SUPER QUANTUM DISCORD OF X STATES UNDER LOCAL NONDISSIPATIVE CHANNELS*

Ming-yang Shen, Qian Yu, Zhuo-yi Tian & Yuan-hong Tao#
Department of Mathematics, College of Science, Yanbian University, Yanji, Jilin 13302

ABSTRACT

In this paper, we analytically evaluate the super quantum discord of a family of X states with four parameters under three kinds of local nondissipative channels: bit flip channel, phase flip channel and bit phase flip channel. Besides, we make a comparious of the three results and get the conclusion that different channels make different impact on the quantumness.

Keywords: local nondissipative channels; super quantum discord; weak measurement.

I. Introduction

Quantum discord is a fundemental and significant notion of quantum correlation, which is defined as the discrepancy between mutual information and maximum classical mutual information. The quantum discord via von Neumann measurements on one side is given by[1]

$$D_{AB} = \min_{\{\Pi_k^B\}} \sum_k p_k S(\rho \mid \{\Pi_k^B\}) + S(\rho^B) - S(\rho^{AB}).$$

Recently, many scholars have studied quantum discord under different measurements and, among which the weak measurement has been improved exceedingly important. Weak measurement was firstly introduced by Aharonov, Albert, and Vaidman (AAV)[2] in 1988. The weak measurement operators are given by [3]

$$P(+x) = \sqrt{\frac{(1-\tanh x)}{2}}\Pi_0 + \sqrt{\frac{(1+\tanh x)}{2}}\Pi_1,$$

$$P(-x) = \sqrt{\frac{(1+\tanh x)}{2}}\Pi_o + \sqrt{\frac{(1-\tanh x)}{2}}\Pi_1,$$

where x is the measurement strength parameter, Π_0 and Π_1 are two orthogonal projectors with $\Pi_0 + \Pi_1 = I$, moreover, the weak measurement operators satisfy two conditions: (i) $P^+(+x)P(+x) + P^+(-x)P(-x) = I$; (ii) $\lim_{x\to\infty} P(+x) = \Pi_0, \lim_{x\to\infty} P(-x) = \Pi_1$.

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#Corresponding author:Yuan-hong Tao, E-mail:taoyuanhong12@126.com.

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Weak measurement is so useful for high-precision measurements that much progress has been made in this fanscinating field. It has been applied to study arbitrary probe[4], entangled probe[5] and a qubit probe[6], to observe the spin Hall effect in light[7], to detect very small transverse beam deflections[8] and to examine the feedback control of quantum systems in the presense of noise[9].

Based on weak measurement, the super quantum discord for bipartite quantum state is defined as a new concept[10]. Wang et al[11]. evaluated the super quantum discord for Werner states and pointed out that super quantum discord is greater than quantum discord. Jing et al[12]. have evaluated the super quantum discord for general two qubit X states in terms of a one-variable function and over nontrivial regions of a seven dimensional manifold. Li. et. al[13]. studied the super quantum discord for Werner states and Bell diagonal states. He also found some different properties between super quantum discord and quantum discord and give a new way to compare them.

Besides different measurements cause differences in the final quantumness, different channels also change quantum information. The state under local environments can be represented with a completely positive trace-preserving map, which can be written in the operator-sum representation[14]:

$$\rho=\sum_{i,j}E_i^{(A)}E_j^{(B)}\rho_0E_j^{(B)^+}E_i^{(A)^+},$$

where $E_i^{(k)}(k=A,B)$ is the Kraus operator which is always used to represent the channel A or B, and $\sum_i E_i^{(k)}E_i^{(k)^+}=1$. In this paper, we will demonstrate three kinds of Markovian noise channels on each subsystem in the dynamics, which are bit flip channel, phase flip channel and bit phase flip channel, they can be represented with the following Kraus operators respectively:

$$E_0^{b-f}=\sqrt{1-\frac{p}{2}}\begin{pmatrix}1&0\\0&1\end{pmatrix},\quad E_0^{b-f}=\sqrt{\frac{p}{2}}\begin{pmatrix}0&1\\1&0\end{pmatrix};$$

$$E_0^{p-f}=\sqrt{1-\frac{p}{2}}\begin{pmatrix}1&0\\0&1\end{pmatrix},\quad E_0^{p-f}=\sqrt{\frac{p}{2}}\begin{pmatrix}1&0\\0&-1\end{pmatrix};$$

$$E_0^{bp-f}=\sqrt{1-\frac{p}{2}}\begin{pmatrix}1&0\\0&1\end{pmatrix},\quad E_0^{bp-f}=\sqrt{\frac{p}{2}}\begin{pmatrix}0&-i\\i&0\end{pmatrix}.$$

here, $0\le p\le 1$ is the probability that the noise act on the qubit. Wang et al.[11] calculated the super quantum discord of Bell-diagonal states that with three parameters under phase flip channel. Li et al.[13] studied the super quantum discord for X states with six parameters under bit flip channel. Wang et al.[15] calculated the maximal Holevo quantity of X states under bit flip channel, phase flip channel, bit phase flip channel and generalized amplitude damping channel. Eftekhari[16] evaluated the super quantum discord of X states with four parameters under phase flip channel.

This paper evaluate the super quantum discord of the X state with four parameters under three kinds of channels. It is arranged as follows: in Sec. II, we meticulously evaluate the super quantum discord under bit flip channel, and in Sec. III, we calculate it under phase flip channel and bit phase flip channel. In Sec. IV, we make a comparison of the super quantum discord of the X state under these three channels with different measurement strength and different probability. In Sec. V, we make final conclusion.

II. Super quantum discord for X states with four parameters under bit flip channel

The super quantum discord for bipartite quantum state with weak measurement on subsystem B is given by[10]

$$D_w\left(\rho^{AB}\right)=\min_{\{P^B(x)\}} S_w\left(A|\left\{P^B(x)\right\}\right)+S\left(\rho^B\right)-S\left(\rho^{AB}\right),$$

where $S(\rho)=-tr(\rho\log\rho)$ is the von Neumann entropy, ρ^B is the reduced density matrices of ρ^{AB}, $\{P^B(x)\}$ is weak measurement performed on subsystem B, and

$$S_w\left(A|\left\{P^B(x)\right\}\right)=p(+x)S\left(\rho^A|P^B(x)\right)+p(-x)S\left(\rho^A|P^B(-x)\right),$$

$$\mathrm{p}(\pm x)=\mathrm{tr}\left[\left(\mathrm{I}\otimes\mathrm{P}^B(\pm x)\right)\rho^{AB}\left(\mathrm{I}\otimes\mathrm{P}^B(\pm x)\right)\right],$$

$$\rho^A|P^B(\pm\mathrm{x})=\frac{tr_B\left[\left(I\otimes P^B(\pm\mathrm{x})\right)\rho^{AB}\left(I\otimes P^B(\pm x)\right)\right]}{tr\left[I\otimes P^B(\pm x)\rho^{AB}\left(I\otimes P^B(\pm x)\right)\right]}.$$

Let the computational basis of Hilbert space $C^2\otimes C^2$ be $\{|00\rangle,|01\rangle,|10\rangle,|11\rangle\}$, then any state can be parameterized as[14]

$$\rho_0=\frac{1}{4}\left(I+I\otimes\vec{a}\,\vec{\sigma}+\vec{b}\,\vec{\sigma}\otimes I+\sum_{j=1}^{3}c_j\sigma_j\otimes\sigma_j\right),$$

here, $\vec{a}=(a_1,a_2,a_3)$, $\vec{b}=(b_1,b_2,b_3)$, $a_i,b_i,c_i,i=1,2,3$ are all real numbers. σ_j $j=1,2,3$ are Pauli matrices with $\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix}$, $\sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix}$ and $\sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$.

Now, we consider the following two qubit state with four parameters:

$$\rho_0{}^{AB}=\frac{1}{4}(I+I\otimes s\cdot\sigma_3+c_1\sigma_1\otimes\sigma_1+c_2\sigma_2\otimes\sigma_2+c_3\sigma_3\otimes\sigma_3),$$

here, the four parameters c_1, c_2, c_3, s satisfy $|c_1|<|c_2|<|c_3|<1$ and $0<|s|<1-|c_3|$.

Obviously, the state ρ_0^{AB} under the bit flip channel will be

$$\rho_1^{AB}=\frac{1}{4}\left(I\otimes I+(1-p)I\otimes s\sigma_3+c_1\sigma_1\otimes\sigma_1+(1-p)^2c_2\sigma_2\otimes\sigma_2+(1-p)^2c_3\sigma_3\otimes\sigma_3\right), \qquad (1)$$

then the eigenvalues of (1) are as follows

$$\lambda_{1,2}=\frac{1}{4}\left(1-(1-p)^2c_3\pm\sqrt{(1-p)^2s^2+\left(c_1+(1-p)^2c_2\right)^2}\right),$$

$$\lambda_{3,4}=\frac{1}{4}\left(1+(1-p)^2c_3\pm\sqrt{(1-p)^2s^2+\left(c_1-(1-p)^2c_2\right)^2}\right).$$

So the entropy of (1) is,

$$S\left(\rho_1^{AB}\right)=-\sum_{i=1}^{4}\lambda_i\log\lambda_i$$

$$=2-\frac{1}{4}\Bigg[\left(1-(1-p)^2c_3+\sqrt{(1-p)^2s^2+\left(c_1+(1-p)^2c_2\right)^2}\right)\log\left(1-(1-p)^2c_3+\sqrt{(1-p)^2s^2+\left(c_1+(1-p)^2c_2\right)^2}\right)$$

$$+\left(1-(1-p)^2c_3-\sqrt{(1-p)^2s^2+\left(c_1+(1-p)^2c_2\right)^2}\right)\log\left(1-(1-p)^2c_3-\sqrt{(1-p)^2s^2+\left(c_1+(1-p)^2c_2\right)^2}\right)$$

$$+\left(1+(1-p)^2c_3+\sqrt{(1-p)^2s^2+\left(c_1-(1-p)^2c_2\right)^2}\right)\log\left(1+(1-p)^2c_3+\sqrt{(1-p)^2s^2+\left(c_1-(1-p)^2c_2\right)^2}\right)$$

$$+\left(1+(1-p)^2c_3-\sqrt{(1-p)^2s^2+\left(c_1-(1-p)^2c_2\right)^2}\right)\log\left(1+(1-p)^2c_3-\sqrt{(1-p)^2s^2+\left(c_1-(1-p)^2c_2\right)^2}\right)\Bigg].$$

Since the partial trace operator is $\rho_1^B=tr_A(\rho_1^{AB})=\frac{1}{2}\begin{pmatrix}1+(1-p)s & 0\\ 0 & 1-(1-p)s\end{pmatrix}$,

$$S\left(\rho^B\right)=1-\frac{1+(1-p)s}{2}\log\left(1+(1-p)s\right)-\frac{1-(1-p)s}{2}\log\left(1-(1-p)s\right).$$

Any weak measurement operators on $C^2\otimes C^2$ can be written as

$$I\otimes P(\pm x)=\sqrt{\frac{1\mu\tanh x}{2}}I\otimes V\Pi_0V^{+}+\sqrt{\frac{1\pm\tanh x}{2}}I\otimes V\Pi_1V^{+}.$$

for any unitary $V\in U(2)$, moreover, any unitary V can be represented as $V=tI+i\vec{y}\cdot\vec{\sigma}$, with $t\in R$, $\vec{y}=(y_1,y_2,y_3)\in R^3$ and $t^2+y_1^2+y_2^2+y_3^2=1$.

Obviously,

$$V^{+}\sigma_1 V=\left(t^2+y_1^2-y_2^2-y_3^2\right)\sigma_1+2\left(ty_3+y_1y_2\right)\sigma_2+2\left(-ty_2+y_1y_3\right)\sigma_3,$$

$$V^{+}\sigma_2 V=2\left(-ty_3+y_1y_2\right)\sigma_1+2\left(t^2+y_2^2-y_1^2-y_3^2\right)\sigma_2+2\left(ty_1+y_2y_3\right)\sigma_3,$$

$$V^{+}\sigma_3 V=2\left(ty_2+y_1y_3\right)\sigma_1+2\left(-t\,y_1+y_2y_3\right)\sigma_2+\left(t^2+y_3^2-y_1^2-y_2^2\right)\sigma_3,$$

$$\pi_0\sigma_3\pi_0=\pi_0,\ \ \pi_1\sigma_3\pi_1=-\pi_1,\ \ \pi_j\sigma_k\pi_j=0 \text{ for } \ j=0,1,\ \ k=1,2.$$

Assume that

$$z_1=2\left(-ty_2+y_1y_3\right),\ \ z_2=2\left(t\,y_1+y_2y_3\right),\ \ z_3=\left(t^2+y_3^2-y_1^2-y_2^2\right),$$

$$\eta_0=\sqrt{\frac{1+\tanh x}{2}},\ \ \eta_1=\sqrt{\frac{1-\tanh x}{2}}.$$

In order to figure out $S_w(\rho_1^A\,|\,\{P^B(x)\})$, we evaluate $p(\pm x)$ at first.

$$\left(I\otimes P^B(+x)\right)\rho_1^{AB}\left(I\otimes P^B(+x)\right)$$

$$=\frac{1}{4}[I\otimes V\left(\eta_0\Pi_0+\eta_1\Pi_1\right)V^{+}]\cdot[I\otimes I+(1-p)I\otimes s\cdot\sigma_3+c_1\sigma_1\otimes\sigma_1$$

$$+(1-p)^2c_2\sigma_2\otimes\sigma_2+(1-p)^2c_3\sigma_3\otimes\sigma_3]\cdot[I\otimes V\left(\eta_0\Pi_0+\eta_1\Pi_1\right)V^{+}]$$

$$=\frac{1}{4}I\otimes V\left(\eta_0\Pi_0+\eta_1\Pi_1\right)V^{+}V\left(\eta_0\Pi_0+\eta_1\Pi_1\right)V^{+}+\frac{(1-p)s}{4}I\otimes V\left(\eta_0\Pi_0+\eta_1\Pi_1\right)V^{+}\sigma_3V\left(\eta_0\Pi_0+\eta_1\Pi_1\right)V^{+}$$

$$+\frac{c_1}{4}\sigma_1\otimes V\left(\eta_0\Pi_0+\eta_1\Pi_1\right)V^{+}\sigma_1V\left(\eta_0\Pi_0+\eta_1\Pi_1\right)V^{+}+\frac{(1-p)^2c_2}{4}\sigma_1\otimes V\left(\eta_0\Pi_0+\eta_1\Pi_1\right)V^{+}\sigma_2V\left(\eta_0\Pi_0+\eta_1\Pi_1\right)V^{+}$$

$$+\frac{(1-p)^2c_3}{4}\sigma_3\otimes V\left(\eta_0\Pi_0+\eta_1\Pi_1\right)V^{+}\sigma_3V\left(\eta_0\Pi_0+\eta_1\Pi_1\right)V^{+}.$$

Then

$$p(+x)=tr\left[\left(I\otimes P^B(\pm x)\right)\rho_1^{AB}\left(I\otimes P^B(\pm x)\right)\right]$$

$$=\frac{1}{4}\times 2\times tr\left(\eta_0^2\Pi_0^2+\eta_1^2\Pi_1^2+(1-p)sz_3(\eta_0^2\Pi_0^2-\eta_1^2\Pi_1^2)\right)=\frac{1}{2}\left(1-(1-p)sz_3\tanh x\right).$$

Similarly, $p(-x)=\dfrac{1}{2}\left[1+(1-p)sz_3\tanh x\right]$.

Thus,

$$\rho_1{}^A \mid P^B(+x) = \frac{1}{p^B(+x)} tr_B[(I \otimes P^B(\pm x))\rho_1{}^{AB}(I \otimes P^B(\pm x))]$$

$$= \frac{2}{(1-(1-p)sz_3\tanh x)} \mathrm{tr}_B\left[\frac{1}{4} I \otimes (\eta_0^2\Pi_0^2 + \eta_1^2\Pi_1^2) + \frac{(1-p)s}{4} z_3 I \otimes (\eta_0^2\Pi_0^2 - \eta_1^2\Pi_1^2)\right.$$

$$+ \frac{c_1}{4} z_1\sigma_1 \otimes (\eta_0^2\Pi_0^2 - \eta_1^2\Pi_1^2) + \frac{c_2(1-p)^2}{4} z_2\sigma_2 \otimes (\eta_0^2\Pi_0^2 - \eta_1^2\Pi_1^2)$$

$$\left. + \frac{c_3(1-p)^2}{4} z_3\sigma_3 \otimes (\eta_0^2\Pi_0^2 - \eta_1^2\Pi_1^2)\right].$$

Since, $\eta_0^2 + \eta_1^2 = 1, \eta_0^2 - \eta_1^2 = -\tanh x$, then

$$\rho_1{}^A \mid P^B(+x) = \frac{[I - \tanh x((1-p)sz_3 I + c_1\sigma_1 z_1 + (1-p)^2 c_2\sigma_2 z_2 + (1-p)^2 c_3\sigma_3 z_3)]}{2(1-(1-p)sz_3 \tanh x)},$$

Similarly,

$$\rho_1{}^A \mid P^B(-x) = \frac{[I + \tanh x((1-p)sz_3 I + c_1\sigma_1 z_1 + (1-p)^2 c_2\sigma_2 z_2 + (1-p)^2 c_3\sigma_3 z_3)]}{2(1+(1-p)sz_3 \tanh x)}.$$

Let

$$M = \tanh x((1-p)sz_3 I + c_1\sigma_1 z_1 + (1-p)^2 c_2\sigma_2 z_2 + (1-p)^2 c_3 z_3\sigma_3),$$

$$\phi = (1-p)sz_3, \quad \theta = \sqrt{|c_1 z_1|^2 + |c_2 z_2(1-p)^2|^2 + |c_3 z_3(1-p)^2|^2},$$

and the eigenvalues of $\frac{1}{2}(I+M)$ and $\frac{1}{2}(I+M)$ are λ_5, λ_6 and λ_7, λ_8 respectively,

i.e.,

$$\lambda_5 = \frac{1+\tanh x(\phi+\theta)}{2(1-\phi\tanh x)}, \quad \lambda_6 = \frac{1+\tanh x(\phi-\theta)}{2(1-\phi\tanh x)},$$

$$\lambda_7 = \frac{1-\tanh x(\phi+\theta)}{2(1+\phi\tanh x)}, \quad \lambda_8 = \frac{1-\tanh x(\phi-\theta)}{2(1+\phi\tanh x)}.$$

So

$$S_w(A \mid P^B(\pm x)) = \sum_{i=5}^{8} \lambda_i \log_2 \lambda_i$$

$$= \frac{1}{4}\left[(1+\tanh x(\phi+\theta))\log_2\left(\frac{1+\tanh x(\phi+\theta)}{2(1-\phi\tanh x)}\right) + (1+\tanh x(\phi-\theta))\log_2\left(\frac{1+\tanh x(\phi-\theta)}{2(1-\phi\tanh x)}\right)\right.$$

$$(1+\tanh x(-\phi+\theta))\log_2\left(\frac{1+\tanh x(-\phi+\theta)}{2(1+\phi\tanh x)}\right)+(1+\tanh x(-\phi-\theta))\log_2\left(\frac{1+\tanh x-(\phi-\theta)}{2(1+\phi\tanh x)}\right)\Bigg].$$

By use of the domain scope of logarithmic function $f(\phi,\theta)$, we obtain the range of ϕ and θ: $0\le|c_1|\le\theta\le|c_3|\le1$, $-1<\phi<1$.

We can verify that $f(-f,q)=f(f,q)$, then the graph of $f(f,q)$ is symmetrical with respect to the q-axis, furthermore,

$$\frac{\partial f}{\partial\theta}=-\frac{1}{2}\log_2\left[\frac{[(1+\tanh x\cdot\theta)^2-(\tanh x\cdot\phi)^2](1+\phi\tanh x)^2}{[(1-\tanh x\cdot\theta)^2-(\tanh x\cdot\phi)^2](1-\phi\tanh x)^2}\right]<0,\ 0<q<1,$$

$$\frac{\partial f}{\partial\phi}=-\frac{1}{2}\log_2\left[\frac{[(1+\tanh x\cdot\phi)^2-(\tanh x\cdot\theta)^2](1+\phi\tanh x)^2}{[(1-\tanh x\cdot\phi)^2-(\tanh x\cdot\theta)^2](1-\phi\tanh x)^2}\right]<0,\ 0<\phi<1,$$

hence $f(f,q)$ is a monotonic decreasing function. When $q=|c_3|$, we can obtain $\phi=|s|$. So $f(f,q)$ can obtain the minimum at the point $(|s|,|c_3|)$, and the minimum of $f(f,q)$ is given by

$$\min_{\{\Pi_i^B\}}S_w(A|\{P^B(x)\})=$$

$$-\frac{1}{4}\Bigg[(1+\tanh x(c_3+s))\log_2\left(\frac{1+\tanh x(c_3+s)}{2(1-s\tanh x)}\right)+(1+\tanh x(c_3-s))\log_2\left(\frac{1+\tanh x(c_3-s)}{2(1-s\tanh x)}\right)+$$

$$(1+\tanh x(-c_3+s))\log_2\left(\frac{1+\tanh x(-c_3+s)}{2(1+s\tanh x)}\right)+(1+\tanh x(-c_3-s))\log_2\left(\frac{1+\tanh x(-c_3-s)}{2(1+s\tanh x)}\right)\Bigg]$$

At last, we get value of super quantum discord of ρ_1^{AB} under bit flip channel:

$$D_w(\rho_1^{AB})=\min_{\{P^B(x)\}}S_w(A|\{P^B(x)\})+S(\rho_1^B)-S(\rho_1^{AB})$$

$$=\frac{1}{4}\Bigg[\left(1-(1-p)^2c_3+\sqrt{(1-p)^2s^2+(c_1+(1-p)^2c_2)^2}\right)\log\left(1-(1-p)^2c_3+\sqrt{(1-p)^2s^2+(c_1+(1-p)^2c_2)^2}\right)$$

$$+\left(1-(1-p)^2c_3-\sqrt{(1-p)^2s^2+(c_1+(1-p)^2c_2)^2}\right)\log\left(1-(1-p)^2c_3-\sqrt{(1-p)^2s^2+(c_1+(1-p)^2c_2)^2}\right)$$

$$+\left(1+(1-p)^2c_3+\sqrt{(1-p)^2s^2+(c_1-(1-p)^2c_2)^2}\right)\log\left(1+(1-p)^2c_3+\sqrt{(1-p)^2s^2+(c_1-(1-p)^2c_2)^2}\right)$$

$$+\left(1+(1-p)^2c_3-\sqrt{(1-p)^2s^2+\left(c_1-(1-p)^2c_2\right)^2}\right)\log\left(1+(1-p)^2c_3-\sqrt{(1-p)^2s^2+\left(c_1-(1-p)^2c_2\right)^2}\right)\Big]$$

$$-\frac{1+(1-p)s}{2}\log(1+(1-p)s)-\frac{1-(1-p)s}{2}\log(1-(1-p)s)$$

$$-\frac{1}{4}\Big[(1+\tanh x(c_3+s))\log_2\left(\frac{1+\tanh x(c_3+s)}{1-s\tanh x}\right)+(1+\tanh x(c_3-s))\log_2\left(\frac{1+\tanh x(c_3-s)}{1-s\tanh x}\right)$$

$$(1+\tanh x(-c_3+s))\log_2\left(\frac{1+\tanh x(-c_3+s)}{1+s\tanh x}\right)+(1+\tanh x(-c_3-s))\log_2\left(\frac{1+\tanh x(-c_3-s)}{1+s\tanh x}\right)\Big]. \quad (2)$$

III. Super quantum discord for X states with four parameters under phase flip channel and bit phase flip channel

In this section, we consider the influence of two channels: the phase flip channel and bit phase flip channel on super quantum discord.

Firstly, if the state ρ_0^{AB} undergoes the phase flip channel, we have

$$\rho_2^{AB}=\frac{1}{4}(I\otimes I+I\otimes s\cdot\sigma_3+(1-p)^2c_1\sigma_1\otimes\sigma_1+(1-p)^2c_2\sigma_2\otimes\sigma_2+c_3\sigma_3\otimes\sigma_3), \quad (3)$$

similar to the calculation of Sec.II, the super quantum discord of (3) can be obtained asa follow,

$$D_w\left(\rho_2^{AB}\right)=\min_{\{P^B(x)\}}S_w\left(A|\{P^B(x)\}\right)+S\left(\rho_2^{B}\right)-S\left(\rho_2^{AB}\right)$$

$$=\frac{1}{4}[\left(1-c_3+\sqrt{s^2+(1-p)^4(c_1+c_2)^2}\right)\log\left(1-c_3+\sqrt{s^2+(1-p)^4(c_1+c_2)^2}\right)$$

$$+\left(1-c_3-\sqrt{s^2+(1-p)^4(c_1+c_2)^2}\right)\log\left(1-c_3-\sqrt{s^2+(1-p)^4(c_1+c_2)^2}\right)$$

$$+\left(1+c_3+\sqrt{s^2+(1-p)^4(c_1-c_2)^2}\right)\log\left(1+c_3+\sqrt{s^2+(1-p)^4(c_1-c_2)^2}\right)$$

$$+\left(1+c_3-\sqrt{s^2+(1-p)^4(c_1-c_2)^2}\right)\log\left(1+c_3-\sqrt{s^2+(1-p)^4(c_1-c_2)^2}\right)]$$

$$-\frac{1+s}{2}\log(1+s)-\frac{1-s}{2}\log(1-s)$$

$$-\frac{1}{4}\Big[(1+\tanh x(c_3+s))\log_2\left(\frac{1+\tanh x(c_3+s)}{1-s\tanh x}\right)+(1+\tanh x(c_3-s))\log_2\left(\frac{1+\tanh x(c_3-s)}{1-s\tanh x}\right)$$

$$(1+\tanh x(-c_3+s))\log_2\left(\frac{1+\tanh x(-c_3+s)}{1+s\tanh x}\right)+(1+\tanh x(-c_3-s))\log_2\left(\frac{1+\tanh x(-c_3-s)}{1+s\tanh x}\right)\Bigg]. \quad (4)$$

Remark. The above result is same as the results calculated by Eftekhari el at.[16]

Next, if the X state ρ_0^{AB} undergoes the bit phase flip channel, we have

$$\rho_3^{AB}=\frac{1}{4}(I\otimes I+(1-p)I\otimes s\cdot\sigma_3+(1-p)^2c_1\sigma_1\otimes\sigma_1+c_2\sigma_2\otimes\sigma_2+(1-p)^2c_3\sigma_3\otimes\sigma_3), \quad (5)$$

And the super quantum discord of (5) can be calculated respectively as,

$$D_w\left(\rho_3^{AB}\right)=\min_{\{P^B(x)\}}S_w\left(A\,|\{P^B(x)\}\right)+S\left(\rho_3^{B}\right)-S\left(\rho_3^{AB}\right)$$

$$=\frac{1}{4}\Bigg[\left(1-(1-p)^2c_3+\sqrt{(1-p)^2s^2+((1-p)^2c_1+c_2)^2}\right)\log\left(1-(1-p)^2c_3+\sqrt{(1-p)^2s^2+((1-p)^2c_1+c_2)^2}\right)$$

$$+\left(1-(1-p)^2c_3-\sqrt{(1-p)^2s^2+((1-p)^2c_1+c_2)^2}\right)\log\left(1-(1-p)^2c_3-\sqrt{(1-p)^2s^2+((1-p)^2c_1+c_2)^2}\right)$$

$$+\left(1+(1-p)^2c_3+\sqrt{(1-p)^2s^2+((1-p)^2c_1-c_2)^2}\right)\log\left(1+(1-p)^2c_3+\sqrt{(1-p)^2s^2+((1-p)^2c_1-c_2)^2}\right)$$

$$+\left(1+(1-p)^2c_3-\sqrt{(1-p)^2s^2+((1-p)^2c_1-c_2)^2}\right)\log\left(1+(1-p)^2c_3-\sqrt{(1-p)^2s^2+((1-p)^2c_1-c_2)^2}\right)\Bigg]$$

$$-\frac{1+(1-p)s}{2}\log(1+(1-p)s)-\frac{1-(1-p)s}{2}\log(1-(1-p)s)$$

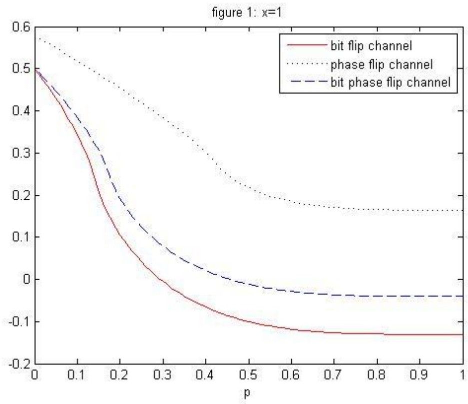
$$-\frac{1}{4}\Bigg[(1+\tanh x(c_3+s))\log_2\left(\frac{1+\tanh x(c_3+s)}{1-s\tanh x}\right)+(1+\tanh x(c_3-s))\log_2\left(\frac{1+\tanh x(c_3-s)}{1-s\tanh x}\right)$$

$$(1+\tanh x(-c_3+s))\log_2\left(\frac{1+\tanh x(-c_3+s)}{1+s\tanh x}\right)+(1+\tanh x(-c_3-s))\log_2\left(\frac{1+\tanh x(-c_3-s)}{1+s\tanh x}\right)\Bigg]. \quad (6)$$

IV. Comparisons

To compare the super quantum discord of X state ρ_0^{AB} under three channels, we draw the graphs of $c_1=0.2, c_2=0.4, c_3=0.6, s=0.3$ when $x=1$ in figure 1 and $x=5$ in figure 2, we can find that the super quantum discord of the X state when $c_1=0.2, c_2=0.4, c_3=0.6, s=0.3$ is vary from channel to channel if $x=1$ or $x=5$. And the order of super quantum discord under different channels from big to small is phase flip channel, bit phase flip channel, bit flip channel. i.e., $D_\omega(\rho_2^{AB}) \geq D_\omega(\rho_3^{AB}) \geq D_\omega(\rho_1^{AB})$. But the values of the later two is the same when $p=0$. In addition, the greater probability leads to the smaller super quantum discord.

In figure 3 and figure 4, we assume the probability that noise act on the qubit is $p=0.5$ and $p=0.3$ respectively, and then we draw the graphs about super quantum discord and measurement strength. It can be seen that with the increasing of measurement strength, super quantum discord is monotonic decreasing in the three channels and the order of it from big to small is still phase flip channel, bit phase flip channel, bit flip channel, i.e., $D_\omega(\rho_2^{AB}) \geq D_\omega(\rho_3^{AB}) \geq D_\omega(\rho_1^{AB})$. Besides, when $x=0$, the value is the same under bit flip channel and bit phase flip channel. As a result, we can use different channels to obtain different targets.



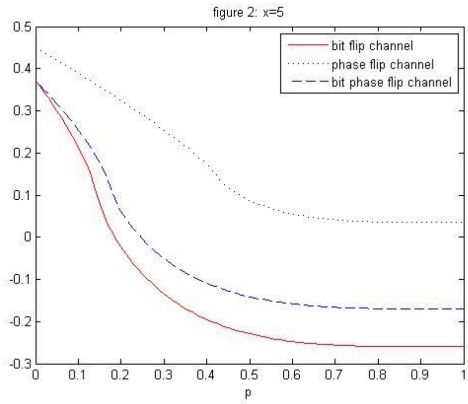
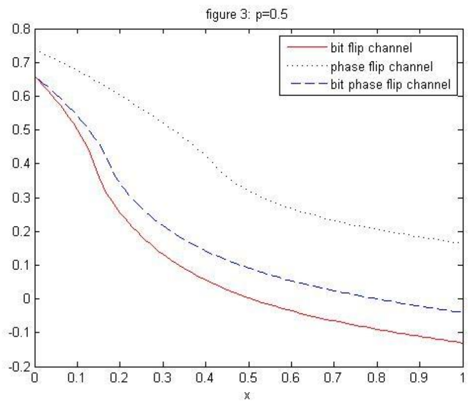


Figure 1. Super quantum discord of X states under the bit flip channel, phase flip channel and bit phase flip channel when $c_1=0.2, c_2=0.4, c_3=0.6, s=0.3, x=1$.

Figure 2. Super quantum discord of X states under the bit flip channel, phase flip channel and bit phase flip channel when $c_1=0.2, c_2=0.4, c_3=0.6, s=0.3, x=5$.



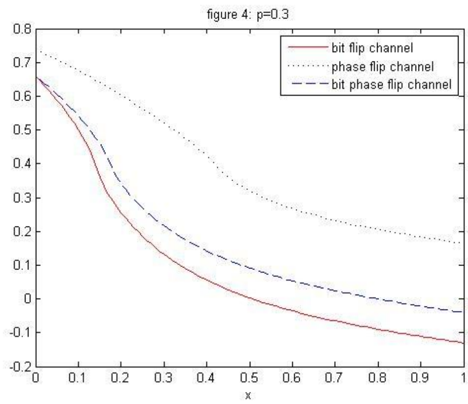


Figure 3. Super quantum discord of X state ($c_1 = 0.2, c_2 = 0.4, c_3 = 0.6, s = 0.2$) under the bit flip channel, phase flip channel and bit phase flip channel when $p = 0.5$.

Figure 4. Super quantum discord of X state ($c_1 = 0.2, c_2 = 0.4, c_3 = 0.6, s = 0.2$) under the bit flip channel, phase flip channel and bit phase flip channel when $p = 0.3$.

V. Conclusion

In this paper, we evaluate the super quantum discord of a class of X states ρ_0^{AB} with four parameters under bit flip channel, phase flip channel and bit phase flip channel. And we compare the three values of super quantum discord and find that with the increasing of measure strengh and probability, the value of super quantum discord is decreasing, moreover, the order of the value of super quantum discord under different channels from big to small is phase flip channel, bit phase flip channel, bit flip channel.

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