# DISCRETE PROGRAM FOR MINIMIZATION OF BOIL-OFF-GAS IN LNG PLANT 

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#### Abstract

The research on minimization of boil-off-gas in refrigerated vessels of liquefied natural gas plant is facilitated with third degree polynomial function. Technically, the mode of operations, safety control of gaseous emissions, and transport of highly volatile products vapour pressure effects forms the main thrust of the polynomial model which this research is based on. Minimization of boil-off-gas due to loading-unloading operations from storage vessels acceptable working pressure was determined through the Newton-Raphson iteration of the polynomial functions. The iterations gave successive degree of convergence on the working Pressures of 5, 10, 25, 100, 150 kPas . The best operating working pressure of 150 kPa was achieved to minimize loss of boil-offgas to barest minimum. Compressive stress, hydrostatic stress of each shell course calculations with corrosion allowance of 3 mm gave a constant value of 160.67 MPa and 180.75 MPa pressure at API 650 is the mechanical integrity recommended.


Keywords: Boil-off-gas, Newton-Raphson-iteration, polynomial function, mechanical-integrity, pressure-volume effects, convergence.

## INTRODUCTION

Boil-off-gas BOG is a rare phenomenon in chemical engineering teachings, but however, occurs in gas liquefaction process plant. Boil-off gas in liquefied natural gas plant, is liquefied natural gas loosed to the atmosphere due to heat-ingress and heat-in-leaks to the refrigerated storage vessels and almost all equipment used to process LNG that are well insulated (Wordu and Boma, 2013).

## LIQUEFACTION OF NATURAL GAS

The Department of Energy (DOE, 2008), liquefied natural gas (LNG) is natural gas that has been cooled to the point that it condenses to a liquid, which occurs at a temperature of approximately $161^{\circ} \mathrm{C}$ and at atmospheric pressure (Wordu and Uzono, 2013). Liquefaction is done using a refrigeration process in a liquefaction plant. The Liquefaction reduces the volume by approximately 600 times, making it more economical to transport between continents. The LNG is transported by special made ships to terminals, and then stored at atmospheric pressure in super-insulated tanks. However the ship cargo tanks, storage tanks and almost all equipment used to process LNG are well insulated, there is always some heat leak into the LNG. Heat entering the LNG, referred to as "heat-
in-leak" causes the LNG to warm up. To keep the pressure and the temperature constant heat adsorbed by the LNG has to be released by boiling off some of the liquid-to-gas. This is known as auto-refrigeration (Energy Information Administration Natural Gas, 2005).

## BOIL-OFF GAS GENERATION

Most cryogenic systems experience boil-off, because like LNG, LPG (liquefied Petroleum Gas), another alternative to LNG which is produced during the refining of oil or separating gas streams from reservoirs are stored at cryogenic temperatures. So there is continuous boil-off of small fraction or portion of LNG due to warming during transport and storage. This boil-off gas is generated primarily due to heat leakage from the atmosphere through tank insulation, unloading and recirculation line insulation.

Boil-off gas BOG is a phenomenon which deals with the vaporization of volatile Liquefied Natural Gas (LNG) stored in tanks which is kept at a reduced temperature of $-161^{\circ} \mathrm{C}$ at atmospheric pressure. It is an inherent part of natural gas liquefaction, transportation and gasification. It varies depending on site temperature, climate condition, integrity of insulation and plant operating mode (Wordu and Boma, 2013).

The LNG gasification plant is mainly composed of LNG storage tanks, LNG pumps, LNG vaporizers and gas pipeline. During normal operation, boil-off gas is produced in the tanks and by heat transfer from the surroundings. During ship unloading, the quantity of vapor in LNG tanks outlet increases significantly as these vapors are a combination of volume is placed in the tanks by the incoming LNG, vapor resulting from the release of energy input by the ship pumps, flash vapor due to the pressure difference between the ship and storage tanks, and vapor generated from heat leak through the unloading arms and transfer lines (Myung et al, 2007).

Boil-off gas can also be experienced in liquid system where temperatures are elevated above the boiling point temperatures of the liquids within the systems; in which this increase in temperature consequently leads to escape of the components of the liquids that are heated up to bubble point (Adom et al, 2010).

Table 1 Natural gas compositions in most parts of the world

| Constituent of <br> gases | Canada <br> (Alberta <br> ) | Western <br> Colorad <br> o | South west <br> Kansas | Bach Ho <br> Field <br> Vietnam | Miska <br> Field <br> Tunisi <br> a | Rio Arriba <br> County <br> New <br> Mexico | Cliffside <br> Field <br> Amarilo, <br> Texas |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Helium | 0.0 | 0.0 | 0.45 | 0.00 | 0.00 | 0.00 | 1.8 |
| Nitrogen | 3.2 | 26.10 | 14.65 | 0.21 | 16.903 | 0.68 | 25.6 |
| Carbon dioxide | 1.7 | 42.66 | 0.0 | 0.06 | 13.588 | 0.82 | 0.0 |
| Hydrogen <br> Sulphide | 3.3 | 0.0 | 0.0 | 0.0 | 0.092 | 0.0 | 0.0 |
| Methane | 77.1 | 29.98 | 72.89 | 70.85 | 63.901 | 96.91 | 65.8 |
| Ethane | 6.6 | 0.55 | 6.27 | 13.41 | 3.349 | 1.33 | 3.8 |
| Propane | 3.1 | 0.28 | 3.74 | 7.5 | 0.960 | 0.19 | 1.7 |
| Butanes | 2.0 | 0.21 | 1.38 | 4.02 | 0.544 | 0.05 | 0.8 |
| Pentanes <br> heavier$\&$ | 3.0 | 0.25 | 0.62 | 2.64 | 0.630 | 0.02 | 0.5 |

Tabular mol \% data on wet basis.( $1.3 \mathrm{~mol} \%$ water).
Source: U.S Bureau of Mines (1972) and Jones et al (1999)
A storage tank is generally not pumped completely dry when emptied. The vapor above the remaining liquid will expand to fill the void space at the liquid's vapor pressure at storage temperature. As the tank fills, vapors are compressed into smaller void space until the set pressure on the vent/relief system is reached. Some filling losses are associated with the liquid expansion into the tank. Vapors emitted from a storage tank's vents and/or relief valves are generated in two ways (Wordu and Boma, 2013).

- Tank vapors forced out during filling operations (displacement losses)
- Vapors generated by liquid vaporization stored in the tank.

The vaporization and displacement losses are basically sources of boil-off-gases BOG generations of interest for gas processing activities.

Displacement losses are losses made up of combined loss from filling and emptying and is considered a working loss or displacement loss. As liquid level increases, the pressure inside the vessel exceeds the relief pressure and vapors are then expelled from the vessel. During emptying of liquids product evaporative loss occurs, and air is drawn into the tank during liquid removal, it becomes saturated with organic vapors and expands, thereby exceeding the vapor space capacity (Alireza, 2004); (Wordu and Boma, 2013).

In terms of vaporization losses, vapors are generated by heat gained through the shell, bottom and roof. The total heat input is the algebraic sum of the radiant, conductive and convective heat transfer. This type of loss is especially prevalent where light hydrocarbon liquids are stored in full pressure or refrigerated storage. This is less prevalent but still quite common in crude oil and finished product storage tanks. These vapors may be recovered by using the vapor recovery system.

In order for boil-off to occur, it indicates that the gas is a real gas and not an ideal gas based on the assumptions of a phase change and (Greater intermolecular attraction between molecules of gases for a real gas and little or no intermolecular attractions between gas molecules for an ideal gas).It is interesting to know that an ideal gas gives a rough accuracy for weakly polar gases at low pressures and moderate temperatures and real gases give a good accuracy for gases at high pressures and low temperatures (Wordu and Abey, 2013).

## MATERIALS AND METHOD

The minimization of discrete program research is facilitated by the Thrust polynomial function equation (2) expressed in pressure-volume of the BOG loss in the plant at design pressure range of $P<250 \mathrm{KPa}$ and vapour pressure at liquid temperature $P_{v}<100 \mathrm{KPa}$. Table 2 is experimentally determined tuned coefficients of $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ to evaluate the polynomial equation and mathematical techniques of calculus minimum and maximum and Newton-Raphson numerical method calculating for convergence.

A polynomial function expressed in pressure is a corresponding equation to Kamerlingh Onnes equation expressed in volume which uses a power series.

$$
\begin{equation*}
\frac{P V}{R T}=1+\frac{B}{V}+\frac{C}{V^{2}}+\frac{D}{V^{3}}+\ldots \ldots \tag{1}
\end{equation*}
$$

Although very complicated procedure to calculate the coefficients, theoretical expressions were developed for each of the coefficients. The equation of state predicts the PVT (pressure-volumetemperature) behavior of a real gas with a remarkable accuracy over a wide range.

The coefficients B, C, D etc. are functions of temperature only.

Tuned coefficients ( $A_{x}, B_{x}, C_{x}$ and $\left.D_{x}\right)$ in Table 2 below were used for obtaining the coefficients, $a, \mathrm{~b}, \mathrm{c}$ and d which are derived through polynomial equations in the $3^{\text {rd }} \operatorname{order}\left(\mathrm{A}_{\mathrm{x}}+\mathrm{B}_{\mathrm{x}} \mathrm{P}_{\mathrm{v}}+\mathrm{C}_{\mathrm{x}} \mathrm{P}_{\mathrm{v}}{ }^{2}\right.$ $+D_{x} P_{v}^{3}$ ), with x denoting either $1,2,3$ or 4 .

Table 2 Experimentally determined tuned coefficients used for polynomial model.

| X | $\mathrm{A}_{\mathrm{x}}$ | $\mathrm{B}_{\mathrm{x}}$ | $\mathrm{C}_{\mathrm{x}}$ | $\mathrm{D}_{\mathrm{x}}$ |
| :--- | :---: | :---: | :---: | :---: |
| 1 | $1.96692469015064 \mathrm{E}-1$ | $-2.997855656735 \mathrm{E}-3$ | $1.5038580989 \mathrm{E}-5$ | $-2.4675901 \mathrm{E}-8$ |
| 2 | $-3.6068872437314 \mathrm{E}-2$ | $5.96919209208 \mathrm{E}-4$ | $-3.065251613 \mathrm{E}-6$ | $5.109185 \mathrm{E}-9$ |
| 3 | $1.725348224825 \mathrm{E}-3$ | $-2.5923852687 \mathrm{E}-5$ | $1.30357956 \mathrm{E}-7$ | $-2.174 \mathrm{E}-10$ |
| 4 | $-1.26233316566389 \mathrm{E}-5$ | $1.833674141261 \mathrm{E}-7$ | $-9.137656612 \mathrm{E}-10$ | $1.5231731 \mathrm{E}-12$ |

Source: Gas Processors and Suppliers Association Data Book, $12^{\text {th }}$ Edition, Tulsa, Oklahoma, 2004. (Alireza, B; 2004).

## Range:

Working Pressure, $P<250 \mathrm{KPa}$, and Vapor pressure at liquid temperature, $\mathrm{P}_{\mathrm{v}}<100 \mathrm{KPa}$.

## Thermodynamic polynomial model and determination of coefficients

Using a new proposed correlation with the polynomial equation (2), boil-off-gas minimum pressure can be investigated starting with calculus of maximum and minimum value functions, where four coefficients $\alpha^{0}, \beta, \gamma$ and $\chi$ are applied to relate the filling/unloading losses from storage containers in percent of liquid pumped in with working pressure in KPa .

$$
\begin{equation*}
\text { Loss }=P V=\alpha^{0}+\beta \mathbf{P}^{1}+\gamma \mathbf{P}^{2}+\chi \mathbf{P}^{3} \tag{2}
\end{equation*}
$$

In equation (1) the coefficients $a, \mathrm{~b}, \mathrm{c}$ and d are replaced with $\alpha^{\mathrm{o}}, \beta, \gamma$ and $\chi$ respectively and are constants of temperature only. The coefficients $\alpha^{0}, \beta, \gamma$ and $\chi$ were evaluated in the polynomial equations applying calculus for Min and Max values as stated below.
$\mathrm{dv} / \mathrm{dp}=0$, at stationary point,
$d^{2} v / d p^{2}=\min$ or max value
For brevity, the stepwise logical program sequence from calculus stage extending to the Newton-
Raphson iterative method for 5 kpa and 150 kpa are reported.

## For working Pressure of $\mathbf{5} \mathbf{~ k P a}$

Determination of $\alpha^{0}$
Equating $\alpha^{0}$ with the given polynomial.
$\alpha^{0}=A_{x}+B_{x} P_{v}+C_{x} P_{v}{ }^{2}+D_{x} P_{v}{ }^{3}$
Substituting the tune coefficients with $\mathrm{x}=1$ and $\mathrm{P}_{\mathrm{v}}=100 \mathrm{kPa}$
$\alpha^{0}=0.196692469015064-2.997855656735 \times 10^{-3}(100)+1.50338580989 \times 10^{-5}(100)^{2}-$
$2.4675901 \times 10^{-8}(100)^{3}=0.022616812$.
Determination of $\beta$

Equating $\beta \mathrm{P} 1$ with the given polynomial with $\mathrm{x}=2$ and $\mathrm{P}_{\mathrm{v}}=100 \mathrm{kpa}$ and $\mathrm{P}=5 \mathrm{kpa}$.
$\beta P=A_{x}+B_{x} P_{v}+C_{x} P_{v}{ }^{2}+D_{x} P_{v}{ }^{3}$
$\beta(5)=-3.6068872437314 \times 10^{-2}+5.96919209298 \times 10^{-4}(100)-3.065251613 \times 10^{-6}(100)^{2}+$
$5.109185 \times 10^{-9}(100)^{3}=-0.000384056686$.
Determination of $\gamma$
Equating $\gamma \mathrm{P}^{2}$ with the given polynomial with $\mathrm{x}=3$ and $\mathrm{P}_{\mathrm{v}}=100 \mathrm{kpa}$ and $\mathrm{P}=5 \mathrm{kpa}$.
$\gamma \mathrm{P}^{2}=\mathrm{A}_{\mathrm{x}}+\mathrm{B}_{\mathrm{x}} \mathrm{P}_{\mathrm{v}}+\mathrm{C}_{\mathrm{x}} \mathrm{P}_{\mathrm{v}}{ }^{2}+\mathrm{D}_{\mathrm{x}} \mathrm{P}_{\mathrm{v}}{ }^{3}$
$\gamma(5)^{2}=1.725348224825 \times 10^{-3}-2.5923852687 \times 10^{-5}(100)+1.30357956 \times 10^{-7}-2.174 \times 10^{-10}$
$(100)^{3}=0.000008765700064$
Determination of $\chi$

Equating $\chi \mathrm{P}^{3}$ with the given polynomial with $\mathrm{x}=3$ and $\mathrm{P}_{\mathrm{v}}=100 \mathrm{kPa}$ and $\mathrm{P}=5 \mathrm{kPa}$.
$\chi \mathrm{P}^{3}=\mathrm{A}_{\mathrm{x}}+\mathrm{B}_{\mathrm{x}} \mathrm{P}_{\mathrm{v}}+\mathrm{C}_{\mathrm{x}} \mathrm{P}_{\mathrm{v}}{ }^{2}+\mathrm{D}_{\mathrm{x}} \mathrm{P}_{\mathrm{v}}{ }^{3}$
$\chi(5)^{3}=-1.26233316566389 \times 10^{-5}+1.833674141261 \times 10^{-7}(100)-9.137656612 \times 10^{-10}(100)^{2}+$ $1.5231731 \times 10^{-12}(100)^{3}=-0.0000000152086152$.

Substituting, values of coefficients $\alpha^{\circ}, \beta, \gamma$ and $\chi$ into equation

Hence; equation (1), correlating the boil-off volume is given as:

$$
\begin{align*}
& \text { Loss }=\mathrm{Pv}=0.022616812-0.000384056686 \mathrm{P}+0.000008765700064 \mathrm{P}^{2}- \\
& 0.0000000152086152 \mathrm{P}^{3} \tag{2}
\end{align*}
$$

## MINIMIZATION OF BOIL-OFF VOLUME USING CALCULUS

From EQ 2,

Making V the subject of the formula,
$\mathrm{V}=0.022616812 \mathrm{P}^{-1}-0.000384056686+0.00000876570064 \mathrm{P}-$
$0.0000000152086152 \mathrm{P}^{2}$

## SOLVING FOR MAXIMUM AND MINIMUM VALUES OF A POLYNOMIAL

$\mathrm{dv} / \mathrm{dp}=0$; stationary point, $\mathrm{d}^{2} \mathrm{v} / \mathrm{dp}^{2}=\mathrm{min}$ or max.

Hence,

Differentiating V w.r.t P
$\mathrm{dv} / \mathrm{dp}=-0.022616812 \mathrm{P}^{-2}+0.00000876570064-0.0000000304172304 \mathrm{P}$
At stationary point $\mathrm{dv} / \mathrm{dp}=0$,
Hence,

## RULE:

Solving with Newton Raphson iteration formula, EQ (5) to get minimum value of P , for working pressures for $5 \mathrm{kpa}, 10 \mathrm{kpa}, 25 \mathrm{kpa}, 100 \mathrm{kpa}$ and 150 kpa .

Obtain minimum value of P, after several iterations using Newton Raphson iteration formula.
Using working pressure of $5 \mathrm{kpa}, \mathrm{P}_{0}$ was taken as $1, \mathrm{f}(\mathrm{p})$ was obtained after getting the $1^{\text {st }}$ derivative i.e. dv/dp which gave $-2.259 \times 10^{-2}$ and $f^{\prime}(P)$ was obtained as $1.74 \times 10^{-5}$.

For working Pressure of 5kpa
The $2^{\text {nd }}$ derivative was obtained by differentiating EQ4 to get EQ6 and P value was obtained after several iterations to get a minimum pressure and is substituted into EQ6.
$\mathrm{dv} / \mathrm{dp}=-0.022616812 \mathrm{P}^{-2}+0.00000876570064-0.0000000304172304 \mathrm{P}=0$
Multiply through by $\mathrm{P}^{2}$
$=-0.022616812+0.00000876570064 \mathrm{P}^{2}-0.00000000304172304 \mathrm{P}^{3}=0$
Or, $=-0.0000000304172304 \mathrm{P}^{3}+0.00000876570064 \mathrm{P}^{2}-0.022616812$

Hence, $\mathrm{f}(\mathrm{P})=-0.0000000304172304 \mathrm{P}^{3}+0.00000876570064 \mathrm{P}^{2}-0.022616812$

SOLVING USING NEWTON RAPHSON ITERATION FORMULA

|  |  | $\begin{aligned} & \mathrm{P}_{1}=\mathrm{P}_{0}- \\ & \underline{f(P)} \end{aligned}$ |
| :---: | :---: | :---: |
| $\mathrm{d}^{2} \mathrm{v} / \mathrm{dp}^{2}=4.52 \times 10^{-2} \mathrm{P}^{-3}-3.04 \times 10^{-8}$ | (6) for 5 kpa |  |
| $\mathrm{d}^{2} \mathrm{v} / \mathrm{dp}^{2}=4.52 \times 10^{-2} \mathrm{P}^{-3}-3.808 \times 10^{-9}$ | (7) for 10 kpa |  |
| $\mathrm{d}^{2} \mathrm{v} / \mathrm{dp}^{2}=4.52 \times 10^{-2} \mathrm{P}^{-3}-2.437 \times 10^{-8}$ | (8) for 25 kpa |  |
| $\mathrm{d}^{2} \mathrm{v} / \mathrm{dp}^{2}=4.52 \times 10^{-2} \mathrm{P}^{-3}-3.808 \times 10^{-12}$ | (9) for 100 kpa |  |
| $\mathrm{d}^{2} \mathrm{v} / \mathrm{dp}^{2}=4.52 \times 10^{-2} \mathrm{P}^{-3}-1.128 \times 10^{-12}$ | (10) for 150 kpa |  |

(5)

IF EQ6 gives a (-ve) value, then $\mathrm{P}>250$ or high P , hence
GO TO EQ3
E.g For working pressure of 5 kpa , Substitute Value of $\mathrm{P}=278.75$ into EQ3 to get minimum volume ( $\mathrm{V}_{\mathrm{MIN}}$ )

Multiply $\mathrm{V}_{\text {MIN }}$ value with the corresponding vapor pressures $\left(\mathrm{P}_{\mathrm{v}}\right)$ given say,
$10 \mathrm{kpa}, 20 \mathrm{kpa}, 30 \mathrm{kpa}, 40 \mathrm{kpa}, 50 \mathrm{kpa}, 60 \mathrm{kpa}, 70 \mathrm{kpa}, 80 \mathrm{kpa}, 90 \mathrm{kpa}$ and 100 kpa to get the various equivalent losses.
$\mathrm{P}_{\mathrm{v}} * \mathrm{~V}_{\mathrm{MIN}}=$ Loss, then plot graph of Loss against $\mathrm{P}_{\mathrm{v}}($ Vapor pressure at liquid temperature $)$.
Do same for working pressures of $10 \mathrm{kpa}, 25 \mathrm{kpa}, 100 \mathrm{kpa}$ and 150 kpa and see the results of the different curves on graph plotted.

## RESULTS OBTAINED Second derivation

For working Pressure of 10 kpa
$\alpha^{0}=2.26 \times 10^{-2}$
$\beta=-1.919 \times 10^{-4}$
$\gamma=2.186 \times 10^{-6}$
$\chi=-1.904 \times 10^{-9}$
$\mathrm{PV}=2.26 \times 10^{-2}-1.919 \times 10^{-4} \mathrm{P}+2.186 \times 10^{-6} \mathrm{P}^{2}-1.904 \times 10^{-9} \mathrm{P}^{3}(11)$
$\mathrm{V}=2.26 \times 10^{-2} \mathrm{P}^{-1}-1.919 \times 10^{-4}+2.186 \times 10^{-6} \mathrm{P}-1.904 \times 10^{-9} \mathrm{P}^{2}(12)$
$\mathrm{dv} / \mathrm{dp}=-2.26 \times 10^{-2} \mathrm{P}^{-2}+2.186 \times 10^{-6}-3.808 \times 10^{-9} \mathrm{P}$
Multiply through EQ13 by $\mathrm{P}^{2}$
$\mathrm{dv} / \mathrm{dp}=-2.26 \times 10^{-2}+2.186 \times 10^{-6} \mathrm{P}^{2}-3.808 \times 10^{-9} \mathrm{P}^{3}=0$
or,
$\mathrm{f}(\mathrm{p})=-3.808 \times 10^{-9} \mathrm{P}^{3}+2.186 \times 10^{-6} \mathrm{P}^{2}-2.26 \times 10^{-2}$.
GO TO EQ5
$\mathrm{P}_{1}=\mathrm{P}_{0}-\frac{f(P)}{f^{\prime}(P)}$

## For working Pressure of 150kpa

$\alpha^{0}=2.26 \times 10^{-2}$
$\beta=-1.279 \times 10^{-5}$
$\gamma=9.7155 \times 10^{-9}$
$\chi=-5.6414 \times 10^{-13}$
$\mathrm{PV}=2.26 \times 10^{-2}-1.279 \times 10^{-5} \mathrm{P}+9.7155 \times 10^{-9} \mathrm{P}^{2}-5.6414 \times 10^{-13} \mathrm{P}^{3}(20)$
$\mathrm{V}=2.26 \times 10^{-2} \mathrm{P}^{-1}-1.279 \times 10^{-5}+9.7155 \times 10^{-9} \mathrm{P}-5.6414 \times 10^{-13} \mathrm{P}^{2}(21)$
$\mathrm{dv} / \mathrm{dp}=-2.26 \times 10^{-2} \mathrm{P}^{-2}+9.7155 \times 10^{-9}-1.12828 \times 10^{-12} \mathrm{P}$
Multiply through EQ22 by $\mathrm{P}^{2}$
$\mathrm{dv} / \mathrm{dp}=-2.26 \times 10^{-2}+9.7155 \times 10^{-9} \mathrm{P}^{2}-1.12828 \times 10^{-12} \mathrm{P}^{3}=0$
or,
$\mathrm{f}(\mathrm{p})=-1.12828 \times 10^{-12} \mathrm{P}^{3}+9.7155 \times 10^{-9} \mathrm{P}^{2}-2.26 \times 10^{-2}$.
GO TO EQ5
$\mathrm{P}_{1}=\mathrm{P}_{0}-\frac{f(P)}{f^{\prime}(P)}$

Table 3 Results program for 5 KPa
PROCEDURES IN DETERMINING COEFFICIENTS $\alpha^{o}, \beta, \gamma$ and $\chi$ FOR WORKING PRESSURE OF 5 KPa

DETERMINING COEFFICEINTS $\alpha^{0}, \beta, \gamma$ and $\chi$ FOR WORKING PRESSURE OF 5kpa

| $\alpha^{\mathbf{0}}$ | Ax | BxPv | CxPv ^2 | $\mathrm{DxPv}^{\wedge} 3$ |
| :--- | :---: | :---: | :---: | :---: |
|  | $2.261681223156 \mathrm{E}-02$ | $1.97 \mathrm{E}-01$ | $-3.00 \mathrm{E}-03$ | $1.50 \mathrm{E}-05$ |
| $\beta$ |  |  | $-2.47 \mathrm{E}-08$ |  |
|  | $-3.840565275028 \mathrm{E}-04$ | $-3.61 \mathrm{E}-02$ | $5.97 \mathrm{E}-04$ | $-3.07 \mathrm{E}-06$ |
|  |  |  |  | $5.11 \mathrm{E}-09$ |
| $\Gamma$ |  |  |  |  |
|  | $8.765700645000 \mathrm{E}-06$ | $1.73 \mathrm{E}-03$ | $-2.59 \mathrm{E}-05$ | $1.30 \mathrm{E}-07$ |
| X |  |  | $-2.17 \mathrm{E}-10$ |  |
|  |  |  |  |  |
|  | $-1.520859004823 \mathrm{E}-08$ | $-1.26 \mathrm{E}-05$ | $1.83 \mathrm{E}-07$ | $-9.14 \mathrm{E}-10$ |
|  |  |  |  | $1.52 \mathrm{E}-12$ |

SOLVING USING NEWTON RAPHSON ITERATION FORMULA WITH WORKING PRESSURE OF 5KPA

|  |  |  | $\mathrm{F}\left(\mathrm{P}_{0}\right)$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $-3.04 \mathrm{E}-08$ | $8.77 \mathrm{E}-06$ | $-2.26 \mathrm{E}-02$ | -0.02259127 |  |
|  |  |  | $\mathrm{~F}^{\prime}\left(\mathrm{P}_{0}\right)$ |  |
| $-3.04 \mathrm{E}-08$ | $8.77 \mathrm{E}-06$ | $-2.26 \mathrm{E}-02$ | $1.74 \mathrm{E}-05$ |  |
|  |  |  | P 1 |  |
|  |  |  | 1296.45986 |  |
|  |  |  |  |  |
|  |  |  | $\mathrm{~F}(\mathrm{P} 1)$ |  |


| -3.04E-08 | 8.77E-06 | -2.26E-02 | $-5.15 \mathrm{E}+01$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 1)$ |  |
| -3.04E-08 | 8.77E-06 | -2.26E-02 | -1.31E-01 |  |
|  |  |  | P2 |  |
|  |  |  | 901.7490742 |  |
|  |  |  |  |  |
|  |  |  | F(P2) |  |
| -3.04E-08 | 8.77E-06 | -2.26E-02 | -1.52E+01 |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 2)$ |  |
| -3.04E-08 | $8.77 \mathrm{E}-06$ | -2.26E-02 | -5.84E-02 |  |
|  |  |  | P3 |  |
|  |  |  | 641.4931603 |  |
|  |  |  |  |  |
|  |  |  | F(P3) |  |
| -3.04E-08 | 8.77E-06 | -2.26E-02 | -4.44E+00 |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 3)$ |  |
| -3.04E-08 | $8.77 \mathrm{E}-06$ | -2.26E-02 | -2.63E-02 |  |
|  |  |  | P4 |  |
|  |  |  | 472.5440068 |  |
|  |  |  |  |  |
|  |  |  | F(P4) |  |
| -3.04E-08 | $8.77 \mathrm{E}-06$ | -2.26E-02 | $-1.27 \mathrm{E}+00$ |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 4)$ |  |
| -3.04E-08 | 8.77E-06 | -2.26E-02 | -1.21E-02 |  |
|  |  |  | P5 |  |
|  |  |  | 367.1606506 |  |
|  |  |  |  |  |
|  |  |  | F(P5) |  |
| -3.04E-08 | 8.77E-06 | -2.26E-02 | -3.46E-01 |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 5)$ |  |
| $-3.04 \mathrm{E}-08$ | 8.77E-06 | -2.26E-02 | -5.86E-03 |  |
|  |  |  | P6 |  |
|  |  |  | 308.1497836 |  |
|  |  |  |  |  |
|  |  |  | F(P6) |  |
| -3.04E-08 | 8.77E-06 | -2.26E-02 | -7.98E-02 |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 6)$ |  |
| -3.04E-08 | $8.77 \mathrm{E}-06$ | -2.26E-02 | -3.26E-03 |  |
|  |  |  | P7 |  |
|  |  |  | 283.6465285 |  |
|  |  |  |  |  |


|  |  |  | F(P7) |  |
| :---: | :---: | :---: | :---: | :---: |
| -3.04E-08 | 8.77E-06 | $-2.26 \mathrm{E}-02$ | -1.12E-02 |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 7)$ |  |
| -3.04E-08 | 8.77E-06 | -2.26E-02 | -2.37E-03 |  |
|  |  |  | P8 |  |
|  |  |  | 278.9265854 |  |
|  |  |  |  |  |
|  |  |  | F(P8) |  |
| -3.04E-08 | 8.77E-06 | -2.26E-02 | -3.78E-04 |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 8)$ |  |
| -3.04E-08 | 8.77E-06 | -2.26E-02 | -2.21E-03 |  |
|  |  |  | P9 |  |
|  |  |  | 278.7552927 |  |
|  |  |  |  |  |
|  |  |  | F(P9) |  |
| -3.04E-08 | $8.77 \mathrm{E}-06$ | -2.26E-02 | -4.89E-07 |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 9)$ |  |
| -3.04E-08 | $8.77 \mathrm{E}-06$ | -2.26E-02 | -2.20E-03 |  |
|  |  |  | P10 |  |
|  |  |  | 278.7550704 |  |
|  |  |  |  |  |
|  |  |  |  |  |

$\mathbf{P}=\mathbf{2 7 8 . 7 5} \mathrm{kpa}$ is the minimum working pressure value obtained after several iterations, for 5kpa

| 5kpa |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
| Differentiating EQ4 | EQ6 |  |  |  |
| Substitute Value of P into EQ6 |  |  |  |  |
|  |  |  |  |  |
| IF EQ6 gives a (-ve) value, then $\mathrm{P}>250$ or high P , hence |  |  |  |  |
| GO TO EQ3 |  |  |  |  |
| Substitute Value of P into EQ3 to get minimum volume( $\mathrm{V}_{\mathrm{MIN}}$ ) |  |  |  |  |
|  |  |  |  |  |
| 3.2.2 MINIMIZATION OF THE BOIL-OFF VOLUME USINGCALCULUS. |  |  |  |  |
|  |  |  |  |  |
| EQ 3; $\mathrm{V}_{\mathrm{MIN}, \mathrm{m}^{3}}$ | $\begin{array}{r} 8.10748 \mathrm{E}- \\ 05 \\ \hline \end{array}$ | -3.84E-04 | 0.002441894 | $0.001181728$ |
| $9.57 \mathrm{E}-04$ |  |  |  |  |
| GRAPH |  |  |  |  |
| Loss $=\mathrm{P}_{\mathrm{v}} * \mathrm{~V}_{\mathrm{MIN}}$ | $\mathrm{P}_{\mathrm{v}}$ | $\mathrm{V}_{\text {MIN }}$ |  |  |


| $9.57 \mathrm{E}-03$ | 10 | $9.57 \mathrm{E}-04$ |  |  |
| ---: | ---: | ---: | :--- | :--- |
| $1.91 \mathrm{E}-02$ | 20 | $9.57 \mathrm{E}-04$ |  |  |
| $2.87 \mathrm{E}-02$ | 30 | $9.57 \mathrm{E}-04$ |  |  |
| $3.83 \mathrm{E}-02$ | 40 | $9.57 \mathrm{E}-04$ |  |  |
| $4.79 \mathrm{E}-02$ | 50 | $9.57 \mathrm{E}-04$ |  |  |
| $5.74 \mathrm{E}-02$ | 60 | $9.57 \mathrm{E}-04$ |  |  |
| $6.70 \mathrm{E}-02$ | 70 | $9.57 \mathrm{E}-04$ |  |  |
| $7.66 \mathrm{E}-02$ | 80 | $9.57 \mathrm{E}-04$ |  |  |
| $8.61 \mathrm{E}-02$ | 90 | $9.57 \mathrm{E}-04$ |  |  |
| $9.57 \mathrm{E}-02$ | 100 | $9.57 \mathrm{E}-04$ |  |  |

TABLE 4 Results of program for working pressure of 5 KPa

| For working Pressure of 5kpa |
| :--- |
| $\alpha^{0}=2.26 \times 10^{-2}$ |
| $\beta=-3.84 \times 10^{-4}$ |
| $\gamma=8.76 \times 10^{-6}$ |
| $\chi=-1.52 \times 10^{-8}$ |
| $\mathrm{PV}=2.26 \times 10^{-2}-3.84 \times 10^{-4} \mathrm{P}+8.76 \times 10^{-6} \mathrm{P}^{2}-1.52 \times 10^{-8} \mathrm{P}^{3}$ |
| $\mathrm{~V}=2.26 \times 10^{-2} \mathrm{P}^{-1}-3.84 \times 10^{-4}+8.76 \times 10^{-6} \mathrm{P}-1.52 \times 10^{-8} \mathrm{P}^{2}$ |
| $\mathrm{dv} / \mathrm{dp}=-2.26 \times 10^{-2} \mathrm{P}^{-2}+8.76 \times 10^{-6}-3.04 \times 10^{-8} \mathrm{P}$ |
| Multiply through EQ13 by $\mathrm{P}^{2}$ |
| dv/dp $=-2.26 \times 10^{-2}+8.76 \times 10^{-6} \mathrm{P}^{2}-3.04 \times 10^{-8} \mathrm{P}^{3}=0$ |
| or, |
| $\mathrm{f}(\mathrm{p})=-3.04 \times 10^{-8} \mathrm{P}^{3}+8.76 \times 10^{-6} \mathrm{P}^{2}-2.26 \times 10^{-2}$. |
| GO TO EQ 5 |
| $P_{1}=P_{0}=P_{o}-\frac{f(P)}{f(P)}$ |

## For working Pressure of 150 kpa

$\alpha^{0}=2.26 \times 10^{-2}$
$\beta=-1.279 \times 10^{-5}$
$\gamma=9.7155 \times 10^{-9}$
$\chi=-5.6414 \times 10^{-13}$
$\mathrm{PV}=2.26 \times 10^{-2}-1.279 \times 10^{-5} \mathrm{P}+9.7155 \times 10^{-9} \mathrm{P}^{2}-5.6414 \times 10^{-13} \mathrm{P}^{3}(20)$
$\mathrm{V}=2.26 \times 10^{-2} \mathrm{P}^{-1}-1.279 \times 10^{-5}+9.7155 \times 10^{-9} \mathrm{P}-5.6414 \times 10^{-13} \mathrm{P}^{2}(21)$
$\mathrm{dv} / \mathrm{dp}=-2.26 \times 10^{-2} \mathrm{P}^{-2}+9.7155 \times 10^{-9}-1.12828 \times 10^{-12} \mathrm{P}$
Multiply through EQ22 by $\mathrm{P}^{2}$
$\mathrm{dv} / \mathrm{dp}=-2.26 \times 10^{-2}+9.7155 \times 10^{-9} \mathrm{P}^{2}-1.12828 \times 10^{-12} \mathrm{P}^{3}=0$
or,
$\mathrm{f}(\mathrm{p})=-1.12828 \times 10^{-12} \mathrm{P}^{3}+9.7155 \times 10^{-9} \mathrm{P}^{2}-2.26 \times 10^{-2}$.
GO TO EQ5
$\mathrm{P}_{1}=\mathrm{P}_{0}-\frac{f(P)}{f^{\prime}(P)}$
Different values for coefficients were obtained for different working pressures of $10 \mathrm{kpa}, 25 \mathrm{kpa}$, 100 kpa and 150 kpa .
TABLE 5 COMPUTER PROGRAM FOR 150 KPa CONVERGENCE

| Model coefficients and iterating for convergence 150 KPa |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\alpha^{0}$ | Ax | BxPv | $\mathrm{CxPv}^{\wedge} 2$ | DxPv^3 |
| $2.261681223156 \mathrm{E}-02$ | $1.97 \mathrm{E}-01$ | $\begin{array}{r} \hline-3.00 \mathrm{E}- \\ 03 \end{array}$ | $1.50 \mathrm{E}-05$ | -2.47E-08 |
| $\beta$ |  |  |  |  |
| -1.280188425009E-05 | -3.61E-02 | $5.97 \mathrm{E}-04$ | -3.07E-06 | $5.11 \mathrm{E}-09$ |
|  |  |  |  |  |
| $\gamma$ |  |  |  |  |
| $9.739667383333 \mathrm{E}-09$ | $1.73 \mathrm{E}-03$ | $\begin{array}{r} -2.59 \mathrm{E}- \\ 05 \end{array}$ | 1.30E-07 | $-2.17 \mathrm{E}-10$ |
|  |  |  |  |  |
| $\chi$ |  |  |  |  |
| -5.632811128975E-13 | -1.26E-05 | $1.83 \mathrm{E}-07$ | -9.14E-10 | $1.52 \mathrm{E}-12$ |


| SOLVING USING NEWTON RAPHSON ITERATION FORMULA WITH WORKING PRESSURE OF 150KPA |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\mathrm{F}\left(\mathrm{P}_{0}\right)$ |  |
| -1.13E-12 | $9.72 \mathrm{E}-09$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -0.02259999 |  |
|  |  |  | $\mathrm{F}^{\prime}\left(\mathrm{P}_{0}\right)$ |  |
| -1.13E-12 | $9.72 \mathrm{E}-09$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | $1.94 \mathrm{E}-08$ |  |
|  |  |  | P1 |  |
|  |  |  | 1163293.051 |  |
|  |  |  |  |  |
|  |  |  | F(P1) |  |
| $-1.13 \mathrm{E}-12$ | $9.72 \mathrm{E}-09$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -8.73E-03 |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 1)$ |  |
| -1.13E-12 | $9.72 \mathrm{E}-09$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | $1.95 \mathrm{E}-05$ |  |
|  |  |  | P2 |  |
|  |  |  | 1163740.64 |  |
|  |  |  |  |  |
|  |  |  | F(P2) |  |
| -1.13E-12 | $9.72 \mathrm{E}-09$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | $-1.77 \mathrm{E}+06$ |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 2)$ |  |
| $-1.13 \mathrm{E}-12$ | $9.72 \mathrm{E}-09$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | $-4.56 \mathrm{E}+00$ |  |
|  |  |  | P3 |  |
|  |  |  | 776788.5972 |  |
|  |  |  |  |  |
|  |  |  | F(P3) |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | $-1.77 \mathrm{E}+06$ |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 3)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | $-6.86 \mathrm{E}+00$ |  |
|  |  |  | P4 |  |
|  |  |  | 518500.058 |  |
|  |  |  |  |  |
|  |  |  | F(P4) |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | $-5.25 \mathrm{E}+05$ |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 4)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | -2.26E- | -3.05E+00 |  |


|  |  | 02 |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | P5 |  |
|  |  |  | 346309.2793 |  |
|  |  |  |  |  |
|  |  |  | F(P5) |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | $-1.56 \mathrm{E}+05$ |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 5)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | $-1.35 \mathrm{E}+00$ |  |
|  |  |  | P6 |  |
|  |  |  | 231517.8021 |  |
|  |  |  |  |  |
|  |  |  | F(P6) |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -4.61E+04 |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 6)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -6.02E-01 |  |
|  |  |  | P7 |  |
|  |  |  | 154993.7231 |  |
|  |  |  |  |  |
|  |  |  | F(P7) |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \\ \hline \end{gathered}$ | $-1.37 \mathrm{E}+04$ |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 7)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -2.68E-01 |  |
|  |  |  | P8 |  |
|  |  |  | 103983.0507 |  |
|  |  |  |  |  |
|  |  |  | F(P8) |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | $-4.05 \mathrm{E}+03$ |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 8)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -1.19E-01 |  |
|  |  |  | P9 |  |
|  |  |  | 69984.05457 |  |
|  |  |  |  |  |
|  |  |  | F(P9) |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | $-1.20 \mathrm{E}+03$ |  |


|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 9)$ |  |
| :---: | :---: | :---: | :---: | :---: |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -5.29E-02 |  |
|  |  |  | P10 |  |
|  |  |  | 47330.34504 |  |
|  |  |  |  |  |
|  |  |  | F(P10) |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | $-3.55 \mathrm{E}+02$ |  |
|  |  |  | $\mathrm{F}^{\prime}(10)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \\ \hline \end{gathered}$ | -2.35E-02 |  |
|  |  |  | P11 |  |
|  |  |  | 32246.55234 |  |
|  |  |  |  |  |
|  |  |  | F(P11) |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | $-1.05 \mathrm{E}+02$ |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 11)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} \hline-2.26 \mathrm{E}- \\ 02 \\ \hline \end{gathered}$ | -1.05E-02 |  |
|  |  |  | P12 |  |
|  |  |  | 22219.27397 |  |
|  |  |  |  |  |
|  |  |  | F(P12) |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -3.10E+01 |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 12)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \\ \hline \end{gathered}$ | -4.67E-03 |  |
|  |  |  | P13 |  |
|  |  |  | 15578.56737 |  |
|  |  |  |  |  |
|  |  |  | F(P13) |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -9.11E+00 |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 13)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -2.09E-03 |  |
|  |  |  | P14 |  |
|  |  |  | 11220.46383 |  |
|  |  |  |  |  |
|  |  |  | F(P14) |  |


| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | $-2.65 \mathrm{E}+00$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 14)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -9.48E-04 |  |
|  |  |  | P15 |  |
|  |  |  | 8424.462286 |  |
|  |  |  |  |  |
|  |  |  | $\mathrm{F}(\mathrm{P} 15)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -7.48E-01 |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 15)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -4.42E-04 |  |
|  |  |  | P16 |  |
|  |  |  | 6734.023407 |  |
|  |  |  |  |  |
|  |  |  | F(P16) |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -1.94E-01 |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 16)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -2.24E-04 |  |
|  |  |  | P17 |  |
|  |  |  | 5865.838046 |  |
|  |  |  |  |  |
|  |  |  | F(P17) |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -3.90E-02 |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 17)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -1.37E-04 |  |
|  |  |  | P18 |  |
|  |  |  | 5580.260825 |  |
|  |  |  |  |  |
|  |  |  | F(P18) |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} \hline-2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -3.59E-03 |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 18)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -1.12E-04 |  |
|  |  |  | P18 |  |
|  |  |  | 5548.108081 |  |


|  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | F(P19) |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -4.32E-05 |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 19)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -1.09E-04 |  |
|  |  |  | P19 |  |
|  |  |  | 5547.712259 |  |
|  |  |  |  |  |
|  |  |  | $\mathrm{F}(\mathrm{P} 20)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -6.51E-09 |  |
|  |  |  | $\mathrm{F}^{\prime}(\mathrm{P} 20)$ |  |
| -3.81E-12 | $2.19 \mathrm{E}-08$ | $\begin{gathered} -2.26 \mathrm{E}- \\ 02 \end{gathered}$ | -1.09E-04 |  |
|  |  |  | P20 |  |
|  |  |  | 5547.712199 |  |
|  |  |  |  |  |
|  |  |  |  |  |
| $\mathrm{P}=5547.712 \mathrm{kpa}$ is the minimum pr iterations | essure value ob | ained after |  |  |
|  |  |  |  |  |
| Differentiating EQ22 | EQ10 |  |  |  |
| Substitute Value of P into EQ10 |  |  |  |  |
|  |  |  |  |  |
| IF EQ10 gives a (-ve) value, then $\mathrm{P}>$ hence | $250 \text { or high } \mathrm{P} \text {, }$ |  |  |  |
| GO TO EQ21 |  |  |  |  |
|  |  |  |  |  |
| Substitute Value of P into EQ21 to | minimum vol | me( $\mathrm{V}_{\mathrm{MIN}}$ ) |  |  |
|  |  |  |  |  |
| 3.2.2 MINIMIZATION OF THE B USING CALCULUS. | IL-OFF VOLL |  |  |  |
|  |  |  |  |  |
| EQ 21; $\mathrm{V}_{\mathrm{MIN}, \mathrm{m}^{3}}$ | 4.07375E-06 | $\begin{array}{r} \hline-1.92 \mathrm{E}- \\ 05 \end{array}$ | 0.000121273 | -5.85996E-05 |
| $4.76 \mathrm{E}-05$ |  |  |  |  |
| GRAPH |  |  |  |  |
| Loss $=\mathrm{P}_{\mathrm{v}} * \mathrm{~V}_{\mathrm{MIN}}$ | $\mathrm{P}_{\mathrm{v}}$ | $\mathrm{V}_{\mathrm{MIN}}$ |  |  |
| $4.76 \mathrm{E}-04$ | 10 | 4.76E-05 |  |  |
| $9.52 \mathrm{E}-04$ | 20 | 4.76E-05 |  |  |


| $1.43 \mathrm{E}-03$ | 30 | $4.76 \mathrm{E}-05$ |  |  |
| ---: | ---: | ---: | :--- | :--- |
| $1.90 \mathrm{E}-03$ | 40 | $4.76 \mathrm{E}-05$ |  |  |
| $2.38 \mathrm{E}-03$ | 50 | $4.76 \mathrm{E}-05$ |  |  |
| $2.86 \mathrm{E}-03$ | 60 | $4.76 \mathrm{E}-05$ |  |  |
| $3.33 \mathrm{E}-03$ | 70 | $4.76 \mathrm{E}-05$ |  |  |
| $3.81 \mathrm{E}-03$ | 80 | $4.76 \mathrm{E}-05$ |  |  |
| $4.28 \mathrm{E}-03$ | 90 | $4.76 \mathrm{E}-05$ |  |  |
| $4.76 \mathrm{E}-03$ | 100 | $4.76 \mathrm{E}-05$ |  |  |

## RESULTS AND DISCUSSION

The polynomial model investigated predict perfectly the actual operating pressure from 100 to 150 KPa as reasonable working pressure to minimize BOG in the plant.


Figure 1 shows a composite plot of $5,10,25,100,150 \mathrm{KPa}$


Figure 2: Plot of 3-dimension of 5, 10, 25,100, and 150 KPa

Figure 2 exhibits a 3 -dimensional plot of figure 1; its essence is to show more clarity of the parameters plot.

Figure 1 shows a composite plot of the iterative computer programs for the design range of pressures 5 to 150 kPa . Tables 3, 4, 5 are representative for 10,25 and $100 \mathrm{KPa}, \mathrm{s}$ indicating program logic to predict acceptable minimal working pressure from 100 to 150 KPa as delineated in profiles in figure 1 . ${ }^{\text {‘ }}$

The results showed that at lower working pressures of 5 kpa with which refrigerated tank operates, there is a high rate of vapor loss of Liquefied gas, while at higher working pressures of 100 kpa and 150 kpa there is a minimum volume loss. From the plot, highest amount of vapor loss in vessel occurred with working pressures of $5,10,25 \mathrm{KPa}$; and minimum loss at high operating pressure of 150 kpa and is the acceptable pressure, due to the fact that LNG is compressed at $161^{\circ} \mathrm{C}$ and tends to convert to gas as temperature increases in the refrigerated vessel leading to loss of valuable gas to atmosphere of operation, hence can be hazardous and adequate relief mechanism must be installed for checks. Finally, the mechanical integrity test of corresponding
compressive and hydrostatic stress is maintained at 160.67 and 180.75 with a corrosion allowance of 3 mm .

## NOMENCLATURE



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