3)\log(1+s-c_3)+(1+s+c_3)\log(1+s+c_3)\\&+(1-s-c_3)\log(1-s-c_3)+(1-s+c_3)\log(1-s+c_3)\big].\end{aligned} \tag{30}$$

As an example, for $s=0.3, c_1=0.3, c_2=0.4, c_3=0.56$, the dynamic behavior of OWID of the state under the bit-phase flip channel is depicted in Fig.3. We find that the figure of OWID under the bit-phase flip channel is similar to the figure of OWID under the bit flip channel, and the OWID of the bit-phase flip channel dwindle with increasing probability until $p=0.4$, then the function of OWID start increasing.

Fig. 4 displays the dynamic behavior of OWID under the phase flip channel(solid line), bit flip channel(dotted-dash line) and bit-phase flip channel(dashed line) simultaneously. From Fig.4 we can find that the OWID under the phase flip channel is always positive, and the OWIDs under the bit flip channel and bit-phase flip channel are always negative. And the OWID under the phase flip channel is greater than the OWID under the bit-phase channel, the latter is also greater than the OWID under the bit flip channel.



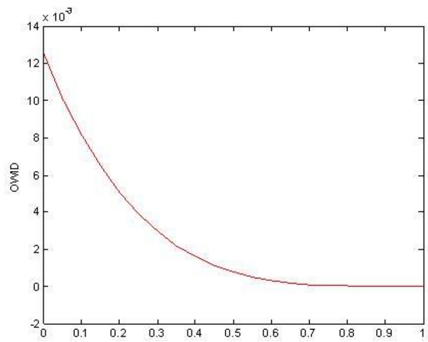


FIG.1.OWID under bit flip channel FIG.1.OWID under phase flip channel

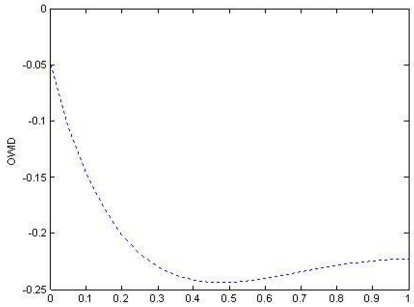


FIG.3.OWID under bit-phase flip channel

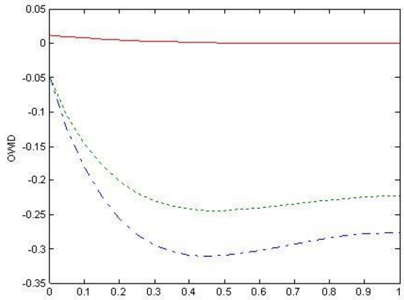


FIG.4.OWID under three channels

IV. CONCLUSION

In this paper, we analytically evaluate the one-way information deficit (OWID) for the X states with four parameters under three kinds of local nondissipative channels. We find that the OWID under the phase flip channel is positive, the OWIDs under the bit flip channel and bit-phase flip channel are negative, and the OWID under the phase flip channel is the greatest among them, the OWID under the bit flip channel is the smallest among them.

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