THE GLOBAL ATTRACTOR FOR A CLASS OF COUPLED KIRCHHOFF-TYPE EQUATIONS

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ABSTRACT

In this paper, we study the longtime behavior of solution to the initial boundary value problem for a generalized Kirchhoff type system equations with strongly damped terms and force terms. At first, we get some priori estimates under the proper assumptions. Further, we prove the existence and uniqueness of the solutions of system equations by the Galerkin's method. At last, we obtain the existence of the global attractor in $H_0^1(\Omega)\times H_0^1(\Omega)\times L^2(\Omega)\times L^2(\Omega)$.

Keyword : The long time behavior of solution, the existence and uniqueness of solutions, the global attractor.
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1. INTRODUCTION

In this paper, we study about with the existence of global attractor for the following generalized Kirchhoff type system equations.

$$u_{tt}-M(\|\nabla u\|^2,\|\nabla v\|^2)\Delta u-\beta\Delta u_t+\alpha(1+|u|^2)^p u=f_1(x)\ , \tag{1.1}$$

$$v_{tt}-M(\|\nabla u\|^2,\|\nabla v\|^2)\Delta v-\beta\Delta v_t+\alpha(1+|v|^2)^p u=f_2(x)\ , \tag{1.2}$$

$$u(x,0)=u_0(x),u_t(x,0)=u_1(x),x\in\Omega,\Omega\in R^2,n=1,2,3\ , \tag{1.3}$$

$$v(x,0)=v_0(x),v_t(x,0)=v_1(x),x\in\Omega\ , \tag{1.4}$$

$$u\,|\,\partial\Omega=v\,|\,\partial\Omega=0,t>0\ . \tag{1.5}$$

where Ω is a bounded domain in R^2 with smooth boundary $\partial\Omega$, $u_0(x),u_1(x),v_0(x),\ v_1(x)$ are certain initial dates, $f_i(x)(i=1,2)$ is a function on $\Omega\times(0,T)$, α,β are constants, then concerning about the assumption of $M(\|\nabla u\|^2,\|\nabla u\|^2)$ will be given latter.

It is well known that Kirchhoff(1883)[1] first studied the following nonlinear vibration of an elastic string for $\delta=f=0$,

$$\rho h\frac{\partial^2 u}{\partial t^2}+\delta\frac{\partial u}{\partial t}=\left\{p_0+\frac{Eh}{2L}\int_0^L\left(\frac{\partial u}{\partial x}\right)^2dx\right\}\frac{\partial^2 u}{\partial x^2}+f;\quad 0\le x\le L,t\ge 0\ , \tag{1.6}$$

where $u=u(x,t)$ is the lateral displacement at the space coordinate x and the time t, ρ the mass density, h the cross-setion area, L the length, E the Young modulus, p_0 the initial axial tension, δ the resistence modulus and f the external force.

Igor Chueshov[2] investigated the long time behavior of solution for the following nonlinear strong damped wave equation of Kirchhoff type.

$$\partial_{tt}u-\sigma(\|\Delta u\|^2)\Delta\partial_t u-\phi(\|\Delta u\|^2)\Delta u+f(u)=h(x)\ , \tag{1.7}$$

Tokio Matsuyama and Ryo Ikehata[3] proved the global existence and the decay estimate of the solution for the nonlinear damped wave equation of Kirchhoff type.

$$u_{tt}-M(\|\nabla u(t)\|_2^2\Delta u+\delta|u_t|^{p-1}u_t=\mu|u|^{q-1}u, \tag{1.8}$$

with compact boundary condition

$$u(x,t)|_{\Omega\alpha}=0,\ \ t>0, \tag{1.9}$$

where $M(s)\in C^1[0,\infty)$, and satisfies $M(s)\geq m_0>0$: $\delta>0$, $\mu\in R$ are constants.

Yunlong Gao, Yunting Sun, Guoguang Lin[4] they studied the longtime behavior of solution to the initial boundary value problem for a class of strong damped Higher-order Kirchhoff type equations:

$$u_{tt}+(-\Delta)^m u_t+\|\nabla^m u\|^{2q}(-\Delta)^m u+g(u)=f(x), \tag{1.10}$$

where $m>1$ is an integer constant and $q>0$ is a positive constant. Moreover, Ω is a bounded domain in R^n with the smooth boundary $\partial\Omega$ and v is the unit outward normal on $\partial\Omega$. $g(u)$ is a nonlinear function specified later.

Guoguang Lin[1],Yunlong Gao[5] they studied the longtime behavior of solution to the initial boundary value problem for a class of strong damped Higher-order Kirchhoff type equations:

$$u_{tt}+(-\Delta)^m u_t+(\alpha+\beta\|\nabla^m u\|^q(-\Delta)^m u+g(u)=f(x), \tag{1.11}$$

where $m>1$ is an integer constant $\alpha>0,\beta>o$ are constants and $q>0$ is a real number. Moreover, Ω is a bounded domain in R^n with the smooth boundary $\partial\Omega$ and v is the unit outward normal on $\partial\Omega$. $g(u)$ is a nonlinear function specified later.

Shun-Tang Wu[6] discussed the existence, asymptotic behavior and blow-up of solutions under some conditions. Furthermore, he gave the decay estimates of the energy function and the estimates for the lifespan of solutions:

$$u_{tt}-M\left(\|\nabla u\|_2^2+\|\nabla v\|_2^2\right)\Delta u+h_1(u_t)=f_1(u)\ ,\quad \text{in}\quad \Omega\times[0,\infty) \tag{1.12}$$

$$v_{tt}-M\left(\|\nabla u\|_2^2+\|\nabla v\|_2^2\right)\Delta v+h_2(v_t)=f_2(v)\ ,\quad \text{in}\quad \Omega\times[0,\infty)\ . \tag{1.13}$$

In 2012, Shun-Tang Wu[7] also investigated the decay estimates of the energy function is exponential or polynomial depending on the exponents of the damping term in both equations by using Nakao's method, under suitable conditions on the nonlinearity of the damping and the source terms and certain initial data in the stable set and for wider class of relaxation functions:

$$u_{tt}-M\left(\|\nabla u\|_2^2+\|\nabla v\|_2^2\right)\Delta u+\int_0^t g_1(t-s)\Delta u(s)ds+|u_t|^{m-1}u_t=f_1(u,v)\ ,\quad \text{in}\quad \Omega\times[0,\infty) \tag{1.14}$$

$$v_{tt}-M\left(\|\nabla u\|_2^2+\|\nabla v\|_2^2\right)\Delta v+\int_0^t g_2(t-s)\Delta v(s)ds+|v_t|^{q-1}v_t=f_2(u,v),\quad \text{in}\quad \Omega\times[0,\infty)\ . \tag{1.15}$$

Motivated by the above work, we intend to study the longtime behavior solution to the initial boundary value problem for a generalized Kirchhoff type system equations with strongly damped terms and force terms.

In this paper, in section 2, we give the preliminaries and important lemmas, in section 3, we prove the existence and uniqueness of the solutions, in section 4, we obtain the finite attractor for the Kirchhoff type wave equations.

2. Preliminaries

In this section for convenience to some places in this paper, we define as follows:

$$g_1(u,v)=\alpha\left(1+|u|^2\right)^p u,\ g_2(u,v)=\alpha\left(1+|v|^2\right)^p v,$$

$$H(\Omega)=L^2(\Omega),$$

$$E_0 = H_0^1(\Omega)\times H_0^1(\Omega)\times L^2(\Omega)\times L^2(\Omega),$$
$$E_1 = \left(H^2(\Omega)\cap H_0^1(\Omega)\right)\times\left(H^2(\Omega)\cap H_0^1(\Omega)\right)\times H_0^1(\Omega)\times H_0^1(\Omega),$$

and we denote the norm and the scalar in H by $(\cdot,\cdot)$ and $\|\cdot\|$ respectively

$$(u,v)=\int_\Omega u(x)v(x)\,dx, \|u\|^2=(u,u).$$

Next, we present some assumptions and notations needed in the proof of our results as follows:

$(G1)M(x,y): R^+\times R^+ \to R^+\setminus\{0\}$, (2.1)

$(G2)M(x,y)=M(x+y)\ge m_0>0$, (2.2)

$(G3)M(x,y)(x+y)\ge \int_0^{x+y} M(s)ds$, (2.3)

$(G4)\, f_i(x)\in L^2(\Omega)(i=1,2), u_0\in H_0^1(\Omega), v_0\in H_0^1(\Omega), u_1, v_1\in L^2(\Omega).$ (2.4)

Lemma 2.1.(Young's inequality[7]) for any $\varepsilon>0$ and $a,b>0$, then

$$ab\le \frac{\varepsilon^p}{p}a^p+\frac{1}{\varepsilon^q q}b^q \quad , \qquad (2.5)$$

where $\frac{1}{p}+\frac{1}{q}=1,(p>1,q>1)$.

Lemma 2.2. (Holder inequality[7]) let $G\in R^n$ is opening set, $G\in R^n$, $p\ge 1, L^p(G)$ repredents p times on G for the collection of measurable function; $p=\infty, L^\infty(G)$ represents bounded on G for the collection of measurable function.

Let $\frac{1}{p}+\frac{1}{q}=1, p>1, q>1, \forall f(x)\in L^p(G)$ and $g(x)\in L^q(G)$, such that

$$\int_G |f(x).g(x)|\le\left(\int_G |f(x)|^p\right)^{\frac{1}{p}}\left(\int_G |g(x)|^q\right)^{\frac{1}{q}} . \qquad (2.6)$$

Lemma2.3.(Poincare's inequality[7])if $\Omega\in R^n$ bounded open subset, such that

$$\|u\|_{L^2(\Omega)}\le \lambda_1^{-\frac{1}{2}}\|\nabla u\|_{L^2(\Omega)}, \forall u\in H_0^1(\Omega), \qquad (2.7)$$

Where λ_1 is the first eigenvalue of $-\Delta$ in Ω with homogeneous Dirichlet boundary condition.

Lemma2.4.(Gronwall's inequality[7]) if $\forall t\in[t_0,+\infty), y(t)\ge 0$ and $\frac{dy}{dt}+gy\le h$, such that

$$y(t)\le y(t_0)e^{-g(t-t_0)}+\frac{h}{g}, t\ge t_0 \quad , \qquad (2.8)$$

where $g>0, h\ge 0$ are constants.

Lemma2.5. Assume $(G_1)\sim(G_3)$ hold, and $(u_0,v_0,p_0,q_0)\in E_0$, $f_i(x)\in L^2(\Omega)(i=1,2)$ Since the solution (u,v,p,q) of the problem(1.1)-(1.5)satisfies $(u,v,p,q)\in E_0$, and

$$(u,u_t)\in L^\infty(0,+\infty;H_0^1(\Omega))\times L^\infty(0,+\infty;L^2(\Omega)), \qquad (2.9)$$

$$(v,v_t)\in L^\infty(0,+\infty;H_0^1(\Omega))\times L^\infty(0,+\infty;L^2(\Omega)), \qquad (2.10)$$

$$\|p\|^2+\|q\|^2+\|\nabla u\|^2+\|\nabla v\|^2\le H_1(0)e^{-\gamma t}+\frac{C}{\gamma}, \qquad (2.11)$$

$$\|\nabla u\|^2+\|\nabla v\|^2+\|p\|^2+\|q\|^2\le H_1(0)e^{-\gamma t}+\frac{C}{\gamma}\ , \qquad (2.12)$$

where λ_1 is the first eigenvalue of $-\Delta$ in Ω with homogeneous Dirchet boundary condition.

Proof. we take the scalar product in L^2 of equation (1.1) with $p=u_t+\varepsilon u$. Then

$$(u_{tt},p)-(M(\|\nabla u\|^2,\|\nabla v\|^2)\ \Delta u,p)-(\beta\Delta u_t,p)+(\alpha(1+|u|^2)^p u,p)=(f_1(x),p) \qquad (2.13)$$

by using Holder inequality, Young inequality and Poincare inequality, we process the items in (2.13), we obtain

$$\begin{aligned}(u_{tt},p)&=(p_t-\varepsilon(p-\varepsilon u),p)\\&=(p_t,p)-\varepsilon(p,p)+\varepsilon^2(u,p)\\&\ge\frac{1}{2}\frac{d}{dt}\|p\|^2-\varepsilon\|p\|^2-\frac{\varepsilon^2}{2}\|u\|^2-\frac{\varepsilon^2}{2}\|p\|^2\\&\ge\frac{1}{2}\frac{d}{dt}\|p\|^2-\frac{\varepsilon^2}{2\lambda_1}\|\nabla u\|^2-\frac{2\varepsilon+\varepsilon^2}{2}\|p\|^2.\end{aligned} \qquad (2.14)$$

$$\begin{aligned}&(-M(\|\nabla u\|^2,\|\nabla v\|^2)\Delta u,p)=(M(\|\nabla u\|^2,\|\nabla v\|^2)\nabla u,\nabla p)\\&=M(\|\nabla u\|^2,\|\nabla v\|^2)(\nabla u,\nabla u_t)+\varepsilon M(\|\nabla u\|^2,\|\nabla v\|^2)(\nabla u,\nabla u)\\&=\frac{1}{2}M(\|\nabla u\|^2,\|\nabla v\|^2)\frac{d}{dt}\|\nabla u\|^2+\varepsilon M(\|\nabla u\|^2,\|\nabla v\|^2)\|\nabla u\|^2.\end{aligned} \qquad (2.15)$$

$$\begin{aligned}&(-\beta\Delta u_t,p)=(-\beta\Delta(p-\varepsilon u),p)\\&=(-\beta\Delta p,p)+(\beta\varepsilon\Delta u,p)\\&=\beta\|\nabla p\|^2-\beta\varepsilon(\nabla u,\nabla p)\\&\ge(\beta\lambda_1-\frac{\beta\varepsilon\lambda_1}{2})\|p\|^2-\frac{\beta\varepsilon}{2}\|\nabla u\|^2.\end{aligned} \qquad (2.16)$$

$$\begin{aligned}&(\alpha(1+|u|^2)^p u,p)=(\alpha(1+|u|^2)^p u,u_t+\varepsilon u)\\&=(\alpha(1+|u|^2)^p u,u_t)+(\alpha(1+|u|^2)^p u,\varepsilon u)\\&=\frac{\alpha}{2(p+1)}\frac{d}{dt}\left(\int_\Omega(1+|u|^2)^{p+1}dx\right)+\alpha\varepsilon\int_\Omega(1+|u|^2)^p|u|^2\,dx\\&\ge\frac{\alpha}{2(p+1)}\frac{d}{dt}\left(\int_\Omega(1+|u|^2)^{p+1}dx\right)+\frac{\alpha\varepsilon}{p+1}\int_\Omega(1+|u|^2)^{p+1}dx-\frac{\Omega}{\alpha\varepsilon(p+1)}.\end{aligned} \qquad (2.17)$$

$$(f_1(x),p)\le\|f_1(x)\|.\|p\|\le\frac{\|f_1(x)\|^2}{2\varepsilon^2}+\frac{\varepsilon^2}{2}\|p\|^2. \qquad (2.18)$$

Integrate (2.14) - (2.18), obtain

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\|p\|^2+\frac{1}{2}M\left(\|\nabla u\|^2,\|\nabla v\|^2\right)\frac{d}{dt}\|\nabla u\|^2+(\beta\lambda_1-\frac{\beta\varepsilon\lambda_1}{2}-\varepsilon-\varepsilon^2)\|p\|^2\\&+\left(\varepsilon M\left(\|\nabla u\|^2,\|\nabla v\|^2\right)-\frac{\varepsilon^2}{2\lambda_1}-\frac{\beta\varepsilon}{2}\right)\|\nabla u\|^2+\frac{\alpha}{2(p+1)}\frac{d}{dt}\left(\int_\Omega\left(1+|u|^2\right)^{p+1}dx\right)\\&+\frac{\alpha\varepsilon}{p+1}\left(\int_\Omega\left(1+|u|^2\right)^{p+1}dx\right)\le\frac{1}{2\varepsilon^2}\|f_1\|^2+\frac{\Omega}{\alpha\varepsilon(p+1)}.\end{aligned} \qquad (2.19)$$

Second, we take the scalar product in L^2 of equation (1.2) with $q=v_t+\varepsilon v$. We obtain

$$(v_{tt},q)-(M(\|\nabla u\|^2,\|\nabla v\|^2)\ \Delta v,q)-(\beta\Delta v_t,q)+(\alpha(1+|v|^2)^p v,q)=(f_2(x),q)\ . \qquad (2.20)$$

Similar (2.14) - (2.18) , we process each items in (2.20), and finishing obtain :

$$\frac{1}{2}\frac{d}{dt}\|q\|^2+\frac{1}{2}M\left(\|\nabla u\|^2,\|\nabla v\|^2\right)\frac{d}{dt}\|\nabla v\|^2+(\beta\lambda_1-\frac{\beta\varepsilon\lambda_1}{2}-\varepsilon-\varepsilon^2)\|q\|^2$$
$$+\left(\varepsilon M\left(\|\nabla u\|^2,\|\nabla v\|^2\right)-\frac{\varepsilon^2}{2\lambda_1}-\frac{\beta\varepsilon}{2}\right)\|\nabla v\|^2+\frac{\alpha}{2(p+1)}\frac{d}{dt}\left(\int_\Omega\left(1+|v|^2\right)^{p+1}dx\right)$$
$$+\frac{\alpha\varepsilon}{p+1}\left(\int_\Omega\left(1+|v|^2\right)^{p+1}dx\right)\le\frac{1}{2\varepsilon^2}\|f_2\|^2+\frac{\Omega}{\alpha\varepsilon(p+1)}.$$

(2.21) By (2.19) and (2.21) we obtain

$$\frac{d}{dt}(\|p\|^2+\|q\|^2)+M\left(\|\nabla u\|^2+\|\nabla v\|^2\right)\frac{d}{dt}(\|\nabla u\|^2+\|\nabla v\|^2)$$
$$+(2\beta\lambda_1-\beta\varepsilon\lambda_1-2\varepsilon-2\varepsilon^2)(\|p\|^2+\|q\|^2)$$
$$+\left(2\varepsilon M\left(\|\nabla u\|^2+\|\nabla v\|^2\right)-\frac{\varepsilon^2}{\lambda_1}-\beta\varepsilon\right)(\|\nabla u\|^2+\|\nabla v\|^2)$$
$$+\frac{\alpha}{(p+1)}\frac{d}{dt}\left(\int_\Omega\left(1+|u|^2\right)^{p+1}dx+\int_\Omega\left(1+|v|^2\right)^{p+1}dx\right) \qquad (2.22)$$
$$+\frac{2\alpha\varepsilon}{p+1}\left(\int_\Omega\left(1+|u|^2\right)^{p+1}dx+\int_\Omega\left(1+|v|^2\right)^{p+1}dx\right)$$
$$\le\frac{1}{\varepsilon^2}(\|f_1\|^2+\|f_2\|^2)+\frac{4\Omega}{\alpha\varepsilon(p+1)}.$$

According to the assumption, we have

$$\|q\|^2+\int_0^{(\|\nabla u\|^2+\|\nabla v\|^2)}M(s)dx+\frac{\alpha}{(p+1)}\left(\int_\Omega\left(1+|u|^2\right)^{p+1}dx+\int_\Omega\left(1+|v|^2\right)^{p+1}dx\right)$$
$$+(2\beta\lambda_1-\beta\varepsilon\lambda_1-2\varepsilon-2\varepsilon^2)(\|p\|^2+\|q\|^2)$$
$$+\left(2\varepsilon M\left(\|\nabla u\|^2+\|\nabla v\|^2\right)-\frac{\varepsilon^2}{\lambda_1}-\beta\varepsilon\right)(\|\nabla u\|^2+\|\nabla v\|^2)$$
$$+\frac{2\alpha\varepsilon}{p+1}\left(\int_\Omega\left(1+|u|^2\right)^{p+1}dx+\int_\Omega\left(1+|v|^2\right)^{p+1}dx\right)$$
$$\le\frac{1}{\varepsilon^2}(\|f_1\|^2+\|f_2\|^2)+\frac{4\Omega}{\alpha\varepsilon(p+1)}=C.$$

(2.23) By assuming (G3), we obtain

$$\frac{d}{dt}(\|p\|^2+\|q\|^2+\int_0^{(\|\nabla u\|^2+\|\nabla v\|^2)}M(s)dx$$
$$+\frac{\alpha}{(p+1)}\left(\int_\Omega\left(1+|u|^2\right)^{p+1}dx+\int_\Omega\left(1+|v|^2\right)^{p+1}dx\right)) \qquad (2.24)$$
$$+(2\beta\lambda_1-\beta\varepsilon\lambda_1-2\varepsilon-2\varepsilon^2)(\|p\|^2+\|q\|^2)+\varepsilon\int_0^{(\|\nabla u\|^2+\|\nabla v\|^2)}M(s)dx$$
$$+\frac{2\alpha\varepsilon}{p+1}\left(\int_\Omega\left(1+|u|^2\right)^{p+1}dx+\int_\Omega\left(1+|v|^2\right)^{p+1}dx\right)\le C.$$

Take appropriate ε, make $2\beta\lambda_1 - \beta\varepsilon\lambda_1 - 2\varepsilon - 2\varepsilon^2 > 0$, and $\gamma = \min\{2\beta\lambda_1 - \beta\varepsilon\lambda_1 - 2\varepsilon - 2\varepsilon^2, \varepsilon\}$, therefore, we have

$$\frac{d}{dt}H_1(t) + \gamma H_1(t) \le C \,, \tag{2.25}$$

where $H_1(t) = \|p\|^2 + \|q\|^2 + \int_0^{\|\nabla u\|^2+\|\nabla v\|^2} M(s)ds + + \frac{\alpha}{(p+1)}\left(\int_\Omega \left(1+|u|^2\right)^{p+1}dx + \int_\Omega \left(1+|v|^2\right)^{p+1}dx\right)$.

Using the Gronwall inequality, we obtain

$$H_1(t) \le H_1(0)e^{-\gamma t} + \frac{C}{\gamma} \,, \tag{2.26}$$

therefore, $\|p\|^2 + \|q\|^2 + \|\nabla u\|^2 + \|\nabla v\|^2 \le H_1(0)e^{-\gamma t} + \frac{C}{\gamma}$. Lemma 2.5 Proof finished.

Lemma 2.6. Under the assumption of lemma 2.5, $(G1)-(G4)$ hold, $(G5)$: $f_1(x), f_2(x) \in H_0^1$, then the solutions (u,v,p,q) of the problems $(1.1)-(1.5)$ satisfies:

$(u,v,p,q) \in E_1$, and $z(t) \le Ce^{-Ct} + C$, (2.27)

where $p = u_t + \varepsilon u, q = v_t + \varepsilon v$, λ_1 is the first eigenvalue of $-\Delta$ in $H_0^1(\Omega)$.

Proof: Taking inner product by $-\Delta p$ with equation (1.1), obtain

$$\left(u_{tt} - M\left(\|\nabla u\|^2, \|\nabla v\|^2\right)\Delta u - \beta\Delta u_t + \alpha\left(1+|u|^2\right)^p u, -\Delta p\right) = \left(f_1(x), -\Delta p\right). \tag{2.28}$$

Following the computation in (2.28) one be one and using Poincare's inequality, Young's inequality, we have

$$\begin{aligned}(u_{tt}, -\Delta p) &= \frac{1}{2}\frac{d}{dt}\|\nabla p\|^2 - \varepsilon\|\nabla p\|^2 + \varepsilon^2(\nabla u, \nabla p)\\ &\ge \frac{1}{2}\frac{d}{dt}\|\nabla p\|^2 - \varepsilon\|\nabla p\|^2 - \frac{\varepsilon^2}{2}\|\nabla u\|^2 - \frac{\varepsilon^2}{2}\|\nabla p\|^2.\end{aligned} \tag{2.29}$$

$$(-M(\|\nabla u\|^2, \|\nabla v\|^2)\Delta u, -\Delta p) = \frac{M(\|\nabla u\|^2 + \|\nabla v\|^2)}{2}\frac{d}{dt}\|\Delta u\|^2 + \varepsilon M(\|\nabla u\|^2 + \|\nabla v\|^2)\|\Delta u\|^2. \tag{2.30}$$

$$\begin{aligned}(-\beta\Delta u_t, -\Delta p) &= \beta\|\Delta p\|^2 - \varepsilon\beta(\Delta u, \Delta p)\\ &\ge \beta\|\Delta p\|^2 - \frac{\varepsilon\beta}{2}\|\Delta u\|^2 - \frac{\varepsilon\beta}{2}\|\Delta p\|^2.\end{aligned} \tag{2.31}$$

$$\left|\left(f_1, -\Delta p\right)\right| \le \frac{\|f_1\|^2}{2\varepsilon} + \frac{\varepsilon}{2}\|\Delta p\|^2 \,. \tag{2.32}$$

Due to

$$(\alpha(1+|u^2|^p u, -\Delta u_t) \le \frac{\alpha\varepsilon^2}{4}\|\Delta u\|^2 + \frac{2^{2p+1}\alpha}{\varepsilon^2}\|u\|^2 + 2^{2p+1}\alpha C\|\nabla u\|^{4p+2}. \tag{2.33}$$

Also interpolate inequality and lemma 2.5 we get:

$$\left|(\alpha(1+|u|^2)^p u, -\varepsilon\Delta u)\right| \le \begin{cases} 2^p\varepsilon\alpha\left|(u,\Delta u\right|, |u|<1,\\ 2^p\varepsilon\alpha\left|(|u|^{2p}u, \Delta u)\right|, |u|\ge 1.\end{cases} \tag{2.34}$$

$$2^p\varepsilon\alpha\left|(u,\Delta u)\right| \le \frac{\varepsilon\alpha}{2}\|\Delta u\|^2 + 2^{p-1}\varepsilon\alpha\|u\|^2 \,. \tag{2.35}$$

$$\begin{aligned}
&2^{p}\varepsilon\alpha\left|(|u|^{2p}u,\Delta u)\right| \\
&\le 2^{p}\varepsilon\alpha\|u\|_{4p+2}^{2p+1}\|\Delta u\| \\
&\le C(\Omega,4p+2)\times 2^{p}\varepsilon\alpha\|\nabla u\|^{2p-1}\|\Delta u\| \\
&\le \frac{\varepsilon\alpha}{8}\|\Delta u\|^{2}+2^{p}C\varepsilon\alpha\|\nabla u\|^{4p+2}.
\end{aligned} \tag{2.36}$$

Thus by (2.34)-(2.36) , obtain :

$$\begin{aligned}
&\left|(\alpha(1+|u|^{2})^{p}u,-\varepsilon\Delta u)\right| \\
&\le \frac{\varepsilon^{2}\alpha}{4}\|\Delta u\|^{2}+2^{2p+1}\alpha\|u\|^{2}+2^{2p+1}\varepsilon\alpha\|\nabla u\|^{4p+2} \\
&\le \frac{\varepsilon\alpha}{4}\|\Delta u\|^{2}+C.
\end{aligned} \tag{2.37}$$

In summary (2.29)- (2.37) obtain

$$\begin{aligned}
&\frac{d}{dt}\left[\|\nabla p\|^{2}+M\left(\|\Delta u\|^{2}+\|\Delta v\|^{2}\right)\|\Delta u\|^{2}-\varepsilon\beta\|\Delta u\|^{2}\right] \\
&-2\varepsilon\|\nabla p\|^{2}+2\varepsilon^{2}(\nabla u,\nabla p)+2\beta\|\Delta p\|^{2}+2\varepsilon M\left(\|\Delta u\|^{2}+\|\Delta v\|^{2}\right)\|\Delta u\|^{2} \\
&-2\beta\varepsilon\|\Delta u\|^{2}-\frac{\alpha\varepsilon^{2}}{4}\|\Delta u\|^{2}-\frac{\alpha\varepsilon^{2}}{2}\|\Delta u\|^{2}-\varepsilon\|\Delta p\|^{2} \\
&\le \frac{\|f_{1}\|^{2}}{\varepsilon}+\|\Delta u\|^{2}\frac{d}{dt}M\left(\|\Delta u\|^{2}+\|\Delta v\|^{2}\right)+C.
\end{aligned} \tag{2.38}$$

Similar, Taking inner product by $-\Delta q$ with equation (1.2), obtain

$$\begin{aligned}
&\frac{d}{dt}\left[\|\nabla q\|^{2}+M\left(\|\Delta u\|^{2}+\|\Delta v\|^{2}\right)\|\Delta u\|^{2}-\varepsilon\beta\|\Delta v\|^{2}\right] \\
&-2\varepsilon\|\nabla q\|^{2}+2\varepsilon^{2}(\nabla u,\nabla q)+2\beta\|\Delta q\|^{2}+2\varepsilon M\left(\|\Delta u\|^{2}+\|\Delta v\|^{2}\right)\|\Delta v\|^{2} \\
&-2\beta\varepsilon\|\Delta v\|^{2}-\frac{\alpha\varepsilon}{4}\|\Delta v\|^{2}-\frac{\alpha\varepsilon}{2}\|\Delta v\|^{2}-\varepsilon\|\Delta q\|^{2} \\
&\le \frac{\|f_{2}\|^{2}}{\varepsilon}+\|\Delta v\|^{2}\frac{d}{dt}M\left(\|\Delta u\|^{2}+\|\Delta v\|^{2}\right)+C.
\end{aligned} \tag{2.39}$$

Plug (2.38) into (2.39), obtain:

$$\begin{aligned}
&\frac{d}{dt}z(t)-2\varepsilon\left(\|\nabla p\|^{2}+\|\nabla q\|^{2}\right)+2\varepsilon^{2}\left((\nabla u,\nabla p)+(\nabla v,\nabla q)\right) \\
&+2\beta\left(\|\Delta p\|^{2}+\|\Delta q\|^{2}\right)+2\varepsilon M\left(\|\Delta u\|^{2}+\|\Delta v\|^{2}\right)\left(\|\Delta u\|^{2}+\|\Delta v\|^{2}\right) \\
&+\left(-2\beta\varepsilon-\frac{\alpha\varepsilon^{2}}{4}-\frac{\alpha\varepsilon^{2}}{2}\right)\left(\|\Delta u\|^{2}+\|\Delta v\|^{2}\right)-\varepsilon\left(\|\Delta p\|^{2}+\|\Delta q\|^{2}\right) \\
&\le \frac{\|f_{1}\|^{2}+\|f_{2}\|^{2}}{\varepsilon}+C\left(\|\nabla u_{t}\|+\|\nabla v_{t}\|\right)z(t),
\end{aligned} \tag{2.40}$$

where

$$z(t)=\|\nabla p\|^{2}+\|\nabla q\|^{2}+M\left(\|\Delta u\|^{2}+\|\Delta v\|^{2}\right)\left(\|\Delta u\|^{2}+\|\Delta v\|^{2}\right)-\varepsilon\beta\left(\|\Delta u\|^{2}+\|\Delta v\|^{2}\right)>0.$$

For (2.40) take ε small enough, we have :

$$\frac{d}{dt}z(t)+\varepsilon z(t)\le\frac{\|f_1\|^2+\|f_2\|^2}{\varepsilon}+C\left(\|\nabla u\|+\|\nabla v\|\right)z(t)\ , \tag{2.41}$$

by Gronwall inequality have :

$$z(t)\le Ce^{-Ct}+C\ . \tag{2.42}$$

3. The existence and uniqueness of solutions

Theorem1 set the given function $f_1, f_2, u_0, v_0, u_1, v_1$ and $f_1, f_2\in H_0^1(\Omega)$, $(u_0,v_0)\in H^2(\Omega)\times H^2(\Omega)$, existence a unique solution for the initial boundary value problem (1.1) - (1.5).

Proof : The existence by lemma 2.5 and lemma 2.6 using Galerkin method can be obtained.

Under prove uniqueness : set $W_1\begin{pmatrix}u_1\\ v_1\end{pmatrix}, W_2\begin{pmatrix}u_2\\ v_2\end{pmatrix}$ are two solutions of (1.1)-(1.5), make $W\begin{pmatrix}u\\ v\end{pmatrix}=W_1\begin{pmatrix}u_1\\ v_1\end{pmatrix}-W_2\begin{pmatrix}u_2\\ v_2\end{pmatrix}$ and $g_1(u,v)=\alpha\left(1+|u|^2\right)^p u, g_2(u,v)=\alpha\left(1+|v|^2\right)^p v$,then two different solutions are satisfied :

$$\begin{aligned}&u_{1tt}-M(\|Du_1\|^2,\|Dv_1\|^2)\Delta u_1-\beta\Delta u_{1t}+g_1(u_1,v_1)=f_1(x),\\&v_{1tt}-M(\|Du_1\|^2,\|Dv_1\|^2)\Delta v_1-\beta\Delta v_{1t}+g_2(u_1,v_1)=f_2(x),\\&u_1(x,0)=u_{01}(x);\ \ u_{1t}(x,0)=u_{11}(x),\\&v_1(x,0)=v_{01}(x);\ \ v_{1t}(x,0)=v_{11}(x).\end{aligned} \tag{3.1}$$

$$\begin{aligned}&u_{2tt}-M(\|Du_2\|^2,\|Dv_2\|^2)\Delta u_2-\beta\Delta u_{2t}+g_1(u_2,v_2)=f_1(x),\\&v_{2tt}-M(\|Du_2\|^2,\|Dv_2\|^2)\Delta v_2-\beta\Delta v_{2t}+g_2(u_2,v_2)=f_2(x),\\&u_2(x,0)=u_{02}(x);\ \ u_{2t}(x,0)=u_{12}(x),\\&v_2(x,0)=v_{02}(x);\ \ v_{2t}(x,0)=v_{12}(x).\end{aligned} \tag{3.2}$$

From (3.1)-(3.2) above, we obtain

$$\begin{cases}u_{tt}-M(\|Du_1\|^2,\|Dv_1\|^2)\Delta u_1+M(\|Du_2\|^2,\|Dv_2\|^2)\Delta u_2-\beta\Delta u_t+g_1(u_1,v_1)-g_1(u_2,v_2)=0,\\ v_{tt}-M(\|Du_1\|^2,\|Dv_1\|^2)\Delta v_1+M(\|Du_2\|^2,\|Dv_2\|^2)\Delta v_2-\beta\Delta v_t+g_2(u_1,v_1)-g_2(u_2,v_2)=0.\end{cases} \tag{3.3}$$

For first equation of (3.3) multiply on both sides with u_t, we get

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\|u_t\|^2+\left[-M(\|Du_1\|^2,\|D^m v_1\|^2)\Delta u_1+M(\|D^m u_2\|^2+\|D^m v_2\|^2)\Delta u_2,u_t\right]\\&+\beta\|Du_t\|^2+\left[g_1(u_1,v_1)-g_1(u_2,v_2),u_t\right]=0\,.\end{aligned} \tag{3.4}$$

For second equation of (3.3) multiply on both sides with v_t, we get

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\|v_t\|^2+\left[-M(\|Du_1\|^2,\|Dv_1\|^2)\Delta v_1+M(\|Du_2\|^2,\|Dv_2\|^2)\Delta v_2,v_t\right]\\&+\beta\|Dv_t\|^2+\left[g_2(u_1,v_1)-g_2(u_2,v_2),v_t\right]=0.\end{aligned} \tag{3.5}$$

Adding (3.4) to (3.5) can obtain

$$\frac{1}{2}\frac{d}{dt}(\|u_t\|^2+\|v_t\|^2)+\left[-M(\|Du_1\|^2,\|Dv_1\|^2)\Delta u_1+M(\|Du_2\|^2,\|Dv_2\|^2)\Delta u_2,u_t\right]$$
$$+\left[-M(\|Du_1\|^2,\|Dv_1\|^2)\Delta v_1+M(\|Du_2\|^2,\|Dv_2\|^2)\Delta v_2,v_t\right]+\beta(\|Du_t\|^2+\|Dv_t\|^2) \quad (3.6)$$
$$+\left[g_1(u_1,v_1)-g_1(u_2,v_2),u_t\right]+\left[g_2(u_1,v_1)-g_2(u_2,v_2),v_t\right]=0.$$

Because by Holder inequality we obtain

$$(-M(\|Du_1\|^2,\|Dv_1\|^2)\Delta(u_1-u_2)-M(\|Du_1\|^2,\|Dv_1\|^2)\Delta u_2+M(\|Du_2\|^2,\|Dv_2\|^2)\Delta u_2,u_t)$$
$$=M(\|Du_1\|^2,\|Dv_1\|^2)\frac{1}{2}\frac{d}{dt}\|Du\|^2+M'(\xi)(\|Du_1\|^2-\|Du_2\|^2+\|Dv_1\|^2-\|Dv_2\|^2)\|\Delta u_2\|\|u_t\| \quad (3.7)$$
$$=M(\|Du_1\|^2,\|Dv_1\|^2)\frac{1}{2}\frac{d}{dt}\|Du\|^2+M'(\xi)((\|Du_1\|+\|Du_2\|)(\|Du_1\|-\|Du_2\|)$$
$$+(\|Dv_1\|+\|Dv_2\|)(\|Dv_1\|-\|Dv_2\|))\|\Delta u_2\|\|u_t\|.$$

Again by Young inequality and the conclusions of lemma2.5 and lemma2.6 obtain

$$M'(\xi)((\|Du_1\|+\|Du_2\|)(\|Du_1\|-\|Du_2\|)+(\|Dv_1\|+\|Dv_2\|)(\|Dv_1\|-\|Dv_2\|))\|\Delta u_2\|\|u_t\|$$
$$\le C(\|Du_1\|-\|Du_2\|+\|Dv_1\|-\|Dv_2\|)\|u_t\|$$
$$\le C(\|D(u_1-u_2)\|\|D(v_1-v_2\|)\|u_t\| \quad (3.8)$$
$$\le C(\|Du\|\|Dv\|)\|u_t\|$$
$$\le C_1(\|Du\|^2+\|Dv\|^2+\frac{\|u_t\|^2}{2}).$$

Similar, can obtain

$$(-M(\|Du_1\|^2,\|Dv_1\|^2)\Delta(v_1-v_2)-M(\|Du_1\|^2,\|Dv_1\|^2)\Delta v_2+M(\|Du_2\|^2,\|Dv_2\|^2)\Delta v_2,v_t)$$
$$=M(\|Du_1\|^2,\|Dv_1\|^2)\frac{1}{2}\frac{d}{dt}\|Dv\|^2+M'(\xi)(\|Du_1\|^2-\|Du_2\|^2+\|Dv_1\|^2-\|Dv_2\|^2)\|\Delta v_2\|\|v_t\| \quad (3.9)$$
$$=M(\|Du_1\|^2,\|Dv_1\|^2)\frac{1}{2}\frac{d}{dt}\|Dv\|^2+M'(\eta)((\|Du_1\|+\|Du_2\|)(\|Du_1\|-\|Du_2\|)$$
$$+(\|Dv_1\|+\|Dv_2\|)(\|Dv_1\|-\|Dv_2\|))\|\Delta v_2\|\|v_t\|.$$

$$M'(\eta)((\|Du_1\|+\|Du_2\|)(\|Du_1\|-\|Du_2\|)+(\|Dv_1\|+\|Dv_2\|)(\|Dv_1\|-\|Dv_2\|))\|\Delta v_2\|\|v_t\|$$
$$\le C(\|Du_1\|-\|Du_2\|+\|Dv_1\|-\|Dv_2\|)\|v_t\|$$
$$\le C(\|D(u_1-u_2)\|\|D(v_1-v_2\|)\|v_t\| \quad (3.10)$$
$$\le C(\|Du\|\|Dv\|)\|v_t\|$$
$$\le C_2(\|Du\|^2+\|Dv\|^2+\frac{\|v_t\|^2}{2}) \ .$$

Again by Young inequality, Poincare inequality and the conclusion of lemma 2.5 and the assumption of lemma 2.6 (G5)

$$
\begin{aligned}
&|(g_1(u_1,v_1)-g_1(u_2,v_2),u_t)| \\
&=|(g_1(u_1,v_1)-(g_1(u_1,v_2)+(g_1(u_1,v_2)-g_1(u_2,v_2),u_t)| \\
&\le C(\|v\|\|u_t\|+\|u\|\|u_t\|) \\
&\le C(\|v\|^2+\|u\|^2+\frac{\|u_t\|^2}{2}) \\
&\le C_3(\lambda_1^{-1}\|Du\|^2+\lambda_1^{-1}\|Dv\|^2+\frac{\|u_t\|^2}{2}).
\end{aligned} \tag{3.11}
$$

Similar,

$$
\begin{aligned}
&|(g_2(u_1,v_1)-g_2(u_2,v_2),v_t)| \\
&\le C_4(\lambda_1^{-1}\|Du\|^2+\lambda_1^{-1}\|Dv\|^2+\frac{\|v_t\|^2}{2}).
\end{aligned} \tag{3.12}
$$

The conclusions of (3.7) - (3.12), (3.6) and Lemma 2.5 and Lemma 2.6 are available

$$
\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\left[M(\|Du_1\|^2,\|Dv_1\|^2)(\|Du\|^2+\|Dv\|^2)+\|u_t\|^2+\|v_t\|^2\right] \\
&\le(C_6\lambda_1^{-1}+C_5)(\|Du\|^2+\|Dv\|^2)+(\frac{C_5+C_6}{2})\|u_t\|^2+\|v_t\|^2+\frac{C_7}{2}(\|Du\|^2+\|Dv\|^2).
\end{aligned} \tag{3.13}
$$

$$
\begin{aligned}
&\frac{d}{dt}\left[M(\|Du_1\|^2,\|Dv_1\|^2)(\|Du\|^2+\|Dv\|^2)+\|u_t\|^2+\|v_t\|^2\right] \\
&\le(2C_5+2C_6\lambda_1^{-1}+C_7)(\|Du\|^2+\|Dv\|^2)+(C_5+C_6)\|u_t\|^2+\|v_t\|^2.
\end{aligned} \tag{3.14}
$$

$K_3=\max\{2C_5+2C_6\lambda_1^{-1}+C_7,C_5+C_6\}$,

$$
\begin{aligned}
&\frac{d}{dt}(M(\|Du_1\|^2,\|Dv_1\|^2)(\|Du\|^2+\|Dv\|^2)+\|u_t\|^2+\|v_t\|^2) \\
&\le K_3(M(\|Du_1\|^2,\|Dv_1\|^2)(\|Du\|^2+\|Dv\|^2)+\|u_t\|^2+\|v_t\|^2),
\end{aligned} \tag{3.15}
$$

make $\psi_3(t)=M(\|Du_1\|^2+\|Dv_1\|^2)(\|Du\|^2+\|Dv\|^2)+\|u_t\|^2+\|v_t\|^2$,

thus

$$\frac{d}{dt}\psi_3(t)\le K_3\psi_3(t) \quad . \tag{3.16}$$

Therefore by Gronwall inequality obtain

$$\psi_3(t)\le\psi_3(0)e^{K_3t} \ . \tag{3.17}$$

Thus obtain

$$M(\|Du_1\|^2,\|Dv_1\|^2)(\|Du\|^2+\|Dv\|^2)+\|u_t\|^2+\|v_t\|^2\le 0, \tag{3.18}$$

where $\|Du\|=\|Dv\|=\|u_t\|=\|v_t\|=0$,

$$u(x,t)=v(x,t)=0\ .$$

In summary $W\begin{pmatrix}u\\v\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$ uniqueness proof finished.

4.The existence of the global attractor

Theorem 2[7]. Make E is Banach space, $\{S(t)\}(t\ge 0)$ is a semigroup operator,

$$S(t):E\to E \quad S(t)\cdot S(\tau)=S(t+\tau)\ ,$$

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$S(0) = I$ is a constant operator. make $S(t)$ satisfies :

(1) $S(t)$ unanimously bounded, which is $\forall R > 0, \|u\|_E \le R$, with a constant $C(R)$ such that,

$$\|S(t)u\|_E \le C(R)(t \in [0, +\infty)).$$

(2) There is a bounded set $B_0 \subset E$, any bounded set $B \subset E$ there is a time t_0, such that

$$S(t)B \subset B_0 (t > t_0)$$

(3) for any $t \ge 0$, $S(t)$ is all a continuous operator. Then the semigroup has a compact overall attractor A.

Theorem 3、suppose that the global smooth solutions of the system equations satisfy Lemma2.5 and lemma 2.6 assumption conditions, then system equations (1.1)-(1.5) exist in a global attractor

$$A = w(B_0) = \bigcap_{\tau \ge 0} \overline{\bigcup_{t \ge \tau} S(t) B_0}.$$

Proof : Need to verify the conditions of theorem 2: (1), (2), (3). Under the assumption of theorem 3, the simegroup $S(t), E = H^1(\Omega) \times H^1(\Omega)$ solution of the existence system equations, $S(t): H^1(\Omega) \times H^1(\Omega) \to H^1(\Omega) \times H^1(\Omega)$ .by lemma 2.5 and lemma 2.6 bounded set for $\forall B \subset H^1(\Omega) \times H^1(\Omega)$ and contains in the ball $\left\{ \|u\|_{H^1(\Omega)} + \|v\|_{H^1(\Omega)} \le R \right\}$.

$$\|S(t)(u_0 + v_0)\|^2_{H^1(\Omega)\times H^1(\Omega)} = \|u\|^2_{H^1(\Omega)} + \|v\|^2_{H^1(\Omega)} \le c_{29}(\|u_0\|^2_{H^1(\Omega)} + \|v_0\|^2_{H^1(\Omega)}) \le c_{29} R, (t \ge 0, (u_0, v_0) \in B),$$

this indicates $\{S(t)\}(t \ge 0)$ in $H^1(\Omega) \times H^1(\Omega)$ unanimously bounded, further from lemma 2.5 and lemma 2.6 have

$$\|S(t)(u_0 + v_0)\|^2_{H^1(\Omega)\times H^1(\Omega)} = \|u\|^2_{H^1(\Omega)} + \|v\|^2_{H^1(\Omega)} \le 2R^2, \text{ for } t \ge t_0 = t_0(R).$$

Thus $B_0 \ge \left\{ (u, v) \in \|u\|_{H^1(\Omega)} + \|v\|_{H^1(\Omega)}, \|u\|_{H^1(\Omega)} + \|v\|_{H^1(\Omega)} \le \sqrt{2}R \right\}$ is bounded set of semigroup $S(t)$.

From Lemma 2.6, have $\|\Delta u\| + \|\Delta v\| < c_{31}, (t > 0)$,

due to $H^2(\Omega) \times H^2(\Omega) \to H^1(\Omega) \times H^1(\Omega)$ is compact embedded, which bounded set of $H^2(\Omega) \times H^2(\Omega)$ is compact set of $H^1(\Omega) \times H^1(\Omega)$, so the semigroup operator

$$S(t): H^1(\Omega) \times H^1(\Omega) \to H^2(\Omega) \times H^2(\Omega),$$

for $t \ge 0$ is completely continuous, then the system equations is in a global attractor $A = w(B_0) = \bigcap_{\tau \ge 0} \overline{\bigcup_{t \ge \tau} S(t) B_0}$. proof finished.

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