THE GLOBAL ATTRACTORS FOR THE HIGHER-ORDER NONLINEAR KIRCHHOFF-TYPE EQUATION WITH NONLINEAR DAMPED TERMS*

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ABSTRACT

In this paper ,we study the long time behavior of solution to the initial boundary value problems for higher -order kirchhoff-type equation with nonlinear strongly dissipation:

$$u_{tt} + (-\Delta)^m u_t + \phi(\|\nabla^m u\|^2)(-\Delta)^m u + h(u_t) = f(x).$$

At first ,we prove the existence and uniqueness of the solution by priori estimate and Galerkin methodthen,then, we establish the existence of global attractors.

Keywords: Higher-order nonlinear Kirchhoff wave equation; The existence and uniqueness; The Global attractors.

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1 Introduction

In this paper we concerned with the long time behavior of solution to the initial boundary value problems for Higher-order Kirchhoff-type equation with nonlinear strongly dissipation :

$$u_{tt} + (-\Delta)^m u_t + \phi(\|\nabla^m u\|^2)(-\Delta)^m u + h(u_t) = f(x), \tag{1.1}$$

$$u(x,t) = 0, \quad \frac{\partial^i u}{\partial v^i} = 0, \quad i = 1,2,\cdots,m-1, x \in \partial\Omega, t \in (0,+\infty). \tag{1.2}$$

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), x \in \Omega. \tag{1.3}$$

Where $\Omega \subset R^n$ is bounded open domain with smooth boundary; v is the outer norm vector; $m > 1$ is a positive integer, and $q > 0$ is a positive constants, $h(u_t)$ is a nonlinear damped , $f(x)$ is a function specified later, $(-\Delta)^m u_t$ is a strongly dissipation.

There have been many researches on the well-positive and the longtime dynamics for Kirchhoff equation.we can see[1-6],F.Li [5] deals with the higher-order kirchhoff-type equation with nonlinear dissipation:

$$u_{tt} + (\int_\Omega |\nabla^m u|^2)^q (-\Delta)^m u + u_t|u_t|^r = |u|^p u, \qquad x \in \Omega, t > 0. \tag{1.4}$$

$$u(x,t) = 0, \quad \frac{\partial^i u}{\partial v^i} = 0, \quad i = 1,2,\cdots,m-1, x \in \partial\Omega, t > 0. \tag{1.5}$$

$$u(x,0) = u_0, \; u_t(t,0) = u_1(x), \quad x \in \Omega. \tag{1.6}$$

In a bounded domain,where $m > 1$ is a positive integer, $p,q,r > 0$ are positive constants and obtain that the solution exists globally if $p \le r$, while if $p > \max\{r,2q\}$,then for any initial data with negative initial energy,the solution blows up at finite time in L^{p+2} norm.

Yang Zhijian, Wang Yunqing [6] also studied the global attractor for the Kirchhoff type equation with a strong dissipation:

$$u_{tt} - M(\|\Delta u\|^2)\Delta u - \Delta u_t + h(u_t) + g(u) = f(x), \quad \text{in } \Omega \times \mathrm{R}^+, \tag{1.7}$$

$$u(x,t)|_{\partial\Omega} = 0, \quad t > 0, \tag{1.8}$$

$$u(x,0) = u_0, \, u_t(t,0) = u_1(x), \quad x \in \Omega. \tag{1.9}$$

Where $M(s) = 1 + s^{\frac{m}{2}}$, $1 \le m \le \dfrac{4}{N-2}$, Ω is a bounded domain in R^N,with smooth boundary $\partial\Omega$, $h(s)$ and $g(s)$ are nonlinear functions,and $f(x)$ is an extrnal force term.it proves that the relative continuous semigroup $S(t)$ possesses in the phase space with low regularity a global attractor which is connected.

Yang zhijian, Cheng Jianling [7] studies the asymptotic behavior of solutions to the Kirchhoff-type equation:

$$u_{tt} - M(\|\nabla u\|^2)\Delta u - \Delta u_t + h(u_t) + g(x,u) = f(x), \quad \text{in } \Omega \times \mathrm{R}^+, \tag{1.10}$$

$$u|_{\partial\Omega} = 0, \quad t > 0. \tag{1.11}$$

$$u(x,0) = u_0, \, u_t(t,0) = u_1(x), \quad x \in \Omega. \tag{1.12}$$

They prove that the related continuous semigroup $S(t)$ posseses in phase $X = (H^2(\Omega) \cap H_0^1) \times H_0^1(\Omega)$ a global attractor. At the end of the paper, an example is shown,which indicates the Existence of nonlinear functions $g(x,u)$ and $h(u_t)$.

Zhang Yan ,Pu Zhi-lin and Chen Bo-tao [8] studied Boundedness of the solution to the Nonlinear Kirchhoff Equation:

$$u_{tt} - M(\|\nabla^m u\|^2)\Delta u + \beta u_t + g(u) = f(x), \quad \text{in } Q = \Omega \times (0,\infty). \tag{1.13}$$

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x) \quad \text{in } x \in \Omega. \tag{1.14}$$

$$u = 0, \text{in } \sum = \Gamma \times (0,\infty). \tag{1.15}$$

here $\|\nabla^m u\|^2 = \sum_{i=1}^{n} \int_\Omega \left|\dfrac{\partial u}{\partial x_i}\right|^2 dx$, $\Delta u = \sum_{i=1}^{n} \dfrac{\partial^2 u}{\partial x_i^2}$, u is the transverse displacement.the function $g \in C^1$ Satisfying the following conditions:

$$\liminf_{|s|\to\infty} \frac{G(s)}{s^2} \ge 0, G(s) = \int_0^s g(r)dr, \tag{1.16}$$

$$\limsup_{|s|\to\infty} \frac{|g'(s)|}{|s|^r} = 0. \tag{1.17}$$

Where $0 \le \gamma < \infty (n = 1,2), 0 \le \gamma < 2 (n = 3), \gamma = 0 (n = 4)$. Furthermore,there exists $C_1 > 0$ such that :

$$\liminf_{|s|\to\infty} \frac{sg(s) - C_1 G(s)}{s^2} \ge 0. \tag{1.18}$$

Zhang Yan studied the asymptotic behavior and existence of the solutions of a nonlinear Kirchhoff equation.

The paper is arranged as follows.in section 2,we state some preliminaries under the assume of Lemma 1and Lemma 2, we get the existence and uniqueness of solution; in section 3,we obtain the global attractors for the problems (1.1)-(1.3).

2 Preliminaries

For convenience,we denote the norm and scalar product in $L^2(\Omega)$ by $\|.\|$ and $(.,.)$; $f = f(x)$, $H^k = H^k(\Omega)$, $H_0^k = H_0^k(\Omega)$, $\|.\| = \|.\|_{L^2}$, $C_i (i = 1,2,\cdots,11)$ are constants.

In this section, we present some materials needed in the proof of our results, state a global existence result, and prove our main result. For this reason, we assume that and notations needed in the proof of our results.For this reason, we assume that

(G_1) Let $\phi(\|\nabla^m u\|^2)$ is a nonnegative C^1-function.satisfying

$$\varepsilon\phi(\|\nabla^m u\|^2)\|\nabla^m u\|^2 \ge \varepsilon\Phi(\|\nabla^m u\|^2) + \frac{1}{4}\varepsilon\|\nabla^m u\|^2, \tag{2.1}$$

and

$$\Phi(\|\nabla^m u\|^2) \ge \max\{2C_3\|\nabla^m u\|^{\frac{1}{\delta}}, \gamma\|\nabla^m u\|^2\}. \tag{2.2}$$

Where $\Phi(s) = \int_0^s \phi(s)ds,\ \ \gamma \ge \varepsilon.$

(G_2) there exist constant $0 < \delta < \frac{1}{2}$, have

$$\|h(s)\|_{H^{-m}} \le C_0 (h(s,s))^{1-\delta},\ h(s)s \ge 0. \tag{2.3}$$

(G_3) there exist constant $0 < \sigma_1 < 1$, have

$$\|h(s)\| \le C_1(1 + \|(-\Delta)^m s\|)^{1-\sigma_1}\ \forall s \in H^{2m} \cap H_0^m\ \|s\| \le R. \tag{2.4}$$

(G_4) there exist constant C_2, have

$$\|h(s_1) - h(s_2)\|_{H^{-m}} \le C_2\|s_1 - s_2\|. \tag{2.5}$$

2.1 the existence and uniqueness of solution

Lemma1 Assume $(G_1) - (G_2)$ hold, and $(u_0, u_1) \in H^m(\Omega) \times L^2(\Omega)$, $f(x) \in L^2(\Omega)$,then the solution $(u, v) \in H^m(\Omega) \times L^2(\Omega)$,and

$$\|(u,v)\|^2 = \|v\|^2 + \|\nabla^m u\|^2 \le \frac{W(0)e^{-\alpha_0 t}}{\alpha_1} + \frac{C_4(1 - e^{-\alpha_0 t})}{\alpha_1\alpha_0}. \tag{2.6}$$

Where $v = u_t + \varepsilon u$, $W(0) = \|v_0\|^2 + \Phi(\|\nabla^m u_0\|^2) - \varepsilon\|\nabla^m u_0\|^2$, $v_0 = u_1 + \varepsilon u_0$, thus there exist R_0 and $t_0 = t_0(\Omega) > 0$,

such that

$$\|(u,v)\|_{H^m \times L^2} = \|\nabla^m u\|^2 + \|v\|^2 \le R_0 (t > t_0). \tag{2.7}$$

Proof. Let $v = u_t + \varepsilon u$ we multiply v with both sides of equation (1.1) and obtain

$$(u_{tt} + (-\Delta)^m u_t + \phi(\|\nabla^m u\|^2)(-\Delta)^m u + h(u_t), v) + (f(u), v) = (f(x), v). \tag{2.8}$$

$$\begin{aligned}&(u_{tt}, v)\\ &= (v_t - \varepsilon u_t, v)\end{aligned}$$

$$= (v_t, v) - \varepsilon(v - \varepsilon u, v)$$
$$= \frac{1}{2}\frac{d}{dt}\|v\|^2 - \varepsilon(v - \varepsilon u, v)$$
$$\geq \frac{1}{2}\frac{d}{dt}\|v\|^2 - \varepsilon\|v\|^2 - \frac{\varepsilon^2}{2\lambda_1^m}\|\nabla^m u\|^2 - \frac{\varepsilon^2}{2}\|v\|^2. \tag{2.9}$$

$$((-\Delta)^m u_t, v)$$
$$= ((-\Delta)^m (v - \varepsilon u), v)$$
$$= \|\nabla^m v\|^2 - \frac{\varepsilon}{2}\frac{d}{dt}\|\nabla^m u\|^2 - \varepsilon^2\|\nabla^m u\|^2. \tag{2.10}$$

$$(\phi(\|\nabla^m u\|^2)(-\Delta)^m u, v)$$
$$= (\phi(\|\nabla^m u\|^2)(-\Delta)^m u, u_t + \varepsilon u)$$
$$= \frac{1}{2}\phi(\|\nabla^m u\|^2)\frac{d}{dt}\|\nabla^m u\|^2 + \varepsilon\phi(\|\nabla^m u\|^2)\|\nabla^m u\|^2, \tag{2.11}$$

according (2.1),we obtain

$$(\phi(\|\nabla^m u\|^2)(-\Delta)^m u, v)$$
$$\geq \frac{1}{2}\frac{d}{dt}\Phi(\|\nabla^m u\|^2) + \varepsilon\Phi(\|\nabla^m u\|^2) + \frac{\varepsilon}{4}\|\nabla^m u\|^2. \tag{2.12}$$

$$(h(u_t), v) = (h(u_t), u_t + \varepsilon u), \tag{2.13}$$

from (2.3),we have

$$\varepsilon|(h(u_t), u)|$$
$$\leq \|h(u_t)\|_{H^{-m}}\|\nabla^m u\|$$
$$\leq \varepsilon C_0 (h(u_t), u_t)^{1-\delta}\|\nabla^m u\|$$
$$\leq \frac{1}{2}(h(u_t), u_t) + C_3\varepsilon^{\frac{1}{\delta}}\|\nabla^m u\|^{\frac{1}{\delta}}, \tag{2.14}$$

so,we get

$$(h(u_t), v) \geq \frac{1}{2}(h(u_t), u_t) - C_3\varepsilon^{\frac{1}{\delta}}\|\nabla^m u\|^{\frac{1}{\delta}} \tag{2.15}$$

$$(f(x), v) \leq \|f(x)\|\|v\| \leq \frac{\|f\|^2}{2\varepsilon^2} + \frac{\varepsilon^2}{2}\|v\|^2. \tag{2.16}$$

From above ,we have

$$\frac{1}{2}\frac{d}{dt}(\|v\|^2 + \Phi(\|\nabla^m u\|^2) - \varepsilon\|\nabla^m u\|^2) + \|\nabla^m v\|^2 - \varepsilon\|v\|^2 - \varepsilon^2\|v\|^2$$

$$+\varepsilon\Phi(\|\nabla^m u\|^2) + \frac{\varepsilon}{4}\|\nabla^m u\|^2 - \frac{\varepsilon^2}{2\lambda^m}\|\nabla^m u\|^2 - \varepsilon^2\|\nabla^m u\|^2 - 2C_3\varepsilon^{\frac{1}{\delta}}\|\nabla^m u\|^{\frac{1}{\delta}}$$

$$\leq \frac{1}{2\varepsilon^2}\|f\|^2. \tag{2.17}$$

By using Poincare inequality,we obtain: $\|\nabla^m v\|^2 \geq \lambda_1{}^m \|v\|^2$, then we have

$$\frac{d}{dt}(\|v\| + \Phi(\|\nabla^m u\|^2) - \varepsilon\|\nabla^m u\|^2) + (2\lambda_1{}^m - 2\varepsilon - 2\varepsilon^2)\|v\|^2$$
$$+2\varepsilon\Phi(\|\nabla^m u\|^2) - \frac{\varepsilon^2}{\lambda^m}\|\nabla^m u\|^2 - 2\varepsilon^2\|\nabla^m u\|^2 + \frac{\varepsilon}{2}\|\nabla^m u\|^2 - 2C_3\varepsilon^{\frac{1}{\delta}}\|\nabla^m u\|^{\frac{1}{\delta}}$$
$$\leq \frac{1}{\varepsilon^2}\|f\|^2,. \qquad (2.18)$$

from (2.18),we have

$$\frac{d}{dt}(\|v\| + \Phi(\|\nabla^m u\|^2) - \varepsilon\|\nabla^m u\|^2) + (2\lambda_1{}^m - 2\varepsilon - 2\varepsilon^2)\|v\|^2$$
$$+\varepsilon\Phi(\|\nabla^m u\|^2) - \frac{\varepsilon^2}{\lambda^m}\|\nabla^m u\|^2 - 2\varepsilon^2\|\nabla^m u\|^2 + \frac{\varepsilon}{2}\|\nabla^m u\|^2 + \varepsilon\Phi(\|\nabla^m u\|^2) - 2C_3\varepsilon^{\frac{1}{\delta}}\|\nabla^m u\|^{\frac{1}{\delta}}$$
$$\leq \frac{1}{\varepsilon^2}\|f\|^2. \qquad (2.19)$$

From (2.2), we have $\varepsilon\Phi(\|\nabla^m u\|^2) - 2C_3\varepsilon^{\frac{1}{\delta}}\|\nabla^m u\|^{\frac{1}{\delta}} \geq 0,$ we take $\kappa_1 = \frac{1}{2}\varepsilon - (\frac{1}{\lambda^m} + 2)\varepsilon^2 \geq 0,$.

we get

$$\frac{d}{dt}(\|v\| + \Phi(\|\nabla^m u\|^2) - \varepsilon\|\nabla^m u\|^2) + (2\lambda_1{}^m - 2\varepsilon - 2\varepsilon^2)\|v\|^2$$
$$+\varepsilon\Phi(\|\nabla^m u\|^2)$$
$$\leq \frac{1}{\varepsilon^2}\|f\|^2. \qquad (2.20)$$

Next we take $\alpha_0 = \min\{2\lambda^m{}_1 - 2\varepsilon - 2\varepsilon^2, \varepsilon\}$ we get

$$\frac{d}{dt}(\|v\|^2 + \Phi(\|\nabla^m u\|^2) - \varepsilon\|\nabla^m u\|^2)$$
$$+\alpha_0(\|v\|^2 + \Phi(\|\nabla^m u\|^2) - \varepsilon\|\nabla^m u\|^2)$$
$$\leq \frac{1}{\varepsilon^2}\|f\|^2. \qquad (2.21)$$

From(2.2),we obtain

$$\|v\|^2 + \Phi(\|\nabla^m u\|^2) - \varepsilon\|\nabla^m u\|^2 \geq 0. \qquad (2.22)$$

Then we have

$$\frac{d}{dt}W(t) + \alpha_0 W(t) \leq C_4, \qquad (2.23)$$

where $W(t) = \|v\|^2 + \Phi(\|\nabla^m u\|^2) - \varepsilon\|\nabla^m u\|^2, C_4 = \frac{1}{\varepsilon^2}\|f\|^2$,by using Gronwall inequality ,we obtain

$$W(t) \leq W(0)e^{-\alpha_0 t} + \frac{C_4(1 - e^{-\alpha_0 t})}{\alpha_0}, \qquad (2.24)$$

where $W(0)=\|v_0\|^2+\Phi(\|\nabla^m u_0\|^2)-\varepsilon\|\nabla^m u_0\|^2$.
From (2.2), we know

$$\begin{aligned}&\|v\|^2+(\gamma-\varepsilon)\|\nabla^m u\|^2\\&\le\|v\|^2+\Phi(\|\nabla^m u\|^2)-\varepsilon\|\nabla^m u\|^2\\&\le W(0)e^{-\alpha_0 t}+\frac{C_4(1-e^{-\alpha_0 t})}{\alpha_0},\end{aligned}\tag{2.25}$$

we take $\alpha_1=\min\{1,\gamma-\varepsilon\}$,so

$$\alpha_1(\|v\|^2+\|\nabla^m u\|^2)\le W(0)e^{-\alpha_0 t}+\frac{C_4(1-e^{-\alpha_0 t})}{\alpha_0}.\tag{2.26}$$

So,we have

$$\|v\|^2+\|\nabla^m u\|^2\le\frac{W(0)e^{-\alpha_0 t}}{\alpha_1}+\frac{C_4(1-e^{-\alpha_0 t})}{\alpha_1\alpha_0},\tag{2.27}$$

so,we obtain

$$\|(u,v)\|^2=\|v\|^2+\|\nabla^m u\|^2\le\frac{W(0)e^{-\alpha_0 t}}{\alpha_1}+\frac{C_4(1-e^{-\alpha_0 t})}{\alpha_1\alpha_0},\tag{2.28}$$

and

$$\overline{\lim_{t\to\infty}}\|(u,v)\|^2_{H^m\times L^2}\le\frac{C_4}{\alpha_1\alpha_0}.\tag{2.29}$$

So,there exist R_0 and $t_0=t_0(\Omega)>0$, such that

$$\|(u,v)\|^2{}_{H^m\times L^2}=\|\nabla^m u\|^2+\|v\|^2\le R_0(t>t_0).\tag{2.30}$$

Lemma2 In addition to the assumptions of Lemma1, and $(G_1)-(G_3)$ hold, if $f\in H^m(\Omega)$,and
$(u_0,u_1)\in H^{2m}(\Omega)\times H^m(\Omega)$,then the solution (u,v) of the problems (1.1)-(1.3) satisfies (u,v) $\in H^{2m}(\Omega)\times H^m(\Omega)$,and

$$\|(u,v)\|^2_{H^{2m}\times H^m}=\|(-\Delta)^m u\|^2+\|\nabla^m v\|^2\le\frac{Y(0)e^{-\beta_0 t}}{\beta_1}+\frac{C_6(1-e^{-\beta_0 t})}{\beta_0\beta_1}.\tag{2.31}$$

Where $(-\Delta)^m v=(-\Delta)^m u_t+\varepsilon(-\Delta)^m u$,and $Y(0)=(\delta-\varepsilon)\|(-\Delta)^m u_0\|^2+\|\nabla^m v_0\|^2$, thus there exist R_1 and $t_1=t_1(\Omega)>0$,such that

$$\|(u,v)\|^2_{H^{2m}\times L^2}=\|(-\Delta)^m u\|^2+\|\nabla^m v\|^2\le R_1(t>t_1).\tag{2.32}$$

Proof. Let $(-\Delta)^m v=(-\Delta)^m u_t+\varepsilon(-\Delta)^m u,$ we multiply $(-\Delta)^m v$ with both sides of equation (1.1) , and obtain

$$(u_{tt}+(-\Delta)^m u_t+\phi(\|\nabla^m u\|^2)(-\Delta)^m u+h(u_t),(-\Delta)^m v)=(f(x),(-\Delta)^m v).\tag{2.33}$$

$$(u_{tt},(-\Delta)^m v)\ge \frac{1}{2}\frac{d}{dt}\|\nabla^m v\|^2-\varepsilon\|\nabla^m v\|^2-\frac{\varepsilon^2}{2\lambda_1^m}\|(-\Delta)^m u\|^2-\frac{\varepsilon^2}{2}\|\nabla^m v\|^2. \quad (2.34)$$

$$\begin{aligned}&((-\Delta)^m u_t,(-\Delta)^m v)\\&=((-\Delta)^m(v-\varepsilon u),(-\Delta)^m v)\\&=\|(-\Delta)^m v\|^2-\frac{\varepsilon}{2}\frac{d}{dt}\|(-\Delta)^m u\|^2-\varepsilon^2\|(-\Delta)^m u\|^2.\end{aligned} \quad (2.35)$$

$$\begin{aligned}&(\phi(\|\nabla^m u\|^2)(-\Delta)^m u,(-\Delta)^m v)\\&=\frac{1}{2}\phi(\|\nabla^m u\|^2)\frac{d}{dt}(\|(-\Delta)^m u\|^2)+\varepsilon\phi(\|\nabla^m u\|^2)\|(-\Delta)^m u\|^2,\end{aligned} \quad (2.36)$$

according Lemma 1,we have $\varepsilon\le\delta_0\le\phi(s)\le\delta_1,\ \delta_2=\{^{\delta_0,\frac{d}{dt}(\|\nabla^m u\|^2)\ge 0}_{\delta_1,\frac{d}{dt}(\|\nabla^m u\|^2)<0}$, we obtain ,

$$\begin{aligned}&(\phi(\|\nabla^m u\|^2)(-\Delta)^m u,(-\Delta)^m v)\\&=\frac{1}{2}\phi(\|\nabla^m u\|^2)\frac{d}{dt}(\|(-\Delta)^m u\|^2)+\varepsilon\phi(\|\nabla^m u\|^2)\|(-\Delta)^m u\|^2\\&\ge\frac{\delta_2}{2}\frac{d}{dt}\|(-\Delta)^m u\|^2+\varepsilon\delta_0\|(-\Delta)^m u\|^2.\end{aligned} \quad (2.37)$$

$$|(h(u_t),(-\Delta)^m v|\le\frac{\|h(u_t)\|^2}{2}+\frac{\|(-\Delta)^m v\|^2}{2}, \quad (2.38)$$

From (2.4), we have

$$\|h(u_t)\|^2\le C_1^{\,2}(1+\|(-\Delta)^m u_t\|)^{2(1-\sigma_1)}, \quad (2.39)$$

By using Young's inequality

$$\|h(u_t)\|^2\le\frac{\sigma_1}{\mu^{\frac{1}{\sigma_1}}}(C_1^{\,2})^{\frac{1}{\sigma_1}}+(1-\sigma_1)\mu^{\frac{1}{1-\sigma_1}}((1+\|(-\Delta)^m u_t\|)^{2(1-\sigma_1)})^{\frac{1}{1-\sigma_1}}, \quad (2.40)$$

and

$$\|h(u_t)\|^2\le C_5+\frac{1}{4}\|(-\Delta)^m v\|^2+\frac{\varepsilon^2}{4}\|(-\Delta)^m u\|^2,$$

and

$$(h(u_t),(-\Delta)^m v)\ge-\frac{C_5}{2}-\frac{\varepsilon^2}{8}\|(-\Delta)^m u\|^2-\frac{5}{8}\|(-\Delta)^m v\|^2. \quad (2.41)$$

Where $C_5:=\frac{\sigma_1}{\mu^{\frac{1}{\sigma_1}}}(C_1^{\,2}(R))^{\frac{1}{\sigma_1}}+2(1-\sigma_1)\mu^{\frac{1}{1-\sigma_1}}$,we take proper μ ,such that : $4(1-\sigma_1)\mu^{\frac{1}{1-\sigma_1}}=\frac{1}{4}$,

$$(f(x),(-\Delta)^m v)\le \frac{\left\|\nabla^m f\right\|^2}{2\varepsilon^2}+\frac{\varepsilon^2\left\|\nabla^m v\right\|^2}{2}. \qquad (2.42)$$

Form above,we have

$$\frac{d}{dt}(\left\|\nabla^m v\right\|^2+\delta_2\left\|(-\Delta)^m u\right\|^2-\varepsilon\left\|(-\Delta)^m u\right\|^2)+(\frac{3\lambda_1^m}{4}-2\varepsilon-2\varepsilon^2)\left\|\nabla^m v\right\|^2$$
$$+2\varepsilon\delta_0\left\|(-\Delta)^m u\right\|^2-(\frac{9\varepsilon^2}{4}+\frac{\varepsilon^2}{\lambda_1^m})\left\|(-\Delta)^m u\right\|^2$$
$$\le\frac{1}{\varepsilon^2}\left\|\nabla^m f\right\|^2+C_5. \qquad (2.43)$$

Then we take proper ε ,let $\frac{3\lambda_1^m}{4}-2\varepsilon-2\varepsilon^2\ge 0,\ \delta_2-\varepsilon\ge 0$. so,we get

$$\frac{d}{dt}(\left\|\nabla^m v\right\|^2+\delta_2\left\|(-\Delta)^m u\right\|^2-\varepsilon\left\|(-\Delta)^m u\right\|^2)+(\frac{3\lambda_1^m}{4}-2\varepsilon-2\varepsilon^2)\left\|\nabla^m v\right\|^2$$
$$\frac{(2\varepsilon\delta_0-\frac{9\varepsilon^2}{4}-\frac{\varepsilon^2}{\lambda_1^m})(\delta_2-\varepsilon)}{\delta_2-\varepsilon}\left\|(-\Delta)^m u\right\|^2$$
$$\le\frac{1}{\varepsilon^2}\left\|\nabla^m f\right\|^2+C_5. \qquad (2.44)$$

Let Taking $\beta_0=\min\{\frac{3\lambda_1^m}{4}-2\varepsilon-2\varepsilon^2,\frac{2\varepsilon\delta_0-\frac{9\varepsilon^2}{4}-\frac{\varepsilon^2}{\lambda_1^m}}{\delta_2-\varepsilon}\}$, $C_6=\frac{1}{\varepsilon^2}\left\|\nabla^m f\right\|^2+C_5$, then

$$\frac{d}{dt}Y(t)+\beta_0 Y(t)\le C_6\ , \qquad (2.45)$$

where $Y(t)=\left\|\nabla^m v\right\|^2+(\delta_2-\varepsilon)\left\|(-\Delta)^m u\right\|^2\ge 0$, by using Gronwall inequality,then

$$Y(t)\le Y(0)e^{-\beta_0 t}+\frac{C_6}{\beta_0}(1-e^{-\beta_0 t}). \qquad (2.46)$$

where $Y(0)=(\delta_2-\varepsilon)\left\|(-\Delta)^m u_0\right\|^2+\left\|\nabla^m v_0\right\|^2$, Let $\beta_1=\min\{1,\delta_2-\varepsilon\}$,we get

$$\beta_1(\left\|\nabla^m v\right\|^2+\left\|(-\Delta)^m u\right\|^2)\le Y(t), \qquad (2.47)$$

so we get

$$\left\|(u,v)\right\|^2_{H^{2m}\times H^m}=\left\|(-\Delta)^m u\right\|^2+\left\|\nabla^m v\right\|^2\le\frac{Y(0)e^{-\beta_0 t}}{\beta_1}+\frac{C_6(1-e^{-\beta_0 t})}{\beta_0\beta_1}, \qquad (2.48)$$

and

$$\overline{\lim_{t\to\infty}}\left\|(u,v)\right\|^2_{H^{2m}\times H^m}\le\frac{C_6}{\beta_0\beta_1}. \qquad (2.49)$$

So,there exist R_1 and $t_1=t_1(\Omega)>0$, such that

$$\left\|(u,v)\right\|^2{}_{H^{2m}\times H^m}=\left\|(-\Delta)^m u\right\|^2+\left\|\nabla^m v\right\|^2\le R_1(t>t_2). \qquad (2.50)$$

Theorem 2.1 Assume $(G_1)-(G_4)$ holds,and Lemma1 Lemma2 holds; the problem (1.1)-(1.3) exists a unique smooth solution

$$(u,v)\in L^{\infty}([0,+\infty);H^{2m}\times H^{m}). \qquad (2.51)$$

Proof. By the Galerkin method, Lemma 1 and Lemma 2,we can easily obtain the existence of solution. Next, we prove the uniqueness of solutions in detail, Assume u,v are two solutions of the problems (1.1)-(1.3). let $w=u-v$,then $w(x,0)=w_0(x)=0$, $w_t(x,0)=w_1(x)=0$. Then two equations subtract and obtain

$$w_{tt}+(-\Delta)^m w_t+\phi(\|\nabla^m u\|^2)(-\Delta)^m u-\phi(\|\nabla^m v\|^2)(-\Delta)^m v+h(u_t)-h(v_t)=0. \quad (2.52)$$

By multiplying above equation by w_t we get

$$(w_{tt}+(-\Delta)^m w_t+\phi(\|\nabla^m u\|^2)(-\Delta)^m u-\phi(\|\nabla^m v\|^2)(-\Delta)^m v+h(u_t)-h(v_t),w_t)=0. \quad (2.53)$$

$$(w_{tt},w_t)=\frac{1}{2}\frac{d}{dt}\|w_t\|^2. \qquad (2.54)$$

$$((-\Delta)^m w_t,w_t)=\|\nabla^m w_t\|^2. \qquad (2.55)$$

$$\begin{aligned}&(\phi(\|\nabla^m u\|^2)(-\Delta)^m u-\phi(\|\nabla^m v\|^2)(-\Delta)^m v,w_t)\\&=(\phi(\|\nabla^m u\|^2)(-\Delta)^m u-\phi(\|\nabla^m u\|^2)(-\Delta)^m v+\phi\|\nabla^m u\|^2(-\Delta)^m v-\phi(\|\nabla^m v\|^2)(-\Delta)^m v,w_t)\\&=\phi(\|\nabla^m u\|^2)(-\Delta)^m w,w_t)+(\phi(\|\nabla^m u\|^2)-\phi(\|\nabla^m v\|^2))(-\Delta)^m v,w_t)\\&=\phi(\|\nabla^m u\|^2)\frac{1}{2}\frac{d}{dt}\|\nabla^m w\|^2+\phi(\|\nabla^m u\|^2)(-\Delta)^m v-\phi(\|\nabla^m v\|^2)(-\Delta)^m v,w_t),\end{aligned} \qquad (2.56)$$

$$\begin{aligned}&\left|(\phi(\|\nabla^m u\|^2)-\phi(\|\nabla^m v\|^2))(-\Delta)^m v,w_t)\right|\\&\le\phi'(\xi)(\|\nabla^m u\|+\|\nabla^m v\|)(\|\nabla^m u\|-\|\nabla^m v\|)((-\Delta)^m v,w_t).\\&\le\left\|\phi'(\|\nabla^m u\|^2)\right\|_\infty(\|\nabla^m u\|+\|\nabla^m v\|)(\|\nabla^m u\|-\|\nabla^m v\|)\|(-\Delta)^m v\|\|w_t\|. \quad (2.57)\end{aligned}$$

According to Lemma1, Lemma2,we have

$$\begin{aligned}&\left|(\phi(\|\nabla^m u\|^2)-\phi(\|\nabla^m v\|^2))(-\Delta)^m v,w_t)\right|\\&\le C_7\|\nabla^m w\|\|w_t\|\\&\le\frac{\mu_2}{2}\|w_t\|^2+\frac{C_7^2}{2\mu_2}\|\nabla^m w\|^2,\end{aligned} \qquad (2.58)$$

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}(\phi(\|\nabla^m u\|^2)\|\nabla^m w\|^2)-\frac{1}{2}\frac{d}{dt}\phi(\|\nabla^m u\|^2)\|\nabla^m w\|^2\\&=\frac{1}{2}\frac{d}{dt}(\phi(\|\nabla^m u\|^2)\|\nabla^m w\|^2)-\phi'(\|\nabla^m u\|^2)\int_\Omega\nabla^m u\nabla^m u_t dx\|\nabla^m w\|^2.\end{aligned} \qquad (2.59)$$

According to Lemma1, Lemma2,we have

$$\phi'(\|\nabla^m u\|^2)\int_\Omega \nabla^m u \nabla^m u_t dx \|\nabla^m w\|^2$$
$$\leq \|\phi'(\xi)\|_\infty \|\nabla^m u\| \|\nabla^m u_t\| \|\nabla^m w\|^2$$
$$\leq C_8 \|\nabla^m w\|^2, \tag{2.60}$$

so, we have

$$\frac{1}{2}\frac{d}{dt}(\phi(\|\nabla^m u\|^2)\|\nabla^m w\|^2) - \frac{1}{2}\frac{d}{dt}\phi(\|\nabla^m u\|^2)\|\nabla^m w\|^2$$

$$\geq \frac{1}{2}\frac{d}{dt}(\phi(\|\nabla^m u\|^2)\|\nabla^m w\|^2) - C_8\|\nabla^m w\|^2. \tag{2.61}$$

From above,we get

$$(\phi(\|\nabla^m u\|^2)(-\Delta)^m u - \phi(\|\nabla^m v\|^2)(-\Delta)^m v, w_t)$$
$$\geq \frac{1}{2}(\phi(\|\nabla^m u\|^2)\|\nabla^m w\|^2) - C_8\|\nabla^m w\|^2 - \frac{\mu_2}{2}\|w_t\|^2 - \frac{C_7^2}{2\mu_2}\|\nabla^m w\|^2. \tag{2.62}$$

$$|(h(u_t) - h(v_t), w_t)|$$
$$\leq \|h(u_t) - h(v_t)\|_{H^{-m}} \|\nabla^m w_t\|$$
$$\leq C_2 \|w_t\| \|\nabla^m w_t\|$$
$$\leq C_9\|w_t\|^2 + \frac{\varepsilon}{2}\|\nabla^m w_t\|^2, \tag{2.63}$$

so,we get

$$(h(u_t) - h(v_t), w_t) \geq -C_9\|w_t\|^2 - \frac{\varepsilon}{2}\|\nabla^m w_t\|^2. \tag{2.64}$$

From above (2.53)-(2.64) , we have

$$\frac{d}{dt}(\|w_t\|^2 + \phi(\|\nabla^m u\|^2)\|\nabla^m w\|^2) + 2\|\nabla^m w_t\|^2 - \mu_2\|w_t\|^2 - \frac{C_7^2}{\mu_2}\|\nabla^m w\|^2$$
$$-2C_8\|\nabla^m w\|^2 - 2C_9\|w_t\|^2 - \varepsilon\|\nabla^m w_t\|^2$$
$$\leq 0 \ . \tag{2.65}$$

Then

$$\frac{d}{dt}(\|w_t\|^2 + \phi(\|\nabla^m u\|^2)\|\nabla^m w\|^2) \leq C_{10}\|\nabla^m w\|^2 + C_{11}\|w_t\|^2 \tag{2.66}$$

where $C_{10} = \frac{C_7^2}{\mu_2} + 2C_8, C_{11} = \mu_2 + 2C_9$.

According to $\phi(\|\nabla^m u\|^2)\|\nabla^m w\| \geq \varepsilon\|\nabla^m w\|^2$,then

.

$$\frac{d}{dt}(\|u_t\|^2 + \phi(\|\nabla^m u\|^2)\|\nabla^m w\|^2) \leq \frac{C_{10}}{\varepsilon}\phi(\|\nabla^m u\|^2)\|\nabla^m w\|^2 + C_{11}\|w_t\|^2. \tag{2.67}$$

Taking $\gamma_1=\max\{\frac{C_{10}}{\varepsilon},C_{11}\}$,we have

$$\frac{d}{dt}(\|w_t\|^2+\phi(\|\nabla^m u\|^2)\|\nabla^m w\|^2)\le\gamma_1(\phi(\|\nabla^m u\|^2)\|\nabla^m w\|^2+\|w_t\|^2), \qquad (2.68)$$

by using Gronwall inequality, we obtain

$$\|w_t\|^2+\phi(\|\nabla^m u\|^2)\|\nabla^m w\|^2\le\gamma_1(\phi(\|\nabla^m u_0\|^2)\|\nabla^m w(0)\|^2+\|w_t(0)\|^2)e^{\gamma_1 t}, \qquad (2.69)$$

therefore

$$u=v. \qquad (2.70)$$

So we get the uniqueness of the solution.

3. **Global attractor**

Theorem3.1 [11] Let E_1 be a Banach space ,and $\{S(t)\}(t\ge 0)$ are the semigroup on E_1. $S(t):E_1\to E_1, S(t+s)=S(t)S(s),(\forall t,s\ge 0), S(0)=I$, where I is a unit operator , set $S(t)$ the follow conditions.

1) $S(t)$ is uniformly bounded, namely $\forall R>0,\|u\|_{E_1}\le R$, it exists a constant $C(R)$, so that $\|S(t)u\|_{E_1}\le C(R),(t\in[0,+\infty)$;

2) it exists a bounded absorbing set $B_0\subset E_1$, namely, $\forall B\subset E_1$, it exists a constant t_0, so that $S(t)B\subset B_0(t\ge t_0)$;

Where B_0 and B are bounded sets.

3) when $t>0, S(t)$ is a completely continuous operator A.

Therefore, the semigroup operator $S(t)$ exists a compact global attract.

Theorem3.2 Under the assume of Lemma 1,Lemma 2,Theorem 3.1, equations(1.1)-(1.3) have global attractor

$$A=w(B_0)=\bigcap_{s\ge 0}\overline{\bigcup_{t\ge s}S(t)B_0}.$$

Where $B_0=\{(u,v)\in H^{2m}(\Omega)\times H^m(\Omega):\|(u,v)\|^2_{H^{2m}\times H^m}=\|u\|^2_{H^{2m}}+\|v\|^2_{H^m}\le R_0+R_1\}$, B_0 is the bounded absorbing set of $H^{2m}(\Omega)\times H^m(\Omega)$ and satisfies

1) $S(t)A=A, t>0$;

2) $\lim_{t\to\infty}dis(S(t)B,A)=0$, here $B\subset H^{2m}\times H^m$ and it is a bounded set,

$$\lim_{t\to\infty}dis(S(t)B,A)=\sup_{x\in B}(\inf_{y\in A}\|S(t)x-y\|_{H^{2m}\times H^m})\to 0, t\to\infty.$$

Proof. Under the conditions of Theorem 3.1, it exists the solution semigroup $S(t)$, $S(t):H^{2m}(\Omega)\times H^m(\Omega)\to H^{2m}(\Omega)\times H^m(\Omega)$, here $E_1=H^{2m}(\Omega)\times H^m(\Omega)$.

1) from Lemma2.1 to Lemma2.2, we can get that $\forall B\subset H^{2m}(\Omega)\times H^m(\Omega)$ is a bounded set that includes in the ball $\{\|(u,v)\|_{H^{2m}\times H^m}\le R\}$,

$$\|S(t)(u_0,v_0)\|^2_{H^{2m}\times H^m}=\|u\|^2_{H^{2m}}+\|v\|^2_{H^m}\le\|u_0\|^2_{H^{2m}}+\|v_0\|^2_{H^m}+C\le R+C,(t\ge 0,(u_0,v_0)\in B).$$

This shows that $S(t)(t\ge 0))$ is uniformly bounded $H^{2m}(\Omega)\times H^m(\Omega)$.

2) furthermore,for any $(u_0,v_0)\in H^{2m}(\Omega)\times H^m(\Omega)$, when $t\ge\max\{t_0,t_1\}$, we have,

$$\|S(t)(u_0,v_0)\|^2_{H^{2m}\times H^m} = \|u\|^2_{H^{2m}} + \|v\|^2_{H^m} \le R_0 + R_1,$$

so we get B_0 is the bounded absorbing set.

3) since $E_1 = H^{2m}(\Omega)\times H^m(\Omega) \mapsto E_0 = H^m(\Omega)\times L^2(\Omega)$ is compact embedded,which means that the bounded set in E_1 is the compact set in E_0 ,so the semigroup operator $S(t)$ exists a compact global attractor A.

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