USING THE METHOD IN SOLVING THE PROBLEMATIC TASKS WITH SIMPLE DIFFERENTIAL EQUATIONS IN TEACHING ON OPTIMAL CONTROL THEMES

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ABSTRACT

The article highlights the methods with examples of the implementation of the problematic objectives of education in teaching on optimal control themes.

Keywords: Subject, object movement subtending, differential equation, the maximum principle, the problematic situation, problematic education.

INTRODUCTION

Researches of physicians, psychologists and educators show that the quality of acquiring information by students and the level of obtaining knowledge and skills are related to the level of students' motivation, depending on their own personal activity. The popularization of the demand for obtaining higher education and the proliferation of personal computers among students are one of the noticeable factors corresponding to the decline of interest in the study of mathematics. Under the circumstances, it is significant to activate independent thinking so that students are capable of acquiring mathematical knowledge, forming skills and abilities, stimulating interest in science and promoting the culture of mathematics. The implementation of case studies in education is one of the most important means to tackle these problems [1].

LITERATURE REVIEW

Conventional but still existing deductive approach is being known as an ***authoritarian pedagogical*** **style** and the Herbart's conception play a primary role in education. There were scholars such as J.J. *Rousseau*, Célestin Freinet opposing public education system created by the works of Komenský, Pestalozzi, etc. to the individual education system groundlessly. However, they also claimed serious objections in their works that following the regulations of the strict teaching structure and universal textbooks limit children's independence and their personal characteristics. On the threshold of the 19th and 20th centuries John Dewey, an American philosopher, proposed the didactic conception of reestablishing student's priority. This concept focuses on theoretical bases of case study teaching. Dewey united the knowledge how to tackle children's daily problems and the workflows. The workflows include below five sequential steps that should be taken by children to create something new: 1) to feel the problem; 2) to determine and to describe it; 3) to propose possible solutions; 4) to draw correct conclusions from the solutions; 5) further observation and experiments whether or not to accept obtained results. At schools a wide number of psychological-pedagogic and methodic researches are dedicated to the ***case study teaching method*** which has also found a good position in school practice. However, in higher education institutions the ***case method*** *is been considered as a new branch for scientific-methodic researches and pedagogical practice* [2].

The student's need for education is important to know the main component of the case method which is the main source of the creating circumstances for a person's mental development. The source of creating this kind of circumstance is "the problematic cases in which students are asked to overcome psychological barriers leading to development of students' knowledge motives and ones which ensures the appearance of needs for more new information to know" [3].

METHODOLOGY

Cases are seemed close to be revealed and as a discomfort border. There is a need for shifting this border and creating the circumstances for cases to be close to be revealed. If the line between known and unknown is drawn firmly, the need for shifting this border will be more active. According to L.D. *Kudryavtsev, the following reasons are the main ones why first year students cannot learn* higher mathematics at an appropriate level and later on apply mathematical methods to solve practical problems: 1) not to be able to distinguish between understandable and non-understandable; 2) to be able to continue conversation: having understood the teacher's questions, not to be able to respond and to form his/her own question; 3) to perceive all information in distorted and even wrong way. Therefore, one of the main issues of the case method is to distinguish what students know and are probably aware. The teacher's task is to instruct students' ability to distinguish information easily and carry out practically and independently [1].

RESULTS

We below show how to achieve the goal of implementing case study teaching method in higher mathematics on the topic "Performance Management". Usually during lectures or in textbooks this topic does not begin with the explanation of problems, that is with the full discussion of any problem. It is first important to mention the concept of management, the field of management, terms of partial continuous management and other necessary explanations. Then, the existence of solution and its uniqueness is determined for management-involved differential equations system, the appropriate solution to the problem of management is determined, the formation of the main problem and the maximum principle of performance management is stated without proof and discussed [p.4,13-29]. After this, 1st, 2nd, 3rd problems are solved by using the results of the not-yet-proven maximum principle. Although the problems were skillfully selected from the easiest to the most difficult ones, the essence of the maximum principle was left without explanation. In some cases, textbooks provide the solution to the above mentioned problems which is given with the help of graphic explanations after the formal proof of the maximum principle. Besides, in university programs, especially technical ones, providing the full proof of the maximum principle is not taken into account. Therefore, in most cases proofs are completely or partly omitted. It is clear that this way is not intended to motivate students to know how to explore a new material because students' need of knowing a new case study is not shaped. In the case method, this topic can be explained in the following way. At the first preparatory stage, students' awareness of the topic is identified. For this purpose, it is useful to explain students how to write differential equations in simple actions to take into consideration optimal control in equations. Often, students can write differential equations in simple actions but have no idea how to take optimal control in equations into account. Then, the question "What happens if to take into account the control function at the right side of governing equation?" is asked. This question can be called as problematic because it gives the opportunity to generalize the concept of "writing action with the differential equations". Thereafter, all responses are discussed, general conclusion is drawn, and it is necessary to have a glance at the background of these problems. The background of optimal control problems will be briefly explained.

A great number of problems in mathematical physics can be described in the form of the maxima of a *function. In this description, there is an opportunity to find the solution to* boundary problems suitable to it and along with this various methods can be applied to solve them. Therefore, a number of problems are usually provided to find some extrema of a *function.*

Conventional extrema problems are given in a simple form or isoperimetric problems of variational calculus.

Besides, in variational calculus the problems asking to minimalize a functional exist along with ***inequalities***, differential, integral equations. Although these types of problems have been known since the past, only in the 20th century the number of researches increased due to the demand for the realization of practical problems and totally new types of problems arouse for scientists who had introduced clarity into the problems.

These problems were named as “optimal control”. Any optimal control problem is characterized by three following elements:

1) The differential equation determines the condition of object and set requirements. Equation and the character of requirements are identified via accepted mathematical model. There is a possibility to distinguish between the groups of variable parameters (coordinates) connected to the given equation, identifier of the equation variables and external change in the group of control functions related to the collection of parts of functional space.
2) The collection belonging the control value – given control’s fields
3) Given functionals related to state coordinates and control.

Optimal control problems consist of determining the control function for a dynamical system to minimize a performance index.

DISCUSSION

Now we can move on the problems. Here are given analysis to problems that can be solved without the use of maximum principle belonging to optimal control. In other words, we propose to begin the discussion of problems relating to the topic of optimal control.

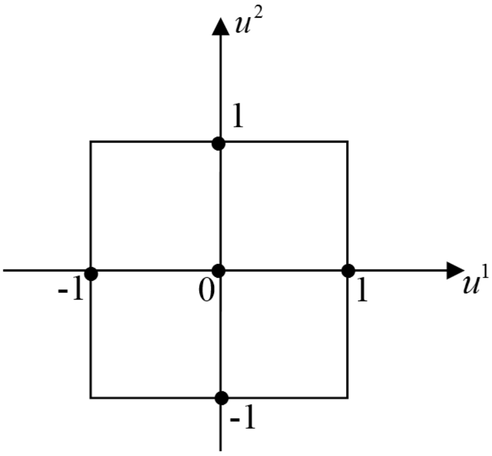
1-Problem: The motion of object is given by the equation $\frac{dx^1}{dt}=u^1, \frac{dx^2}{dt}=u^2$, in which $u^1,\ u^2$ are the parameters of control, $|u^1|\le 1,\ |u^2|\le 1$ are unchangeable functions encouraging the condition. Here is the object’s initial condition $x^1(0)=x_0^1,\ x^2(0)=x_0^2$ ва $x_0^1=x_0^2>0$. You should find such control $u^1=u^1(t),\ u^2=u^2(t)$, so that problem situation should be done $t=T$ for the shortest period of time $x^1(T)=x^2(T)=0$, that is object should be at the beginning for shortest time.

Solution: if $u^1 = u^1(t),\ u^2 = u^2(t)$ are the known functions, having put them in the equation of object motion and having integrated in conditions of initially stated differential equations, we use the following equations for trajectories of object transference:

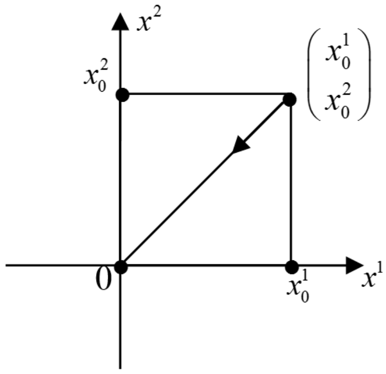
$$\frac{dx_1}{dt} = u^1(t),\ \frac{dx_2}{dt} = u^2(t),\ x^1(0) = x_0^1,\ x^2(0) = x_0^2;$$

$$x^1(t) = x_0^1 + \int_0^t u^1(\tau)d\tau, \qquad x^2(t) = x_0^2 + \int_0^t u^2(\tau)d\tau.$$

Now control integration and initial conditions are depicted in the plane of the coordinate axes (Pictures 1 and 2).



Picture 1



Picture 2

From the 2nd picture, it is noticeable that in order to bring from object's $x_0^1, x_0^2,\ x_0^1 = x_0^2 > 0$ initial condition to the beginning of the coordinate O, it is necessary to direct the control towards it. It is, certainly, probability. Here is the written form of the control: $u^1 = u^1(t) = -1,\ u^2 = u^2(t) = -1,$ in this case the appropriate solution to equation of motion will be following:

$$x^1(t) = x_0^1 + \int_0^t u^1(\tau)d\tau = x_0^1 - \int_0^t 1d\tau = x_0^1 - t;$$

$$x^2(t) = x_0^2 + \int_0^t u^2(\tau)d\tau = x_0^2 - \int_0^t 1d\tau = x_0^2 - t.$$

According to the problem situation, because of $x_0^1 = x_0^2 > 0$, $x^1(t)$ and $x^2(t)$ equals to zero at the given time $t = T = x_0^1 = x_0^2 > 0$. That is,

$$x^1(T) = x_0^1 - T = x_0^1 - x_0^1 = 0,$$

$$x^2(T) = x_0^2 - T = x_0^2 - x_0^2 = 0.$$

It is clear that object goes to the beginning at the time of $T = x_0^1 = x_0^2 > 0$. Now we prove that this time is the shortest one. For this, we will imagine in an adverse way, such controls $u^1 = \bar{u}^1(t),\ u^2 = \bar{u}^2(t)$ and $t = t' < T$ exist; they are put in the equation of the object motion. When the obtained differential equations are solved, there will be $x^1(t') = x^2(t') = 0$. After the control of $u^1 = \bar{u}^1(t)$ is put in the equation of the object motion, the following is acquired:

$$\left|x^1(t')\right| = \left|x_0^1 + \int_0^{t'} u^1(\tau)d\tau\right| = \left|x_0^1 - \int_0^{t'} (-u^1(\tau))d\tau\right| \geq \left|x_0^1\right| - \left|\int_0^{t'} (-u^1(\tau))d\tau\right| \geq x_0^1 - t' > 0.$$

Because according to the imagination, $t = t' < T = x_0^1$. But this is contrary to the $x^1(t') = 0$, so $T = x_0^1$ is the shortest time.

Problem: The motion of object is given by the equation $\frac{dx^1}{dt} = u^1, \frac{dx^2}{dt} = u^2$, in which $u^1,\ u^2$ the parameters of control, $\left|u^1\right| \leq 1,\ \left|u^2\right| \leq 1$ are unchangeable functions encouraging the condition. Here is the object's initial condition $x^1(0) = x_0^1,\ x^2(0) = x_0^2$ and $x_0^1 \geq x_0^2 > 0$. You should find such control $u^1 = u^1(t),\ u^2 = u^2(t)$, so that problem situation should be done $t = T$ for the shortest period of time $x^1(T) = x^2(T) = 0$, that is object should go to the beginning for the shortest time.

Solution: We choose controls in the following form $u^1 = u^1(t) = -1,\ \ u^2 = u^2(t) = -\frac{x_0^2}{x_0^1}$, in this case the solution to the equation of motion will be below:

$$x^1(t) = x_0^1 + \int_0^t u^1(\tau)d\tau = x_0^1 - \int_0^t 1d\tau = x_0^1 - t;$$

$$x^2(t) = x_0^2 + \int_0^t u^2(\tau)d\tau = x_0^2 - \int_0^t \frac{x_0^2}{x_0^1}d\tau = x_0^2 - t\cdot\frac{x_0^2}{x_0^1}.$$

According to problem situation, both these forms equal to zero at the given time $t = T = x_0^1 > 0$. That is,

$$x^1(T) = x_0^1 - T = x_0^1 - x_0^1 = 0,$$

$$x^2(T) = x_0^2 - T\cdot\frac{x_0^2}{x_0^1} = x_0^2 - x_0^1\cdot\frac{x_0^2}{x_0^1} = x_0^2 - x_0^2 = 0.$$

It is clear that object goes to the beginning at the time of $T = x_0^1$. Now we prove that this time is the shortest one. For this, we will imagine in an adverse way, such controls $u^1 = \bar{u}^1(t),\ u^2 = \bar{u}^2(t)$ and $t = t' < T$ exist; they are put in the equation of the object motion. When the obtained differential equations are solved, there will be $x^1(t') = x^2(t') = 0$. After the control of $u^1 = \bar{u}^1(t)$ is put in the equation of the object motion, the following is acquired:

$$\left|x^1(t')\right| = \left|x_0^1 + \int_0^{t'} u^1(\tau)d\tau\right| = \left|x_0^1 - \int_0^{t'} (-u^1(\tau))d\tau\right| \geq \left|x_0^1\right| - \left|\int_0^{t'} (-u^1(\tau))d\tau\right| \geq x_0^1 - t' > 0.$$

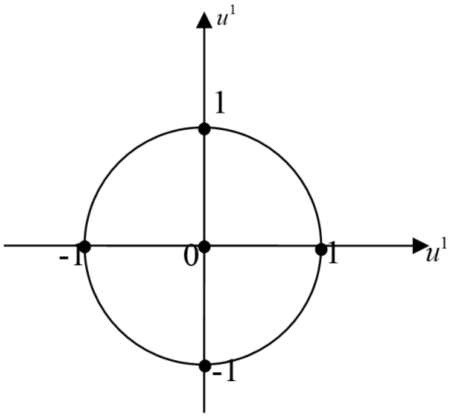
Because, according to the imagined, $t = t' < T = x_0^1$. But this is contrary to the condition of $x^1(t') = 0$, so, $T = x_0^1$ is the shortest time. Thus, the problem has been completely solved.

Problem: The motion of object is given by the equation $\frac{dx}{dt} = u$, $x = \begin{pmatrix} x^1 \\ x^2 \end{pmatrix}$, $u = \begin{pmatrix} u^1 \\ u^2 \end{pmatrix}$, in which u is the parameter of control, $|u| = \sqrt{(u^1)^2 + (u^2)^2} \leq 1$ are unchangeable functions encouraging the condition. Here is the object's initial condition $0 \neq x_0 = \begin{pmatrix} x_0^1 \\ x_0^2 \end{pmatrix}$. You should find such control $u = u(t)$, so that condition should be done $t = T$ for the shortest period of time $x(T) = 0$, that is object should go to the beginning for shortest time.

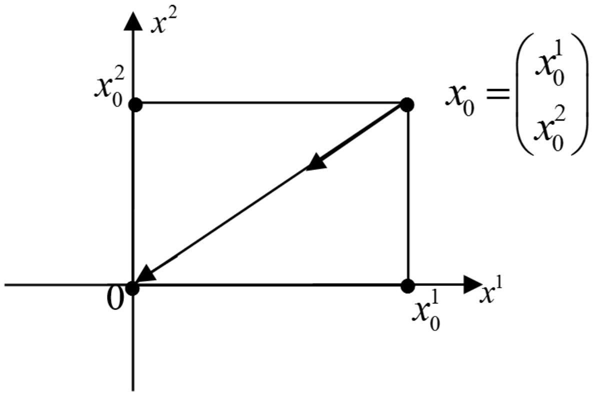
Solution: If $u = u(t)$ are the known functions, having put them in the equation of object motion and having integrated them, we obtain the following:

$$x(t) = x_0 + \int_0^t u(\tau)d\tau.$$

Now control integration and initial conditions are depicted in the plane of the coordinate axes (Pictures 3 and 4).



Picture 3



Picture 4

From the 4th picture, it is noticeable that in order to bring $0 \neq x_0 = \begin{pmatrix} x_0^1 \\ x_0^2 \end{pmatrix}$ from initial condition to the beginning of the coordinate O, it is necessary to direct the control towards it. It is, certainly, probability. Here is the written form of the control: $u = u(t) = -\frac{x_0}{|x_0|}$, in this case the appropriate solution to equation of motion will be following:

$$x(t) = x_0 + \int_0^t u(\tau)d\tau = x_0 + \int_0^t (-\frac{x_0}{|x_0|})d\tau = x_0 - \int_0^t \frac{x_0}{|x_0|}d\tau = x_0 - \frac{x_0}{|x_0|} \cdot t.$$

In the last form we find t which turns into zero:

$$x(t)=x_0-\frac{x_0}{|x_0|}\cdot t=0,\ \frac{x_0}{|x_0|}\cdot(|x_0|-t)=0,\ |x_0|-t=0,\ t=|x_0|=T.$$

So, object goes to the beginning at the time of $t=|x_0|=T$. Now we prove that this time T is the shortest one. For this, we will imagine in an adverse way, such controls $u=\bar{u}(t)$ and $t=t'<T$ exist; they are put in the equation of the object motion. When the obtained differential equations are solved, there will be $x(t')=x(t')=0$. After the control of $u=\bar{u}(t)$ is put in the equation of the object motion, the following is acquired:

$$|x(t')|=\left|x_0+\int_0^{t'}u(\tau)d\tau\right|\geq|x_0|-\left|\int_0^{t'}(-\frac{x_0}{|x_0|})d\tau\right|=|x_0|-\int_0^{t'}1d\tau=|x_0|-t'>0.$$

Because, according to the imagination, $t=t'<T=|x_0|$. But this is contrary to the $x(t')=0$, so, $T=|x_0|$ is the shortest time.

Because, According to the hypothesis $t=t'<T=|x_0|$. It is $x(t')=0$ against the conditions, so that, $T=|x_0|$ would soonest time. Thus, the problem has been completely solved.

CONCLUSION

The problem of optimal control which is above mentioned is the simplest form of control problems. These problems are usually named as “basic” control problems. As it can be seen, they are solved without difficulty, but skills to solve more complicated problems can be initiated and developed with them. You can be introduced with more other problems in the works of Rufus Isaacs. We solved these problems without the help of maximum principles. Even these problems appear simple; you may notice the difficulty-increasing level of proposed problems.

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