ISSN 2056-6018

THE INDIRECT EFFECT IN MULTIPLE MEDIATORS MODEL BY STRUCTURAL EQUATION MODELING

Li-Ju Chen
Department of Business Administration, Far East University, **TAIWAN**

Hsiang-Chin, Hung
I-Shou University, Department of Industrial Engineering and Management, **TAIWAN**

ABSTRACT

As mediator variable can be used to understand the impact of independent variable on dependent variable, and break down interesting causal relationships to determine the possible mechanism causing such relationships, it has become an issue of concern to the researchers. The multiple mediator variable modeling is widely used in social science research. This study proposed an analysis method under the Structural Equation Modeling (SEM) framework to use the function of Bayesian estimation in AMOS to analyze the indirect effect and total effect in the multiple-mediator model. Compared with traditional tools to test the effect of multiple mediators, the proposed methodology had advantages such as high flexibility, high efficiency and ease of use.

Keywords: Indirect Effects, Multiple Mediators, Structural Equation Modeling, Bayesian, AMOS.

INTRODUCTION

There have been many discussions on the effect of mediator variables between independent variable and dependent variable (Baron & Kenny, 1986; Holmbeck, 1997; MacKinnon et al., 2002; Shrout & Bolger, 2002; Edward & Lambert, 2007). These studies attempted to explore whether the relationship between the independent variable and the dependent variable would be different when a third variable is added, and the added third variable is generally known as the interventor. If the interventor is expected to be affected by the independent variable, and affects the dependent variable as well, then such type of interventor would be known as the mediator variable. The effect of independent variable on dependent variable is triggered by the mediator variable. Such effect was termed as the indirect effect (Shrout & Bolger, 2002).

Testing mediating effect can help to understand the impact of the independent variable on the dependent variable, and break down the interesting causal relationships to identify the possible mechanism causing such relationships. Researchers are most interested in the mediation analysis of many causal relationship models, which are very helpful in the theoretical development and testing of possible intervention problems in fields of psychology, sociology and management. Although the multiple- mediator model has strong and large demand in social science fields, it has attracted relatively little attention (Preacher & Hayes, 2008). This study proposed an analysis method under the SEM (Structural equation modeling) framework to use AMOS Bayesian estimation to analyze the indirect effect and total effect in the multiple-mediator model.

LITERATURE REVIEW

Simple Mediation Model

The main purpose of mediation analysis is to check whether the causal effect of the independent variable X on the dependent variable Y is caused by the mediator. Hence, after the addition of the mediator, the part or all relationships between the independent variable and dependent variable should be explained. If the proportion of the indirect effect against the total effect is greater, it indicates a higher mediating effect. The three regression equations of the mediation model proposed by Barron & Kenny (1986) are as follows:

$$Y = c_0 + c' X + e_1 \quad (1)$$

$$M = a_0 + aX + e_2 \quad (2)$$

$$Y = b_0 + cX + bM + e_3 \quad (3)$$

Regression Equation (2) is inputted into Regression Equation (3) to obtain Regression Equation (4)

$$Y = (b_0 + a_0 b) + (c + ab)X + (be_2 + e_3) \quad (4)$$

Compare the coefficients X of Regression Equation (1) and Regression Equation (4) to get

$$c' = c + ab \quad (5)$$

Namely, $c' - c = ab$ (6)

This is the basic equation of the mediation model. Regression Equation (1) can be represented by Figure 1.

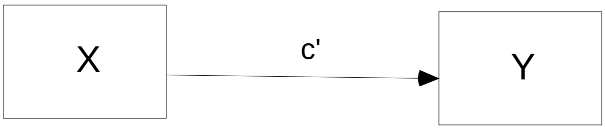

Figure1 X affects Y

Figure 1 shows the impact path of independent variable (X) on the dependent variable (Y), path coefficient c' is also known as the total effect of the independent variable (X) on the dependent variable (Y). Generally, the total effect c' is expected to be significantly different from zero. This study explored whether the impact of the independent variable (X) on the dependent variable (Y) is from another factor, which is termed as the mediator variable represented by M. Hence, mediation analysis at least has three variables including the independent variable (X), dependent variable (Y) and the mediator variable (M). The relationships are often represented by the following path graphs.

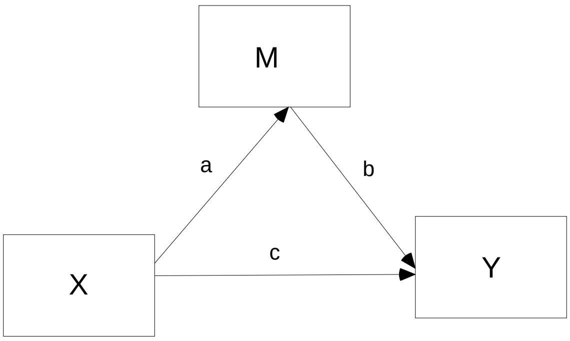

Figure2 Basic mediator model

Figure 2 shows the typical mediation model; path coefficient c is termed as the direct effect of the independent variable (X) on the dependent variable (Y), also known as the effect of the control mediator variable (M) of independent variable (X) on dependent variable (Y), or the residual effect. Path coefficient a is the effect of independent variable (X) on mediator variable (M), also known as the first stage effect. Path coefficient b is the effect of the mediator variable (M) on the dependent variable (Y), also known as the second stage effect.

The multiplication of the first stage effect and second stage effect ab is known as the indirect effect. If the direct effect of independent variable (X) on the dependent variable (Y) after the addition of the mediator variable (M) is insignificant (namely, path coefficient c is significantly), it is known as the full mediation.

Multiple-mediator Model

If there is multiple-mediator variable intervention in the relationships in between the independent variable and dependent variable, it is called the multiple-mediator model, which is helpful to the discussions on relationships in between variables. For example, Aiken et al. (1994) proposed four mediator variable models to discuss the effectiveness of educational courses in increasing breast X-ray screening breast cancer. Raver and Gelfand (2005) proposed three mediator variable models to research the mediating effect of team conflict, team cohesion and team citizenship on team performance by environmental harassment.

Bollen (1987) first proposed the multiple-mediator model. Research established the multiple-mediator model by using SEM, focusing on the definition of total effect and indirect effect, and discussing the calculation of the mediating effect. Brown (1997) similarly focused on the estimation of the mediating effect by SEM, and categorized the effects contained in the multiple mediator variable models into the direct effect, total effect, total indirect effect and individual indirect effect, and proposed the methods to calculate those effects.

The effects of multiple mediator variables can be tested individually and simultaneously. The advantage of simultaneous testing is the ability to learn whether the effect of a mediator and other mediator is independent. However, the differences in concepts of different mediator variables should be determined, while there is no high correlation in between mediator variable.

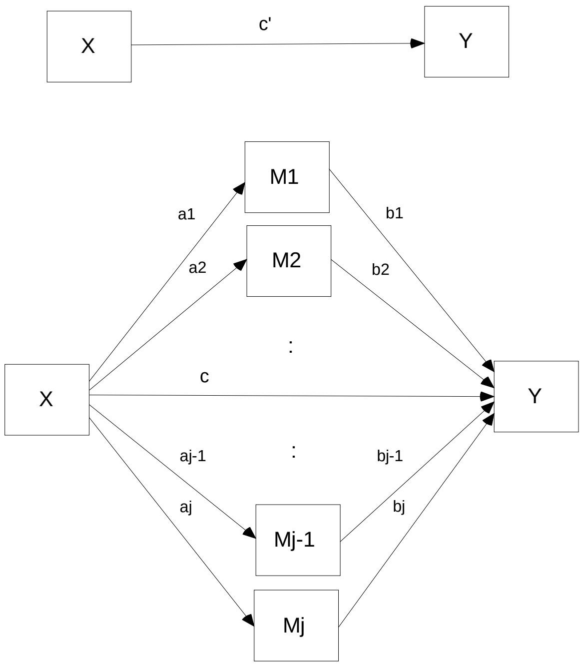

Figure 3 multiple-mediator path diagram

The coefficient c' as illustrated in Figure 3 represents the total effect of X on Y, and the c indicates the direct effect of X on Y, while a_i, b_i (i=1 to j) is the indirect effect of X on Y after j mediator variables. Individual indirect effect is defined as the multiplication of path coefficient ab (Brown, 1997; Fox, 1985). For example, the indirect effect of X on Y by M1 is a_1b_1; the indirect effect of X on Y by M2 is a_2b_2; the indirect effect of X on Y by M3 is a_3b_3. Similarly; the total indirect effect of X on Y by j mediator variables is $(a_1b_1 + a_2b_2 + ... + a_jb_j)$

METHODOLOGY

According to the summary of Mackinnon et al. (2002), the mediation effect testing methodologies can be categorized into four types including the causal-steps test, product-of-coefficients test, difference-in-coefficients test and the resampling method.

Causal-steps tests

The causal-steps test was first proposed by Judd & Kenny (1981) and Baron & Kenny(1986) and can be applied in the single mediator model, as well as in the multiple mediator model by extension. The method requires that the total effect of X on Y should be significant. Mackinnon et al. (2002) also proposed another type of the causal-steps test, which does not require significant total effect of X on Y.

Product-of-coefficients tests

In the single mediator model, the testing of the mediating effect is based on the path coefficient from independent variable to mediator variable in the path model set as a, and the path coefficient from mediator variable to dependent variable set as b. The multiplication of the two coefficients is termed as the indirect effect. Hence, testing the indirect effect is actually the testing of $H_0 : ab = 0$(Alwin & Hauser, 1975; Bollen, 1987; Fox, 1980; Sobel, 1982). Although Sobel testing is the most commonly used mediating effect testing method, it is not the optimal selection if considering the Type I error and testing capability. Sobel assumes that a, b are normally distributed, which is in fact groundless. In particular, when the mediating effect is small (effectiveness=0.14) or medium (effectiveness =0.39), and the number of samples is fewer than 50, the testing power is within 0.4 (Mackinnon et al., 2002). When testing the signification of ab, the standard error of ab should be first identified. Sobel (1982) used the multi-variate delta method to calculate the approximation value of the standard error of $\hat{a}\hat{b}$:

$$z = \frac{\hat{a} \times \hat{b}}{\sqrt{\hat{a}^2 (se(\hat{b}))^2 + \hat{b}^2 (se(\hat{a}))^2}} \qquad (7)$$

Where, $se(\hat{a})$ and $se(\hat{b})$ are respectively the standard errors of $\hat{a}$ and $\hat{b}$. When absolution value of z is bigger than two, then the indirect effect is significant. Namely, the mediator variable has the mediating effect.

Difference-in-coefficients Test

In case of the single mediator model, the difference ($c' - c$) between the coefficient c' and coefficient c can be used in testing the mediating effect. Many studies have proposed the modified the standard error of $c' - c$ (Mackinnon et al., 2002). For example, Freedman & Schatzkin (1992) proposed the following equation:

$$se(c' - c) = \sqrt{se^2(c') + se^2(c) - 2se(c')se(c)\sqrt{1 - \rho^2{}_{XM}}} \qquad (8)$$

Testing of the value of t

$$t = \frac{c'-c}{se(c'-c)} \qquad (9)$$

RESAMPLING

In case of the single mediator model, the product-of-coefficients testing method is used to test the mediating effect ab,, which may result in testing deviation as ab is not normally distributed (Mackinnon et al. 2002). This study proposed a resampling method -Bayes estimation. Amos Bayes estimation is a resampling method using the Markov Chain Monte Carlo Method (MCMC method).

The Bayesian estimation integrated the prior distribution and likelihood of data parameters for estimation of the measurement and structural model before data collection to obtain the posterior distribution of the estimation parameters for statistical inference. Conceptually, posterior distribution (p(θ|y)) is equivalent to the maximum likelihood value of the multiplication of prior distribution of θ and the observation value of y: posterior=prior*likelihood. The feature of this method is that the researchers can definitely use the prior knowledge (previous study or relevant theories) of the model parameters to obtain the posterior distribution of the estimated values of parameters. This method can thus improve parameter estimated values for better applicability in smaller samples and avoiding unreasonable model parameters (or negative variance), or setting parameter function estimation and testing by themselves. The MCMC method can be regarded as an extension of the Bayesian Inference. The feature of the MCMC method is to regard the parameters for estimation and status parameters that cannot be observed as the random variables, and assume they have their own probability distribution. By using the continuous iteration sampling of the Monte Carlo method, this study can simulate the parameter and status variable sampling distribution. With characteristics of large sample, it can converge to the target actual distribution. With the progress of computer calculation and software development, the MCMC method is increasingly popular in use. In recent years, the MCMC method has been widely applied in the simulation study of complex models. It is expected to apply the advantages of the Bayes estimation method in the mediation analysis. The following examples illustrate how the structural equation model software Amos implemented the Bayes estimation.

COMPARATIVE CASE STUDY

This study discussed how to apply the resampling method in the testing of the mediating effect by using the data provided in the article of Preacher to compare the following three methods to confirm that Bayesian estimation can be used in the analysis of the mediating effect.

1. Preacher & Hayes (2008) used the sampling algorithm of the macro grammar of the SPSS software (bootstrap) to obtain the estimated values and confidence intervals of the indirect effect.
2. Edwards & Lambert (2007) used the CNLR (Constrained nonlinear regression) bootstrap of the SPSS software by using the resampling method to determine the multiple path coefficients of the Regression Equations, and input all the path coefficients into one EXCEL report to determine the estimated values and confidence intervals of the indirect effect.

3. This study used the SEM software Amos to implement the Bayes estimation. The custom estimands in the Bayes estimation can implement four arithmetic operations of various path coefficients, and further obtain the estimated values and confidence intervals of various indirect effects or effectiveness gaps.

Preacher & Hayes (2008) discussed the testing of the mediating effect of the multiple-mediator variables using the impact of independent variable of helpfulness on the dependent variable of job satisfaction proposed by Klein et al. (2006). The impact will work through mediator variables including politics, people and performance, and there were a total of 141 samples. The three-mediator path diagram by Preacher and Hayes is shown below.

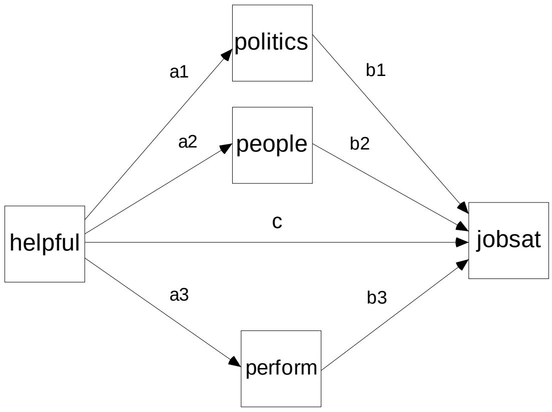

Figure 4 Three-mediator variable model

The three-mediator variable model can obtain the single mediator variable's indirect effect (e.g., a_1b_1, a_2b_2, a_3b_3), and compare the indirect effects internally ($a_1b_1 - a_2b_2$), a1b1-a3b3, a2b2-a3b3). The estimated values and confidence intervals of indirect effect and effectiveness gap obtained by using same data through the SPSS, CNLR and Bayesian estimation methods are shown in Table 1. AMOS custom estimation grammars are shown in Appendix 1. Comparison of the estimated values of the three methods found that the differences are insignificant. The indirect effect after M1 was a1b1, and its estimated value was 0.022. The indirect effect after M2 was a2b2, and the estimated value was 0.080. The indirect effect after M3 was a3b3, and its estimated value was 0.006. The total indirect effect was a1b1+a2b2+a3b3, and its estimated value was 0.107. Regarding the pair comparison of indirect effect, for example, a1b1-a2b2, its estimated value was -0.057.

Table 1 Estimate and C.I. of multiple mediator model

Effect	SPSS Macro			CNLR			AMOS Bayesian		
	Estimate	95%Lower	95%Upper	Estimate	95%Lower	95%Upper	Estimate	95%Lower	95%Upper
M1	0.0222	-0.0173	0.0677	0.0222	-0.0158	0.0698	0.0221	-0.0191	0.0693
M2	0.0796	0.0312	0.1394	0.0796	0.0341	0.1359	0.0796	0.0304	0.1417
M3	0.0057	-0.0201	0.0348	0.0056	-0.0206	0.0343	0.0058	-0.0225	0.0373
Total	0.1074	0.0589	0.1650	0.1074	0.1103	0.3115	0.1074	0.0481	0.1781
M1-M2	-0.057	-0.1420	0.0177	-0.057	-0.1372	0.0195	-0.057	-0.1353	0.0128

	4			4			4		
M1-M3	0.0165	-0.0328	0.0684	0.0165	-0.0318	0.0699	0.0163	-0.0412	0.0775
M2-M3	0.0739	0.0104	0.1498	0.0739	0.0112	0.1451	0.0738	0.0124	0.1470

Note: SPSS bootstrap sample=5000. CNLR bootstrap sample=1000.

CONCLUSIONS

Many scholars (Edwards & Lambert, 2007; Preacher & Hayes, 2008; Williams & MacKinnon, 2008) have proposed the estimation and testing of indirect effect. This study used the SEM Amos software Bayes estimation resampling method to calculate the estimated values and confidence interval of the indirect effect. Bayes estimation did not need to assume whether the path coefficient distribution was normal and its grammar was easy to learn, and it can be easily extended to more complex models. In social science study, latent variables commonly recognized should be used for communications and explanations. Latent variables cannot be observed directly and should be measured by a group of concepts. SEM can discuss the causal relationships not only in between latent variables but also manifest Variable). Meanwhile, the testing of the variable causal model assumptions can meet the needs of researchers.

Appendix 1 Amos syntax of multiple mediator model

```
Public Class CEstimand
        Implements IEstimand

        Public Sub DeclareEstimands() Implements IEstimand.DeclareEstimands
                newestimand("xm1y")
                newestimand("xm2y")
                newestimand("xm3y")
                newestimand("total indirect")
                newestimand("m1-m2")
                newestimand("m2-m3")
                newestimand("m1-m3")
        End Sub

        Public Function CalculateEstimands(ByVal sem As AmosEngine) As String
Implements IEstimand.CalculateEstimands
        estimand("xm1y").value=sem.ParameterValue("a1")*sem.ParameterValue("b1")
        estimand("xm2y").value=sem.ParameterValue("a2")*sem.ParameterValue("b2")
        estimand("xm3y").value=sem.ParameterValue("a3")*sem.ParameterValue("b3")
        estimand("total
indirect").value=estimand("xm1y").value+estimand("xm2y").value+estimand("xm3y").value
   estimand("m1-m2").value=estimand("xm1y").value-estimand("xm2y").value
   estimand("m2-m3").value=estimand("xm2y").value-estimand("xm3y").value
        estimand("m1-m3").value=estimand("xm1y").value-estimand("xm3y").value

                Return ""               'Return an empty string if no error occurred
        End Function

End Class
```

REFERENCES

Alwin D. F., Hauser R. M. (1975), *The decomposition of effects in path analysis*. American Sociological Review. 40, 37–47.

Baron R. M., Kenny D. A. (1986), *The moderator-mediator distinction in social psychological research: Conceptual, strategic, and statistical considerations*. Journal of Personality and Social Psychology. 51, 1173–1182.

Bollen K. A. (1987), *Total, direct and indirect effects in structural equation models*. Sociological Methodology. 17, 37-69.

Brown R. L. (1997), *Assessing specific mediational effects in complex theoretical models*. Structural Equation Modeling. 4, 142-156.

Edwards J. R., Lambert L.S. (2007), *Methods for integrating moderation and mediation: a general analytical framework using moderated path analysis*. Psychological Methods. 12, 1-22.

Holmbeck G. N. (1997), *Toward terminological, conceptual, and statistical clarity in the study of mediators and moderators: Examples from the child-clinical and pediatric psychology literatures*. Journal of Consulting and Clinical Psychology. 65, 599-610.

Judd C. M., Kenny D. A. (1981), *Process Analysis: Estimating mediation in treatment evaluations*. Evaluation Review. 5(5), 602–619.

MacKinnon D. P., Krull J. L., Lockwood C. M. (2000), *Equivalence of the mediation, confounding and suppression effect*. Prevention Science, 1(4), 173-181.

MacKinnon D. P., Lockwood C. M., Hoffman J. M., West S. G., Sheets V. (2002), *A comparison of methods to test mediation and other intervening variable effects*. Psychological Methods. 7(1), 83-104.

Preacher K. J., Hayes A. F. (2008), *Asymptotic and resampling strategies for Assessing and comparing indirect effects in multiple mediator models*. Behavior Research Models. 40(3), 879-891.

Shrout P. E., Bolger N. (2002), *Mediation in experimental and non-experimental studies: New procedures and recommendations*. Psychological methods. 7(4), 422-445.

Sobel, M. E. (1982), *Asymptotic confidence intervals for indirect effects in structural equation models*. In S. Leinhardt (Ed.), Sociological methodology. Washington, DC: American Sociological Association, 290–312.

Taylor A. B., MacKinnon D. P., Tein J. Y. (2008), *Tests of the three-path mediated effect*. Organizational Research Methods. 11(2), 241-269.