# APPLICATION OF ASYMPTOTIC DISTRIBUTION OF MANN-WHITNEY STATISTIC TO DETERMINE THE DIFFERENCE BETWEEN THE SYSTOLIC BLOOD PRESSURE OF MEN AND WOMEN OVER 45 YEARS OF AGE

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## ABSTRACT

We have applied the derived asymptotic distribution of the Mann-Whitney statistic to the systolic blood pressure of different groups; men and women, to determine if there is a difference in their blood pressures. The analysis showed there is no difference between the blood pressures of men and women at $0.05$ and $0.01$ significance levels. The same result was obtained when Mann-Whitney statistic was applied to a smaller sample of the systolic blood pressures of men and women.

**Keywords:** Systolic blood pressure, Man-Whitney Statistic, Asymptotic distribution.

## I. INTRODUCTION

Under the assumption that two groups $X$ and $Y$ are independently and identically distributed with distribution functions $F$ and $G$ not necessarily known, Gurevich (2009), the Mann-Whitney statistic is broadly used to interpret whether there are differences in the distribution of the two groups or differences in their medians. The Mann-Whitney test was developed as a test of stochastic equality, Mann (1947).
The asymptotic distribution of Mann-Whitney remains of interest, and Ferge (1994) suggested a number of tests and evaluates the asymptotic significance level, and compared the asymptotic power for some of these tests. When normality holds, Mann-Whitney test has an asymptotic efficiency of about $3/\pi$ when compared to t-test, Lehamn (1999).
The general conception is that blood pressure increases with age, and that men are more at risk for cardiovascular and renal diseases than their premenopausal women counterparts until after menopause Reckelholff (2001). However, according to Brennan (1986), true variation in blood pressure is related to many factors including physical

activities, emotional state etc.

This paper is aimed at examining the systolic blood pressures of men and women over 45 years of age and use the asymptotic distribution of Mann-Whitney statistic to determine whether there is a significant difference in the systolic blood pressure of the two groups.

## II. Mann – Whitney Statistic

Let $x_1,...,x_m$ and $y_1,...,y_n$ be two independent random samples of sizes $m$ and $n$ respectively from two populations $F$ and $G$. The Mann – Whitney statistic is given by

$$W_{XY} = W_s - \frac{n(n+1)}{2} \qquad 1$$

Where $W_s$ is the sum of the ranks of $y_i$. The null hypothesis $H_0$ is rejected if $W_{XY} > nm - W_{\alpha/2}$.

## III. Deriving the Asymptotic Distribution of Mann – Whitney Statistic

Let the rank of $y_i$ be $R(y_i)$. Thus $W_s = \sum_{i=1}^{n} R(y_i)$

$$E(W_s) = E\left(\sum_{i=1}^{n} R(y_i)\right) = \sum_{i=1}^{n} E(R(y_i))$$

But $$E(R(y_i)) = \frac{1}{N}\sum_{i=1}^{N} R(y_i)$$

$$= \frac{1}{N}\cdot\frac{N(N+1)}{2} = \frac{N+1}{2}$$

Thus $$E(W_s) = \sum_{i=1}^{n}\left(\frac{N+1}{2}\right) = \frac{n(N+1)}{2} \qquad 2$$

Therefore

$$E(W_{XY}) = E(W_s) - \frac{n(n+1)}{2}$$

$$= \frac{n(N+1)}{2} - \frac{n(n+1)}{2} = \frac{nm}{2}$$

$$\text{var}(W_{XY}) = \text{var}(W_s)$$

$$= \operatorname{var}\left(\sum_{i=1}^{n} R(y_i)\right) = \sum_{i=1}^{n} \operatorname{var}\left(R(y_i)\right) + \sum\sum \operatorname{cov}\left(R(y_i), R(y_j)\right)$$

But $\operatorname{var}\left(R(y_i)\right) = E\left(R^2(y_i)\right) - \left(E\left(R(y_i)\right)\right)^2$

$$= \frac{N(N+1)(2N+1)}{6N} - \left(\frac{N+1}{2}\right)^2 = \frac{(N-1)(N+1)}{12}$$

$$\operatorname{cov}\left(R(y_i), R(y_j)\right) = \sum_{\alpha=1}^{N}\sum_{\lambda=1}^{N}\left[k_\alpha - E\left(R(y_i)\right)\right]\left[f_\lambda - E\left(R(y_j)\right)\right] P\left(R(y_i) = k_\alpha, R(y_j) = f_\lambda\right)$$

$$= \frac{1}{N(N-1)} \sum_{\alpha \neq \lambda = 1}^{N} \left(k_\alpha - \frac{N+1}{2}\right)\left(f_\lambda - \frac{N+1}{2}\right) \qquad 3$$

Consider

$$\left[\left(k_1 - \frac{N+1}{2}\right) + \ldots + \left(k_N - \frac{N+1}{2}\right)\right]^2 = \sum_{\alpha=1}^{N}\left(k_\alpha - \frac{N+1}{2}\right)^2 + \sum_{\alpha \neq \lambda = 1}^{N}\left(k_\alpha - \frac{N+1}{2}\right)\left(f_\lambda - \frac{N+1}{2}\right)$$

$$0 = N \operatorname{var}\left(R(y_i)\right) + \sum_{\alpha \neq \lambda = 1}^{N}\left(k_\alpha - \frac{N+1}{2}\right)\left(f_\lambda - \frac{N+1}{2}\right)$$

$$-N \operatorname{var}\left(R(y_i)\right) = \sum_{\alpha \neq \lambda = 1}^{N}\left(k_\alpha - \frac{N+1}{2}\right)\left(f_\lambda - \frac{N+1}{2}\right)$$

Thus

$$\operatorname{cov}\left(R(y_i), R(y_j)\right) = \frac{1}{N(N-1)} \cdot \left[-N \operatorname{var}\left(R(y_i)\right)\right]$$

$$= -\frac{N}{N(N-1)} \cdot \frac{(N-1)(N+1)}{12} = -\frac{(N+1)}{12}$$

$$\operatorname{var}(W_s) = \frac{n(N-1)(N+1)}{12} - \frac{n(n-1)(N+1)}{12}$$

$$= \frac{\left(nN - n^2\right)(N+1)}{12}$$

$$= \frac{nm(N+1)}{12} \qquad 4$$

Under the null hypothesis the statistic

$$\frac{W_{XY} - \frac{nm}{2}}{\sqrt{\frac{nm(N+1)}{12}}} \xrightarrow{d} N(0,1) \qquad 5$$

The null hypothesis is rejected if $W_{XY} > C_{\alpha/2} = \frac{nm}{2} + z_{\alpha/2}\sqrt{\frac{nm(N+1)}{12}}$

**IV Application and discussion**

We have obtained the systolic blood pressure of 8 women and 9 men aged over 45 years, and applied Mann-Whitney test on the pooled data. The statistic, $W_{XY} = 34.5$ and the critical value is $56$. The null hypothesis could not be rejected at a significance level of 0.05, which suggests that there is no difference between the systolic blood pressures of men and women. At a significance level of 0.01, the critical value is $46$, giving the same result. We then increased the sample sizes to 25 women and 26 men, and applied the asymptotic distribution of Mann-Whitney statistic as derived above. The analysis gives $W_{XY} = 275.5$ and $C_{0.025} = 429.0217$. At a significance level of 0.05, the null hypothesis could not be rejected, also suggesting there is no difference in the blood pressure of men and women. The same result holds at a significance level of $0.01$ which gives a critical value of $461.9265$. This agrees with the Mann-Whitney test. Table I gives a summary of the analysis.

Table I: value of calculated statistic and critical values

| | $W_{XY}$ | Critical value | |
|---|---|---|---|
| | | $\alpha = 0.05$ | $\alpha = 0.01$ |
| Mann-Whitney statistic $n = 9, m = 8$ | 34.5 | 56 | 62 |
| Asymptotic distribution $n = 26, m = 25$ | 275.5 | 429.0217 | 461.9265 |

**CONCLUSION**

Asymptotic distribution of Mann-Whitney statistic is a nonparametric method which has been applied to the systolic blood pressures of men and women over the age of 45 to determine if this is a difference in their blood pressures. Based on the data used in this work, there is no significant difference between the systolic blood pressure of men and that of women at significance levels of 0.05 and 0.01.

## REFERENCES

[1] Brennan, M., E. O'Brien and K. O'Malley (1986): "The Effect of Age on Blood Pressure and Heart Rate Variability in Hypertension", Journal of Hypertension, 4(suppl 6): S262-S272

[2] Ferger,D. A. (1994): "On the Power of Nonparametric Change Point-Test", Metrika, vol. 41, pp 227-292

[3] Gurevich, G. (2009): "Asymptotic Distribution of Mann-Whitney Type Statistic for Nonparametric Change Point Problems", Computer Modelling and New Technologies, vol. 13, No. 3, pp 18-26

[4] Lehamn, E. L. (1999): "Elements of Large Sample Theory", Springer, pp. 176-177

[5] Mann, H. B. and D. R. Whitney (1947): "On a Test of Whether One of Two Random Variables is Stochastically Larger than the Other", Ann. Math. Statistics, 18,pp. 50-60

[6] Reckelhoff, J. F. (2001): "Gender Difference in the regulation of Blood Pressure", PubMed, 37(5)