QUANTUM SECRET SHARING VIA FOUR PARTICLE ASYMMETRIC ENTANGLED STATE

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ABSTRACT

A general scheme for quantum secret sharing of an arbitrary single-qubit state with four-qubit asymmetric entangled state is proposed. In this scheme, the sender performs Bell measurement on his particle, the two collaborators and the receiver perform joint unitary operation on the rest particles. Then the two collaborators perform measurement on their particle using mutually orthogonal bases. Thus the receiver can successfully reconstruct the single particle with a certain probability.

Keywords: Quantum secret sharing; asymmetric entangled state; joint unitary operation; Bell measurement.

1 Introduction

Quantum entanglement, as a physical resource, plays an important role in all aspects of quantum information, for instance quantum teleportation[1−6], quantum dense coding[7−8], quantum secure sharing[9−12], etc. Up to now, there are many achievements about quantum entanglement in theory and experiment, which can be seen in the literature[1−12]. In 1999, Hillery et al.[9] proposed the first quantum secure sharing protocol using three or four particle entangled GHZ class as quantum channel. In 2012, Zhang Qunyong et al.[10] proposed a quantum secret sharing scheme of arbitrary single-qubit state with three particle asymmetric entangled state. In 2016, Zhang Jianzhong et al.[11] proposed a scheme of quantum secret sharing based on four-qubit asymmetric entangled state. In 2017, we proposed a scheme for controlled quantum teleportation of an unknown single particle[12], using a special four particle asymmetric entangled state as quantum channel.

In this paper, we intend to discuss a quantum secret sharing of an arbitrary single-qubit state.

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The quantum channel that linking the parties is four particle asymmetric entangled state where the coefficient is arbitrary. There includes four parties, one sender, three agents, where any agent can cooperate with other agents to reconstruct the unknown single-qubit state. We will use a method which is more simple and convenient than that in [12] to present its whole process and we will calculate the total success probability of the scheme.

2 Quantum secret sharing Scheme

Suppose there are four legitimate participants: Alice, Bob, Charlie and David. Among them, Alice is the sender of the message, Bob, Charlie and David are the agents of the message, any agent can cooperate with other agents to reconstruct Alice's message.Now, Alice wants to send an unknown single-particle state $|\psi\rangle_0$ to Bob, Charlie or David,

$$|\psi\rangle_0 = (\alpha|0\rangle + \beta|1\rangle)_0$$

Where α, β are complex numbers, and $|\alpha|^2 + |\beta|^2 = 1$.

Assume Alice, Bob, Charlie and David shared the following four particle asymmetric state

$$|\psi\rangle_{1234} = (a|0011\rangle + b|1101\rangle + c|1010\rangle)_{1234}$$

Where a, b, c are real numbers, $a^2 + b^2 + c^2 = 1$ and $a < b < c$. Particle 1 belongs to Alice, particle 2, 3, 4 belongs to Bob, Charlie and David, respectively.

The total state of the system can be denoted as follows

$$|\zeta\rangle_{01234} = (\alpha|0\rangle + \beta|1\rangle)_0 \otimes (a|0011\rangle + b|1101\rangle + c|1010\rangle)_{1234}$$

In order to teleport state $|\psi\rangle_0$, firstly, Alice performs Bell measurement on particle 0 and 1. Then the state $|\zeta\rangle_{01234}$ will be projected into any one of the following states

$$|\eta_1\rangle = {}_{01}\langle\phi^+|\zeta\rangle_{01234} = \frac{1}{\sqrt{2}}(\alpha a|011\rangle + \beta b|101\rangle + \beta c|010\rangle)_{234}$$

$$|\eta_2\rangle = {}_{01}\langle\phi^-|\zeta\rangle_{01234} = \frac{1}{\sqrt{2}}(\alpha a|011\rangle - \beta b|101\rangle - \beta c|010\rangle)_{234}$$

$$|\eta_3\rangle = {}_{01}\langle\psi^+|\zeta\rangle_{01234} = \frac{1}{\sqrt{2}}(\alpha b|101\rangle + \alpha c|010\rangle + \beta a|011\rangle)_{234}$$

$$\left|\eta_4\right\rangle = {}_{01}\left\langle\psi^- \middle|\zeta\right\rangle_{01234} = \frac{1}{\sqrt{2}}\left(\alpha b|101\rangle + \alpha c|010\rangle - \beta a|011\rangle\right)_{234}$$

Where $\left|\phi^{\pm}\right\rangle_{01} = \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right)_{01}$, $\left|\psi^{\pm}\right\rangle_{01} = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right)_{01}$.

Depending on the difference of the agent who reconstructs the Alice message, we analyze the scheme in the following three cases.

Case 1. Alice appoints David to reconstruct quantum state information.

Based on the above discussion, if Bob and Charlie agree to help David, then they three perform joint unitary transformation on particle 2, 3, 4. If the state of the particle 2, 3, 4 is $|\eta_1\rangle$ or $|\eta_2\rangle$, the joint unitary transformation can be written as

$$\Omega_D = |000\rangle\langle 000| \mp \frac{a}{c}|001\rangle\langle 001| + \sqrt{1-\frac{a^2}{c^2}}|001\rangle\langle 010| + |010\rangle\langle 011| + \sqrt{1-\frac{a^2}{c^2}}|011\rangle\langle 001|$$
$$\pm \frac{a}{c}|011\rangle\langle 010| + |100\rangle\langle 100| \pm |101\rangle\langle 101| + |110\rangle\langle 110| + |111\rangle\langle 111|$$

It's matrix is

$$\Omega_D = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mp\frac{a}{c} & \sqrt{1-\frac{a^2}{c^2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{1-\frac{a^2}{c^2}} & \pm\frac{a}{c} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \pm 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Thus the state of $|\eta_1\rangle$ or $|\eta_2\rangle$ become the following state

$$|\zeta_1\rangle = \frac{1}{\sqrt{2}}\left[a|01\rangle_{23}\left(\alpha|0\rangle + \beta|1\rangle\right)_4 \pm \beta c\sqrt{1-\frac{a^2}{c^2}}|00\rangle_{23}|1\rangle_4 + \beta b|10\rangle_{23}|1\rangle_4\right]$$

Then Bob and Charlie perform the measurement on particle 2, 3 under the bases $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. From the above equation, we can see that only the measurement result of the particle 2, 3 is $|01\rangle$, David can reconstruct the state $|\psi\rangle_0$, and the probability of success is $\frac{a^2}{2}$.

If the state of the particle 2, 3, 4 is $|\eta_3\rangle$ or $|\eta_4\rangle$, the joint unitary transformation can be written as

$$\Omega_{D'} = |000\rangle\langle 000| - \frac{a}{c}|001\rangle\langle 001| + \sqrt{1-\frac{a^2}{c^2}}|001\rangle\langle 010| + \sqrt{1-\frac{a^2}{c^2}}|010\rangle\langle 001| + \frac{a}{c}|010\rangle\langle 010|$$
$$\pm|011\rangle\langle 011| + |100\rangle\langle 100| + |101\rangle\langle 101| + |110\rangle\langle 110| + |111\rangle\langle 111|$$

It's matrix is

$$\Omega_{D'} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{a}{c} & \sqrt{1-\frac{a^2}{c^2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{1-\frac{a^2}{c^2}} & \frac{a}{c} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \pm 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Thus the state of $|\eta_3\rangle$ or $|\eta_4\rangle$ become the following state

$$|\zeta_1'\rangle = \frac{1}{\sqrt{2}}\left[a|01\rangle_{23}(\alpha|0\rangle + \beta|1\rangle)_4 + \alpha c\sqrt{1-\frac{a^2}{c^2}}|00\rangle_{23}|1\rangle_4 + \alpha b|10\rangle_{23}|1\rangle_4\right]$$

Then Bob and Charlie perform the measurement on particle 2, 3 under the bases $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. From the above equation, we can see that only the measurement result of the particle 2, 3 is $|01\rangle$, David can reconstruct the state $|\psi\rangle_0$, and the probability of success is $\frac{a^2}{2}$.

So the total probability of success is $\frac{a^2}{2} \times 4 = 2a^2$.

Case 2. Alice appoints Bob to reconstruct quantum state information.

If Charlie and David agree to help Bob, then they three perform joint unitary transformation on particle 2,3,4. If the state of the particle 2,34 is $|\eta_1\rangle$ or $|\eta_2\rangle$, the joint unitary transformation can be written as

$$\Omega_B = |000\rangle\langle 110| \pm |001\rangle\langle 101| + |010\rangle\langle 100| + |011\rangle\langle 011| + |100\rangle\langle 000| + |101\rangle\langle 111| \mp \frac{a}{c}|110\rangle\langle 001| + \sqrt{1-\frac{a^2}{c^2}}|110\rangle\langle 010| + \sqrt{1-\frac{a^2}{c^2}}|111\rangle\langle 001| \pm \frac{a}{c}|111\rangle\langle 010|$$

It's matrix is

$$\Omega_B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \pm 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \mp\frac{a}{c} & \sqrt{1-\frac{a^2}{c^2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{1-\frac{a^2}{c^2}} & \pm\frac{a}{c} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Thus the state of $|\eta_1\rangle$ or $|\eta_2\rangle$ become the following state

$$|\zeta_2\rangle = \frac{1}{\sqrt{2}}\left[a|11\rangle_{34}\left(\alpha|0\rangle + \beta|1\rangle\right)_2 \pm \beta c\sqrt{1-\frac{a^2}{c^2}}|10\rangle_{34}|1\rangle_2 + \beta b|01\rangle_{34}|0\rangle_2 \right]$$

Then Charlie and David perform the measurement on particle 3, 4 under the bases $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. From the above equation, we can see that only the measurement result of the particle 3, 4 is $|11\rangle$, Bob can reconstruct the state $|\psi\rangle_0$, and the probability of success is $\frac{a^2}{2}$.

If the state of the particle 2, 3, 4 is $|\eta_3\rangle$ or $|\eta_4\rangle$, the joint unitary transformation can be written as

$$\Omega_{B'} = |000\rangle\langle 110| + |001\rangle\langle 101| + |010\rangle\langle 100| + \sqrt{1-\frac{a^2}{c^2}}|011\rangle\langle 001| + \frac{a}{c}|011\rangle\langle 010| - \frac{a}{c}|100\rangle\langle 001| + \sqrt{1-\frac{a^2}{c^2}}|100\rangle\langle 010| + |101\rangle\langle 000| + |110\rangle\langle 111| \pm |111\rangle\langle 011|$$

It's matrix is

$$W_B' = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-\frac{a^2}{c^2}} & \frac{a}{c} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{a}{c} & \sqrt{1-\frac{a^2}{c^2}} & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \pm 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Thus the state of $|\eta_3\rangle$ or $|\eta_4\rangle$ become the following state

$$|\zeta_2'\rangle = \frac{1}{\sqrt{2}}\left[a|11\rangle_{34}(\alpha|0\rangle + \beta|1\rangle)_2 + \alpha c\sqrt{1-\frac{a^2}{c^2}}|00\rangle_{34}|1\rangle_2 + \alpha b|01\rangle_{34}|0\rangle_2\right]$$

Then Charlie and David perform the measurement on particle 3, 4 under the bases $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. From the above equation, we can see that only the measurement result of the particle 3, 4 is $|11\rangle$, Bob can reconstruct the state $|\psi\rangle_0$, and the probability of success is $\frac{a^2}{2}$.

So the total probability of success is $\frac{a^2}{2} \times 4 = 2a^2$.

Case 3. Alice appoints Charlie to reconstruct quantum state information.

If Bob and David agree to help Charlie, then they three perform joint unitary transformation on particle 2,3,4. If the state of the particle 2, 3, 4 is $|\eta_1\rangle$ or $|\eta_2\rangle$, the joint unitary transformation can be written as

$$\begin{aligned}\Omega_C = {} & |000\rangle\langle 100| \pm |001\rangle\langle 101| + |010\rangle\langle 110| + |011\rangle\langle 111| + |100\rangle\langle 000| + |101\rangle\langle 011| \\ & + \sqrt{1-\frac{a^2}{c^2}}|110\rangle\langle 010| \mp \frac{a}{c}|110\rangle\langle 001| + \sqrt{1-\frac{a^2}{c^2}}|111\rangle\langle 001| \pm \frac{a}{c}|111\rangle\langle 010|\end{aligned}$$

It's matrix is

$$\Omega_C=\begin{pmatrix}0&0&0&0&1&0&0&0\\0&0&0&0&0&\pm1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\\1&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&\mp\frac{a}{c}&\sqrt{1-\frac{a^2}{c^2}}&0&0&0&0&0\\0&\sqrt{1-\frac{a^2}{c^2}}&\pm\frac{a}{c}&0&0&0&0&0\end{pmatrix}$$

Thus the state of $|\eta_1\rangle$ or $|\eta_2\rangle$ become the following state

$$|\zeta_3\rangle=\frac{1}{\sqrt{2}}\left[a|11\rangle_{24}\left(\alpha|0\rangle+\beta|1\rangle\right)_3\pm\beta c\sqrt{1-\frac{a^2}{c^2}}|10\rangle_{24}|1\rangle_3+\beta b|01\rangle_{24}|0\rangle_3\right]$$

Then Bob and Charlie perform the measurement on particle 2, 3 under the bases $\{|00\rangle,|01\rangle,|10\rangle,|11\rangle\}$. From the above equation, we can see that only the measurement result of the particle 2, 4 is $|11\rangle$, Charlie can reconstruct the state $|\psi\rangle_0$, and the probability of success is $\frac{a^2}{2}$.

If the state of the particle 2, 3, 4 is $|\eta_3\rangle$ or $|\eta_4\rangle$, the joint unitary transformation can be written as

$$\Omega_{C'}=|000\rangle\langle100|+|001\rangle\langle101|+|010\rangle\langle110|+|011\rangle\langle111|+|100\rangle\langle000|+\sqrt{1-\frac{a^2}{c^2}}|101\rangle\langle001|$$
$$+\frac{a}{c}|101\rangle\langle010|+\sqrt{1-\frac{a^2}{c^2}}|110\rangle\langle010|-\frac{a}{c}|110\rangle\langle001|\pm|111\rangle\langle011|$$

It's matrix is

$$W_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{1-\frac{a^2}{c^2}} & \frac{a}{c} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{a}{c} & \sqrt{1-\frac{a^2}{c^2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \pm 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Thus the state of $|\eta_3\rangle$ or $|\eta_4\rangle$ become the following state

$$|\zeta_3'\rangle = \frac{1}{\sqrt{2}}\left[a|11\rangle_{24}\left(\alpha|0\rangle+\beta|1\rangle\right)_3 + \alpha c\sqrt{1-\frac{a^2}{c^2}}|10\rangle_{24}|1\rangle_3 + \alpha b|01\rangle_{24}|0\rangle_3\right]$$

Then Bob and Charlie perform the measurement on particle 2, 3 under the bases $\{|00\rangle,|01\rangle,|10\rangle,|11\rangle\}$. From the above equation, we can see that only the measurement result of the particle 2, 4 is $|11\rangle$, Charlie can reconstruct the state $|\psi\rangle_0$, and the probability of success is $\frac{a^2}{2}$.

So the total probability of success is $\frac{a^2}{2}\times 4 = 2a^2$.

3 Conclusion

This paper ameliorated the four-particle asymmetric entangled state channel in [12], its coefficients can be any real, satisfying a2 +b2 +c2 = 1. And we analyze the scheme in three situations, one of David, Bob and Charlie is the receiver, and the other is the controller. It is found that the successful probability in each case is 2a2. That is, although the recipients are different, the successful probability is equal. The scheme can be generalized to quantum secret sharing of multi-particle.

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