MATHEMATICAL MODELLING, SIMULATION AND EXPERIMENTAL VERIFICATION OF A PNEUMATIC SYSTEM

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ABSTRACT

In this article we have introduced the mathematical model of a typical pneumatic system, consisting of pneumatic cylinder, proportional electromagnetic valve, connecting tubes. Based on this mathematical model we have built in LabView a simulation program. We compared the results obtained from the simulation with the experimental measurements made in the experimental panel. The matching of the displacement and pressure characteristics in the cylinder chambers indicate the authenticity of the mathematical model which can be used for the constructive improvement of these components as well as for their PID control.

Keywords: Pneumatics, Simulation, Mathematical Model, Pneumatic Cylinder, LabView.

INTRODUCTION

A typical pneumatic system includes a force element (the pneumatic cylinder), a command device (valve), connecting tubes, and position, pressure and force sensors. The external load consists of the mass of external mechanical elements connected to the piston and perhaps a force produced by environmental interaction. A schematic representation of the pneumatic actuator system is shown in Fig. (1), with variables of interest specified for each component.

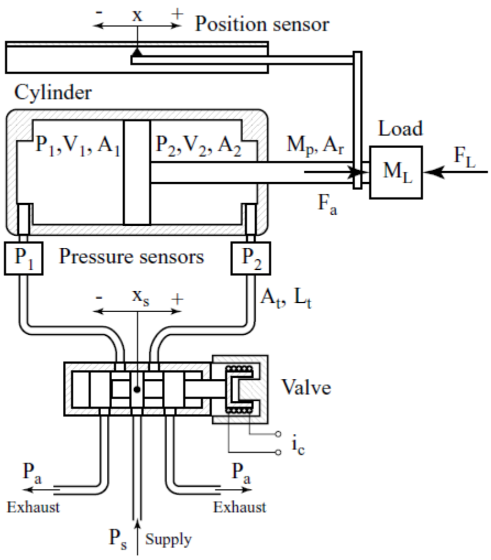


Figure.1. Schematic representation of the pneumatic cylinder-valve system

For our study we have take in consideration one proportional pneumatic electro vale of the type VER 2000 product of the SMC company, One pneumatic cylinder with diameter D=25mm. That valve is of the type 2/3 with 5 holes and can be commanded through the electric signals and it's outlet depend by the intensity of electric current. Fig.2 tells the experimental panel composed by VER 2000 electro valve, pneumatic cylinder, tubes, sensors, electric drivers etc.

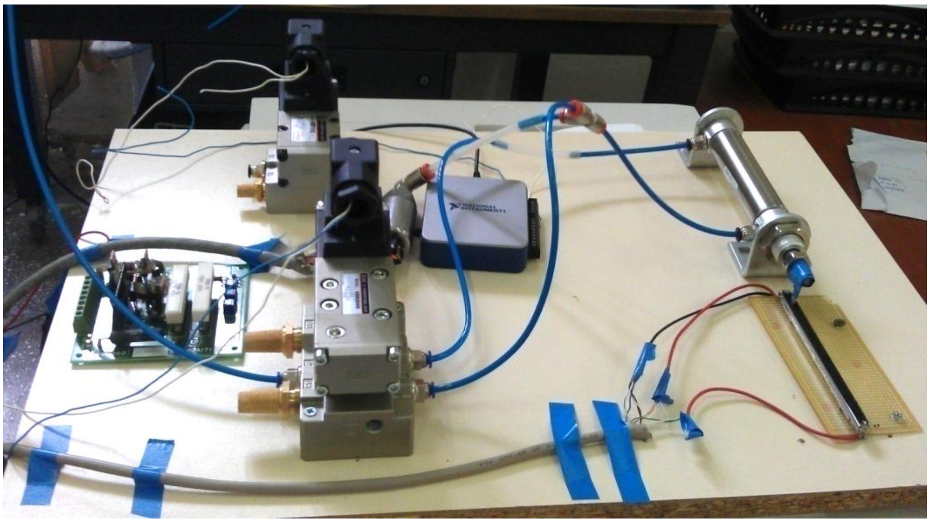


Figure.2. Experimental panel

The construction of this valves presents a number of advantages, such as: almost linear flow characteristics of the air flowing through it, good sealing to internal leaks, the ability to balance both rooms under pressure using only one control signal, Very small internal friction between friction elements.
The pneumatic cylinder is of the type dual acting with diameter D=25mm and the piston rod length L=70mm.

MATHEMATIC MODEL

The equation of motion for the piston-rod-load assembly can be expressed as,

$$(M_L+M_P)\ddot{x}+\beta\dot{x}+F_f+F_L = P_1A1-P_2A_2-P_aA_r \qquad (1)$$

where M_L is the external load mass, M_p is the piston and rod assembly mass, x is the piston position, β is the viscous friction coefficient, F_f is the Coulomb friction force, F_L is the external force, P_1 and P_2 are the absolute pressures in actuator's chambers, P_a is the absolute ambient pressure, A_1 and A_2 are the piston effective areas, and A_r is the rod cross sectional area. The right hand side of Eq. (1) represents the actuator active force, produced by the different pressures acting on the opposite sides of the piston. In order to control the actuator force output, one has to finely tune the pressure levels in the cylinder chambers using the command element (the pneumatic valve). This requires detailed models for the dynamics of pressure in both chambers of the actuator, valve dynamics, and connecting tubes.
For the construction of the mathematical model of the cylinder chambers, we will refer to the connection between the pressure change, the flow mass and the displacement velocity of the cylinder.
The mathematical model for the gas volume in the most general form consists of three equations:

- Equation of state (ideal gas law)
- Conservation of mass (continuity)
- Energy equation

Considering that: a) the gas is perfect, b) the pressure and temperature inside the room are homogeneous, and c) the potential and kinetic energy are negligible. These equations can be written for each cylinder chamber.
Considering the controlled volume V, the density ρ, the mass m, the pressure P and the temperature T, the ideal gas law can be written as:

$$P = \rho RT \qquad (2)$$

Where, R is the ideal gas constant. Referring to the continuity equation, mass flow may be expressed as:

$$\dot{m} = \frac{d}{dt}(\rho V) \qquad (3)$$

Which can also be expressed as,

$$\dot{m}_{in} - \dot{m}_{out} = \dot{\rho}V + \rho\dot{V} \qquad (4)$$

Where, $\dot{m}_{in}$ and $\dot{m}_{out}$ out are the mass flow of the fluid entering and leaving the room
The energy equation can be written as follows:

$$q_{in} - q_{out} + kC_v(\dot{m}_{in}T_{in} - \dot{m}_{out}T) - \dot{W} = \dot{U} \qquad (5)$$

Where q_{in} and q_{out} are the quantities of heat transferred, k is the specific coefficient of heat, C_v is the specific heat at constant volume, T_{in} is the incoming gas temperature, $\dot{W}$ is the change at work, and $\dot{U}$ is the change in the internal energy.
If we consider the process adiabatic $q_{in} - q_{out} = 0$, the change in time of the chamber pressure is,

$$\dot{P} = k\frac{P}{\rho V}(\dot{m}_{in} - \dot{m}_{out}) - k\frac{P}{V}\dot{V} \qquad (6)$$

$$\dot{P} = \frac{RT}{V}(\alpha_{in}\dot{m}_{in} - \alpha_{out}\dot{m}_{out}) - \alpha\frac{P}{V}\dot{V} \qquad (7)$$

Where α , α_{in} and α_{out} taking values between 1 and k, depending on the actual heat transfer during the process. Choosing the origin of the piston displacement at the middle of the stroke, the volume of each chamber can be expressed as,

$$V_i = V_{0i} + A_i(\frac{1}{2}L \pm x) \qquad (8)$$

Where i = 1.2 is the cylinder chambers index, V_{0i} is the inactive volume at the end of stroke and admission ports, A_i is the piston effective area, L is the piston stroke, and x is the piston position. The difference between the piston effective areas for each chamber A_1 and A_2 is due to the piston rod. Substituting Eq. (8) into (7), the time derivative for the pressure in the pneumatic cylinder chambers becomes:

$$\dot{P}_i = \frac{RT}{V_{0i} + A_i\left(\frac{1}{2}L \pm x\right)}(\alpha_{in}\dot{m}_{in} - \alpha_{out}\dot{m}_{out}) - \alpha\frac{PA_i}{V_{0i} + A_i\left(\frac{1}{2}L \pm x\right)}\dot{x} \qquad (9)$$

The first term in the equation represents the effect on pressure of the air flow in or out of the chamber, and the second term accounts for the effect of piston motion.

MATHEMATICAL MODEL OF ELECTRO VALVE

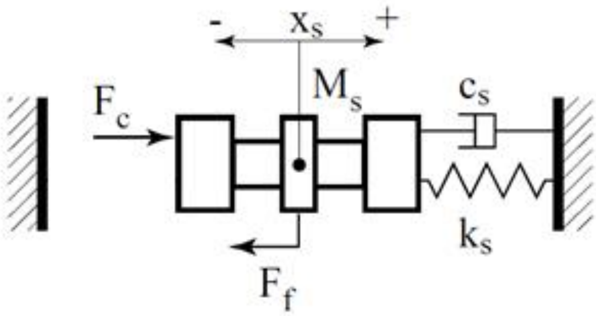


Figure.3. The valve and its elements in balance

During the construction of the mathematical model of the valve we have taken into account two aspects: the dynamics of the movement of the valve elements, and the mass of fluid passing through the variable diameter of the valve hole. Analyzing Figure.3 the movement equation for the electromagnet valve system can be written as follows

$$M_s\ddot{x}_s + c_s\dot{x}_s + F_f + k_s(x_{so} + x_s) = F_c \qquad (10)$$

Where: x_s is the displacement of the floating valve element, x_o is the pre compression of the spring in the equilibrium position, M_s is the system mass, the floating valve element plus the mass of the systems, c_s is the viscosity friction coefficient, F_f is the frictional force, K_s is the spring coefficient and Fc is the force produced by the electromagnet coil.
By simplifying the expression of the forces of the spring we will have:

$$M_s\ddot{x}_s + c_s\dot{x}_s + F_f + k_s x_s = F_c \qquad (11)$$

The friction force F_f can be neglected, considering that the piston (valve inner block) is easily rolling around in the balance position by reducing the friction forces satisfactorily. Referring to the relation of the coil electric power force and neglecting the friction force F_f, the equation (11) is transformed as follows:

$$M_s\ddot{x}_s + c_s\dot{x}_s + k_s x_s = F_c \qquad (12)$$

For the actuating force of the valve electromagnet when passing a given intensity of current we can write:

$$M_s\ddot{x}_s + c_s\dot{x}_s + k_s x_s = 2\pi a N\beta i_c \qquad (13)$$

Where (a) is the coil radius, (N) is the coil number , (β) is the magnetic flux, (i_c) is the currents feeding the coil. The pressure drop during the passage to the valve is relatively high and the gas stream will be treated as printable and turbulent. The equation that expresses the flow of fluid through the valve passage with Av surface is:

$$\dot{m}_v = \begin{cases} C_f\ A_v\ C_1 \frac{P_u}{\sqrt{T}} & if\ \frac{P_d}{P_u} \leq\ P_{cr} \\ C_f\ A_v\ C_1 \frac{P_u}{\sqrt{T}} \left(\frac{P_d}{P_u}\right)^{\frac{1}{k}} \sqrt{1-\left(\frac{P_d}{P_u}\right)^{\frac{(k-1)}{k}}} & if\ \frac{P_d}{P_u} >\ P_{cr} \end{cases} \quad (14)$$

Where $\dot{m}_v$ is the mass flow passing through the valve, C_f is the exhaust coefficient, P_u is the pressure at the valve inlet, P_d is the pressure in the flow direction and

$$C_1 = \sqrt{\frac{k}{R}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}\ ; \quad C_2 = \sqrt{\frac{2k}{R(k-1)}}\ ; \quad P_{cr} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

Are fluid constants , for the air (k=1.4) we have C_1 =0.040418, C_2 =0.156174, and P_{cr}= 0.528. The fluid passage surface in the valve passage hole is determined by the position of the valve piston on the circular hole of the passage on the valve body, as shown in the figure below:

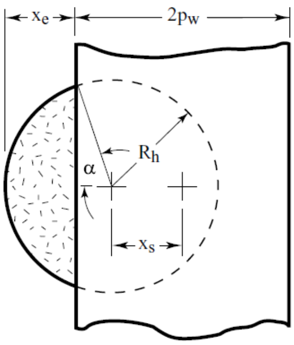


Fig.3 Surface of the hole passage referring the valve piston position

The surface of the circle segment defined by the valve piston position can be expressed as:

$$A_e = 2\int_0^{x_e} \sqrt{R_h^2 - (x_e - R_h)^2}\ dx_e = 2\int_0^{x_e} \sqrt{x_e(2R_h - x_e)^2}\ dx_e \quad (15)$$

Where A_e is the effective fluid flow surface, x_e is the effective displacement of the valve piston and R_h is the hole radius. By integrating the above equation and marking n_h the number of passage holes, the effective passage area of the valve will be:

$$A_v = n_h\ \left(2R_h^2\ arktan\left(\sqrt{\frac{x_e}{2R_h - x_e}}\right) - (R_h - x_e)\sqrt{x_e(2R_h - x_e)}\ \right) \quad (16)$$

The width of the piston rod block ($2p_w$) is slightly larger than the radius of the hole. This is to ensure that the passage is completely closed even in the presence of a small irregularity of the valve elements. Thus the effective displacement of the x_e valve piston will be different from the absolute displacement x_s :

$$x_e = x_s - (p_w - R_h) \quad (17)$$

BUILT PROGRAMS

Based in mathematical model described above we have build e simulation program in LabVIEW, and using the parameters of components in experimental panel we have take the simulation results for displacement of cylinder and the pressure in cylinder chambers.

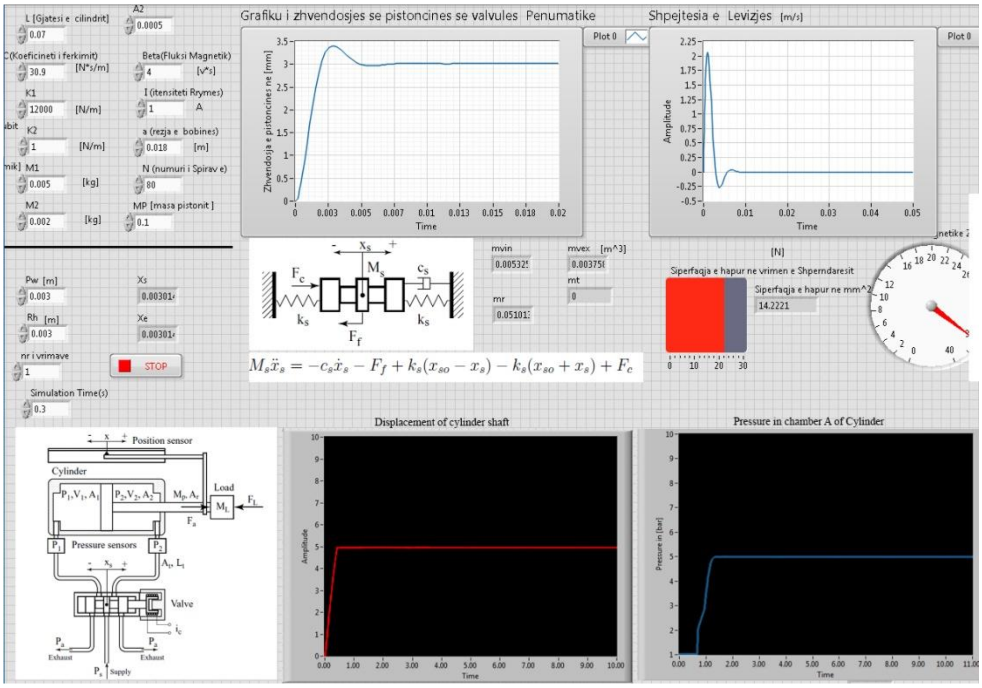
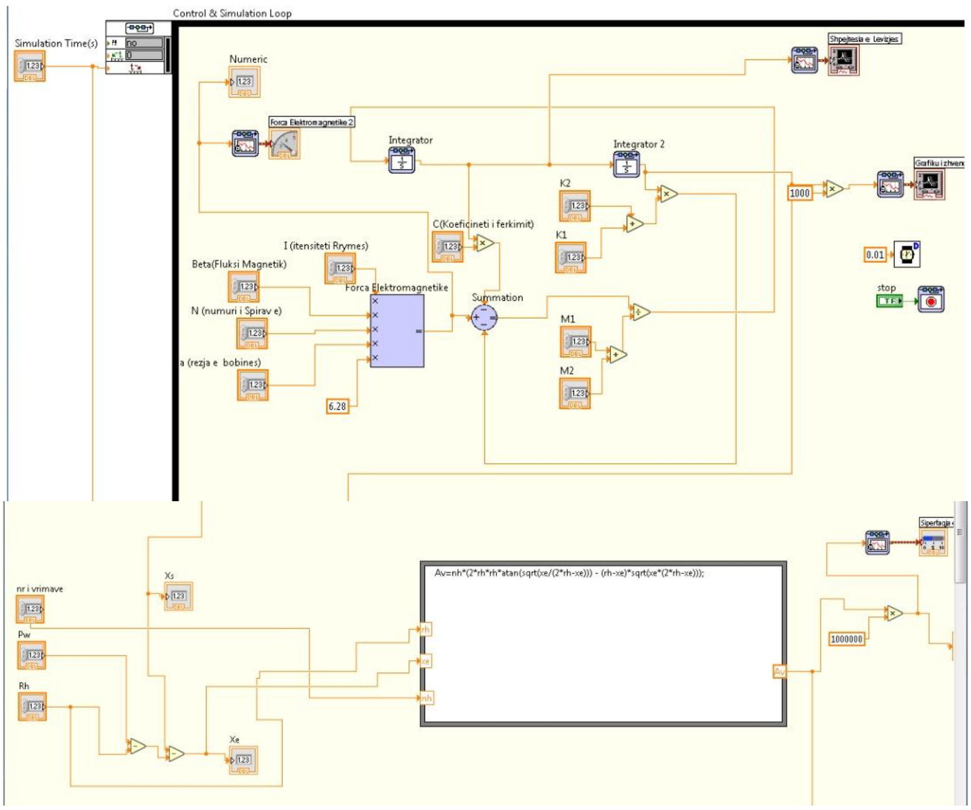
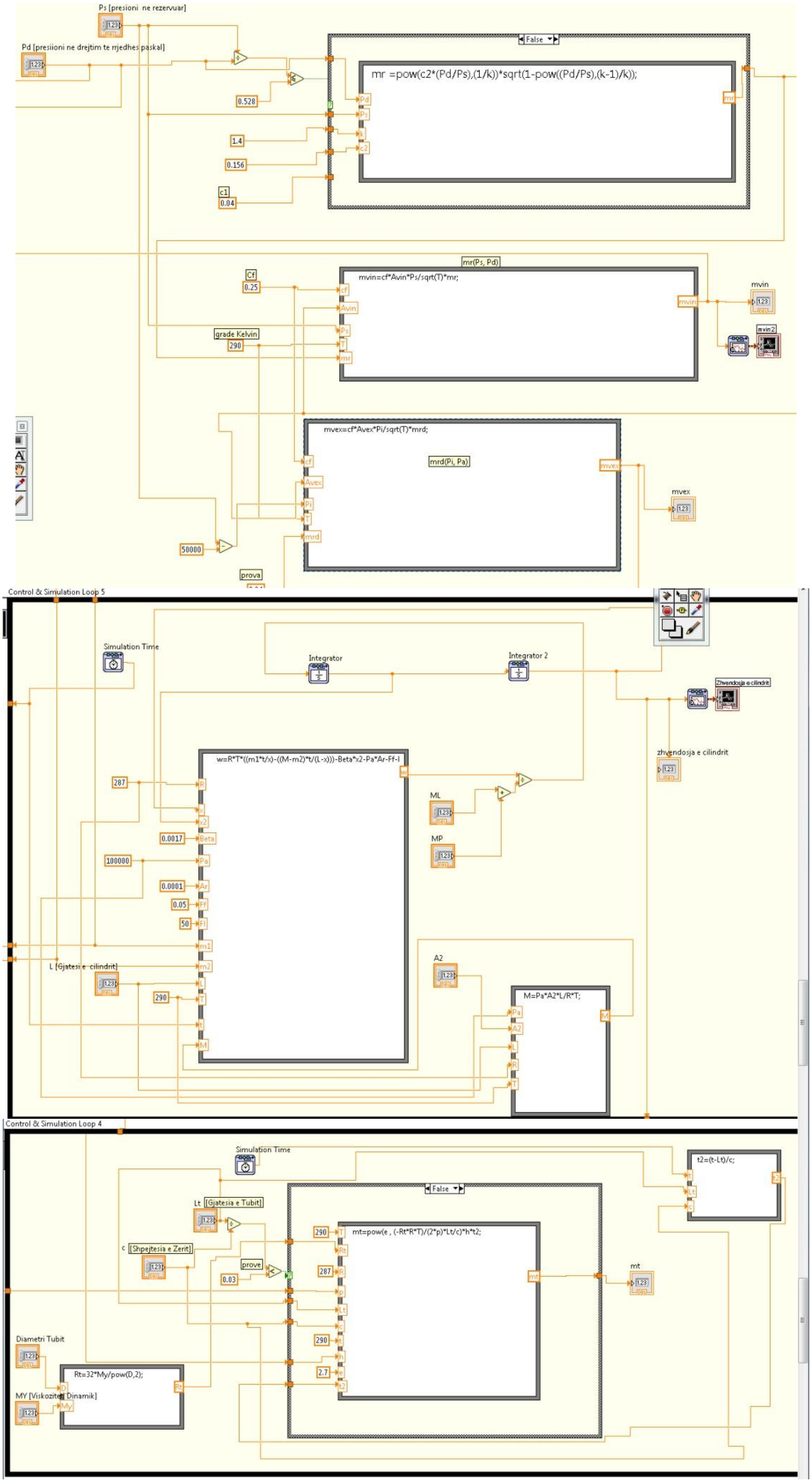


Figure.4. Front Panel of program Builder




mr =pow(c2*(Pd/Ps),(1/k))*sqrt(1-pow((Pd/Ps),(k-1)/k));
mvin=cf*Avin*Ps/sqrt(T)*mr;
mvex=cf*Avex*Pi/sqrt(T)*mrd;
w=R*T*((m1*t/x)-((M-m2)*t/(L-x)))-Beta*x2-Pa*Ar-Ff-I
M=Pa*A2*L/R*T;
t2=(t-Lt)/c;
mt=pow(e , (-Rt*R*T)/(2*p)*Lt/c)*h*t2;
Rt=32*My/pow(D,2);

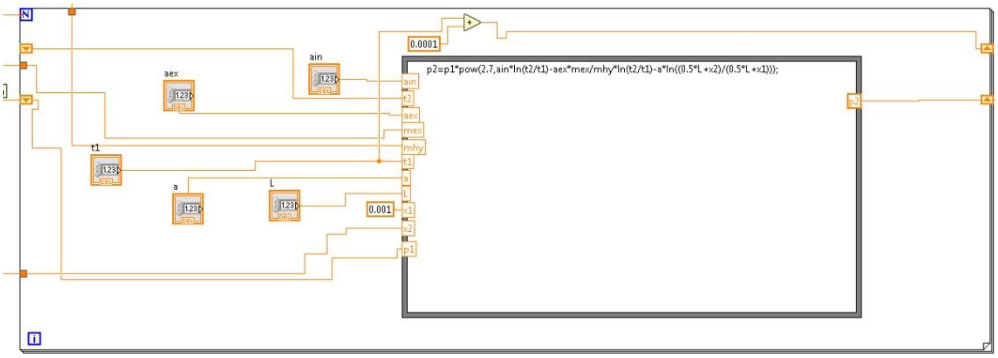


Figure.5. Bock Diagram of program Builder

SIMULATION RESULTS

For the current 1 A in the coil of electro valve and the pressure 5 bar we have take from our program of simulation the graph for displacement of cylinder shaft and pressure in chamber A of cylinder as can be seen se in figure 6 and 7

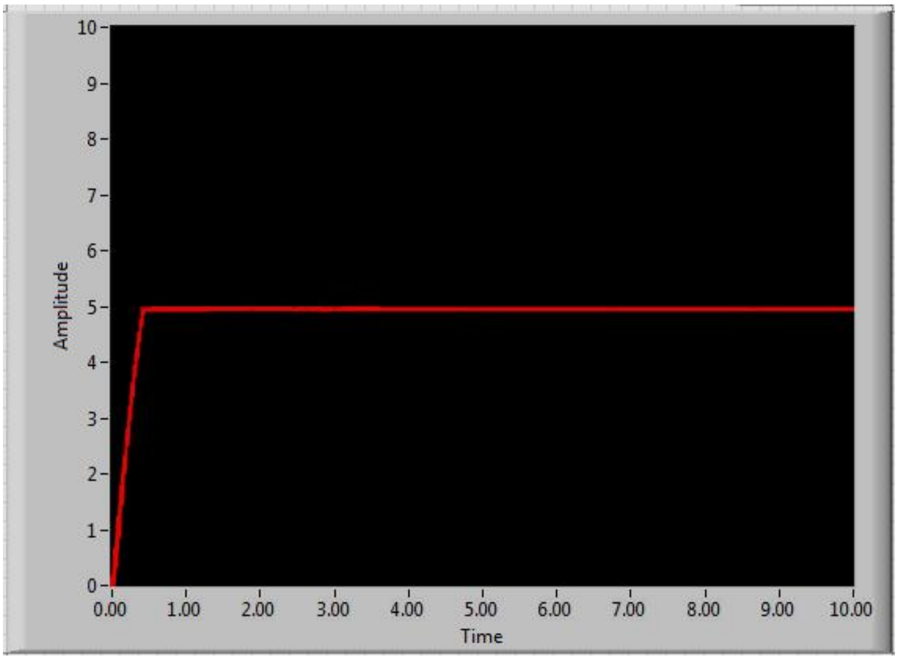


Figure.6. Displacement of Cylinder Shaft

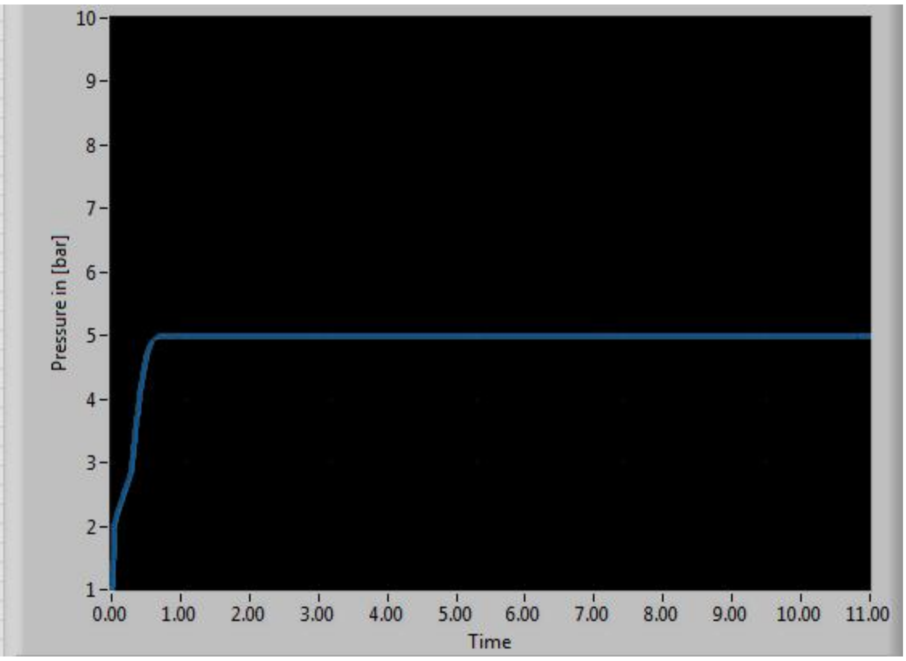


Figure.7. Pressure in the chamber A of cylinder

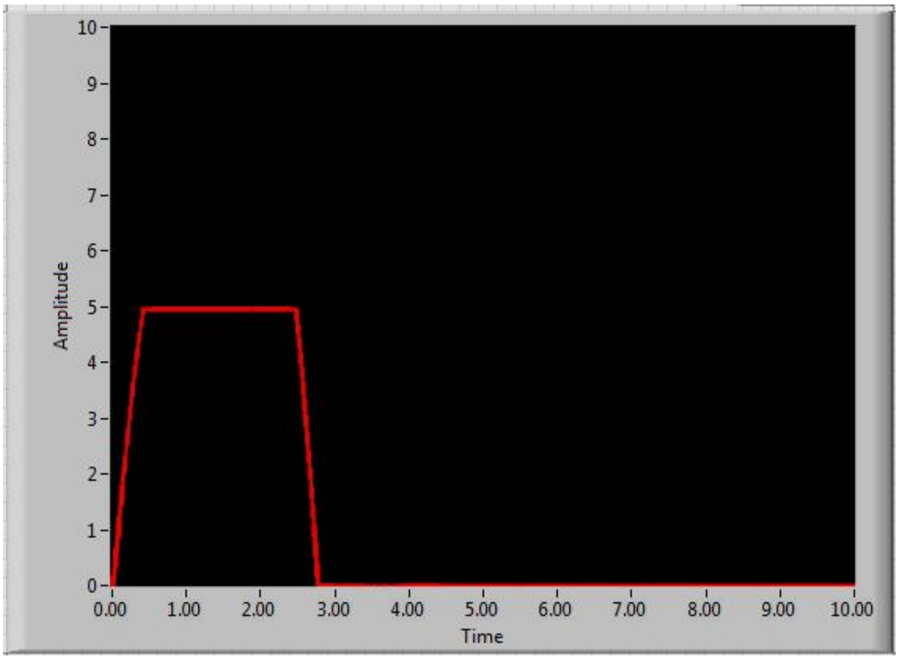


Figure.8. Displacement of Cylinder Shaft for exit and return

EXPERIMENTAL RESULTS

In the experimental panel we have installed a displacement sensor and a pressure sensor of them we have received displacement and pressure graphs as shown in Figures 9 and 10.

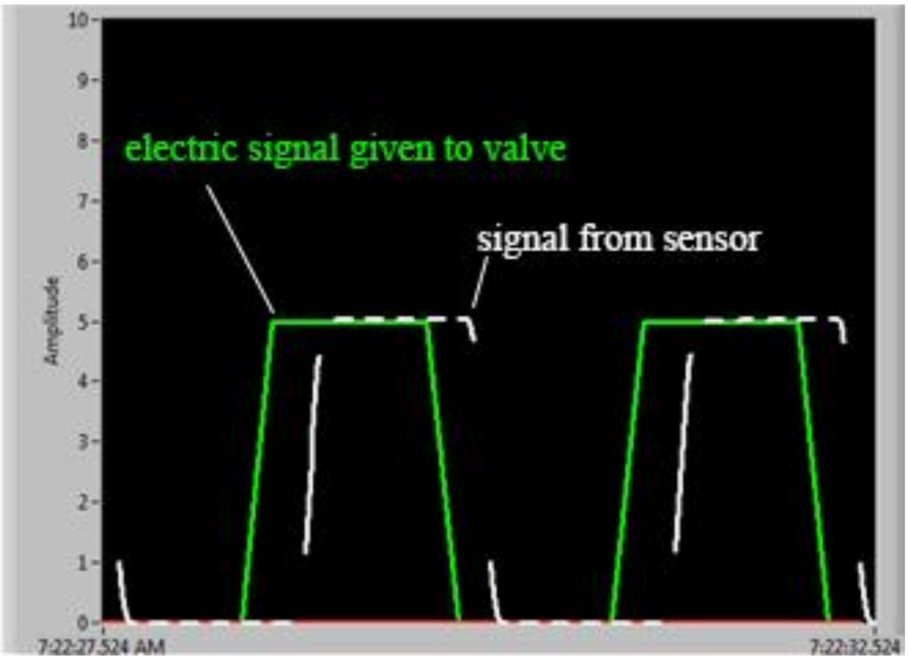


Figure.9. Displacement of cylinder measured from experimental panel

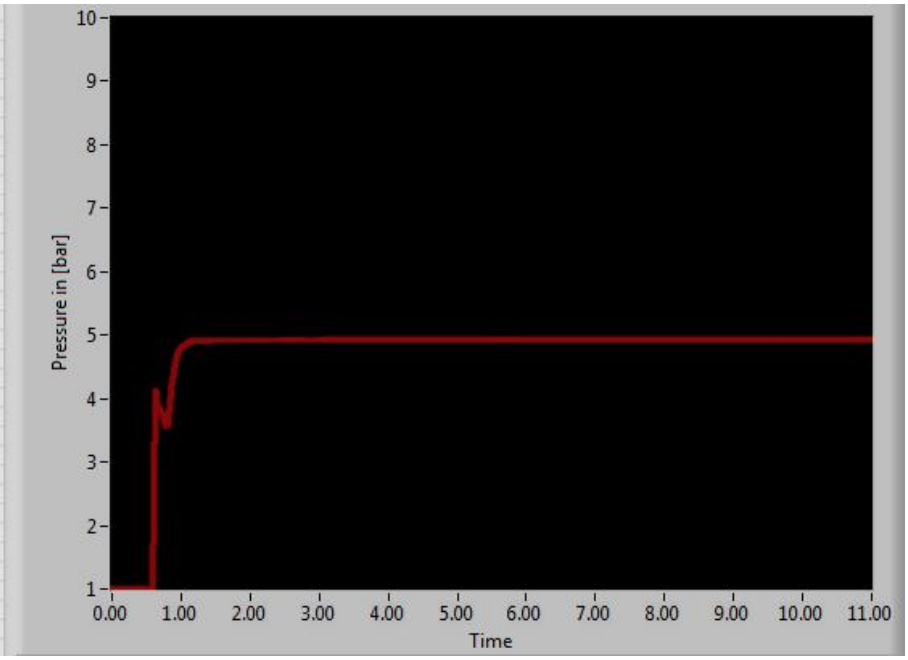


Figure.10. Pressure in the chamber A of cylinder measured from experimental panel

COMPARISON OF RESULTS

Below we are comparing results obtained from numerical simulation and experimental measurements

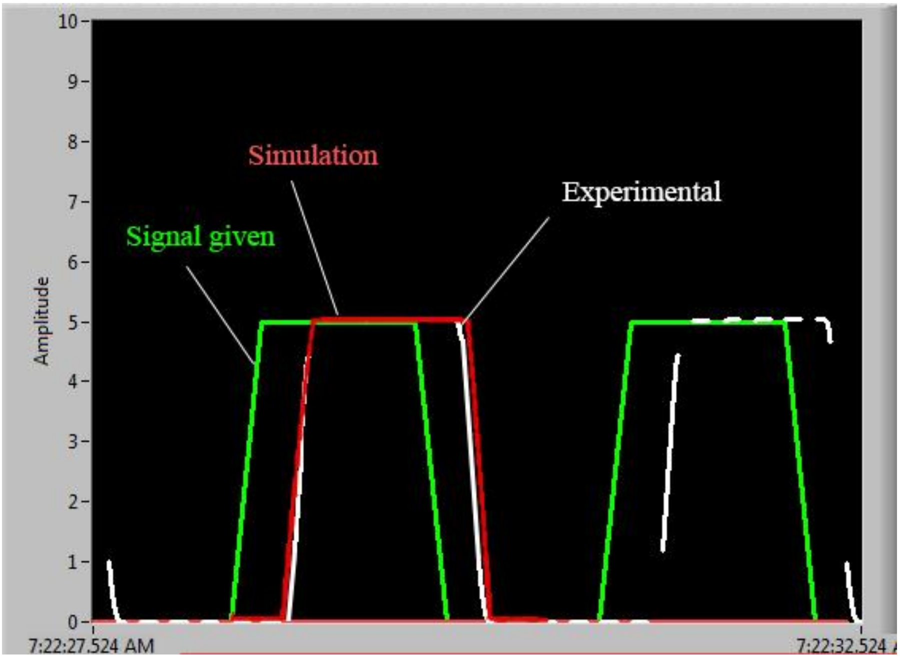


Figure.11. Displacement of cylinder (simulation and experimental)

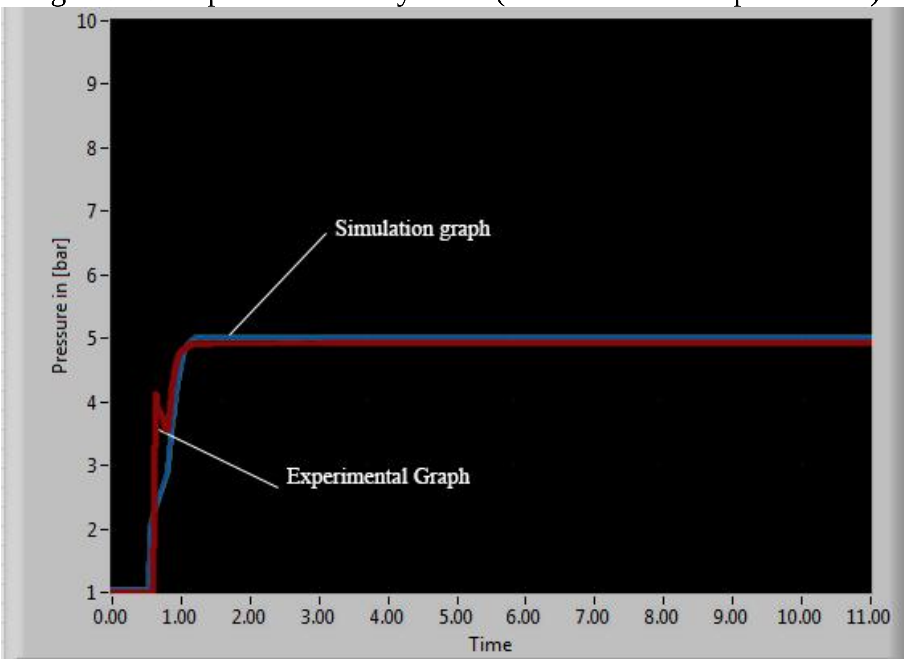


Figure.12. Pressure in chamber A of cylinder (simulation and experimental)

CONCLUSIONS

By comparing the simulation graphs with the experimental measurements graphs, respectively for the displacement of the cylinder shaft and the pressure in the cylinder chamber, we notice a considerable correlation, thus proving the authenticity of the mathematical model and the built program. Since the built program has like inputs the physical parameters of the cylinder valve

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system, provides the possibility of realizing virtual experiments. The built mathematical model can also be used for PID control of these plants.

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