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MATHEMATICAL MODEL AND CONTROL OF SHIP MOTION BASED ON THE FUZZY LOGIC CONTROL

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ABSTRACT

In this paper, the author presents the mathematical model and control of ship motion based on the fuzzy logic control. The goal of the research is to consider the mathematical model and the adaptive synthetic methods basing on the fuzzy logic of the control system of the ship that is negatively affected by the external disturbances to its operation. Consequently, it's considered the problem of design and modelling of the self-tuning auto pilot regulator of the ship (Auto pilot regulator). The paper devotes the experiment on the Offshore service Vessel in Vietnam with the method as: the mathematical models of ship motion, the mathematical model of the auto pilot regulator, synthetic of the ship control algorithms based on the approached fuzzy logic method.

Keywords: Mathematical Models of Ship Motion, Auto Pilot Regulator, Approached Fuzzy Logic Method.

INTRODUCTION

Presently, there are greatly interested in science researches and experimental designs that improve the effect of ship control, these are specially advanced in confidence and cost of equipment. In research of design of the structures and ship control system, it is necessary to calculate fully its operation conditions, as well as to look for the more new effectives devices of ship control. It's necessarily improved the computer system of ship control basing on the application of fuzzy logic and neuron network. This is leading to the most economical effect that is not required the changing of basic function of the ship. Significantly, the ship is currently operated in the competition market that is required to optimize the running cost, improve its reliability, achieve the operative effect and protect the environment.

It's considered the mathematical model and the adaptive synthetic methods basing on the fuzzy logic of the control system of the ship that is negatively affected by the external disturbances to its operation. Consequently, it's considered the problem of design and modelling of the self-tuning auto pilot regulator of the ship (Auto pilot regulator). Normally, the ship is equipped with the auto pilot regulator. The tuning unit is equipped in the auto pilot regulator, but it is impossible to assure the necessary accuracy of the sailing trajectory when the ship is affected by the external disturbances. Up to now, there is no recommendation for the design of the auto pilot regulator that is possibility to optimize the ship control system as the degree of the necessary information accumulation for that purpose.

LITERATURE REVIEW

In this subject, there are researches of authors such as: Zadeh LA (1974); Bolnokin VE, Uy DV and Manh DX (2005); Loc HD (2002), Chai TY and Zhang Tao (1981); Chan PT, Xie WF and Rad AB. (2000); Chang HC, Wang MH (1995); Chang SS, Zadeh LA (1972); Chen CL, Chen WC (1994); Chen FC, Liu CC (1996), Cheng SJ, Chow IS, Malik OP and Hope GS (1986). In this paper, the author presents his experiment on the Offshore service Vessel in

Vietnam with the method as: the mathematical models of ship motion, the mathematical model of the auto pilot regulator, synthetic of the ship control algorithms based on the approached fuzzy logic method.

METHODOLOGY

Mathematical Models of Ship Motion

Firstly, it will be considered the factors in relation to the development of the mathematical model of ship motion that is referred to the effects of these synthetic factors, such as [1]:

- The hydro dynamic characteristics, dimension and the dead weight of ship;
- The disturbance of sea waves;
- The disturbance of the stream, current;
- The disturbance of wind.

Playing the modelling role, M.V HAI DUONG 101 (IMO number 9697727) registered under Vietnamese flag, owned by HAI DUONG COMPANY LIMITED (HADUCO), is chosen as the model ship. This is anchor handling, tug and supply ship with 65 m length, Gross tonnage 2281, equipped with DPS-2, built by Focal Ship Building, China in 2014.
When developing of the mathematical model, it is necessarily used the basic factors of this ship in the condition of external disturbance effects, such as:

- *m*: weight of ship (kgs^2/m);
- $\dot{u}, \dot{v}$: acceleration of ship's speed by the directions *x, y* (m/s^2);
- $\dot{r}$: angle acceleration of ship (l/s^2);
- I_{ZZ}: inertia moment by z co-ordinate axe (kgs^2/m);
- $X_H, Y_H, N_H, X_R, Y_R, N_R$: the hydro-dynamic forces and moments effect on the ship's hull (kg, kgm);
- X_P: the thrust force effect on the ship's propeller (kg);

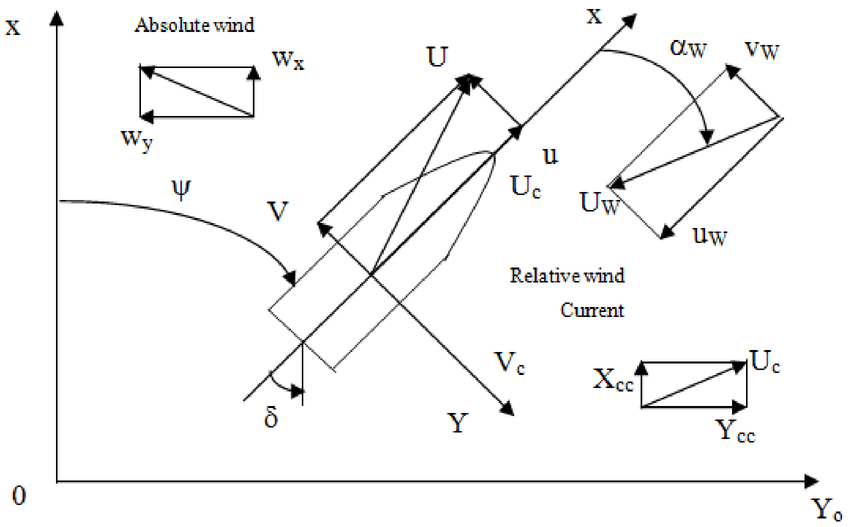

Figure 1: Diagram of ship motion

In the earth co-ordinate, there are:

$$\dot{\psi} = r;\ \dot{X} = u\cos\psi - v\sin\psi;\ \dot{Y} = u\sin\psi + v\cos\psi \qquad (1)$$

And the relation of the rudder angle and rolling angle of the ship:

$$\left.\begin{aligned} \dot{\delta} &= \frac{(\delta' - \delta)}{(|\delta' - \delta| T_{RUD} + a)} \\ \dot{\theta} &= \frac{(\theta_P' - \theta_P)}{(|\theta_P' - \theta_P| T_{FP} + a)} \end{aligned}\right\} \quad (2)$$

Where:

- δ' : the given rudder angle (rad);
- θ_P' : the given rolling angle (rad);
- T_{RUD} : the time constant of ship's rudder (s);
- T_{FP} : The time constant of ship's rolling (s).

The hydro-dynamic forces and moments effect to the ship's hull:

$$\left.\begin{aligned} X_H &= -m_x \dot{u} - \frac{1}{2}\rho S_W C_1 |u| u + \frac{1}{2}\rho L_{PP} d V^2 (X_v' |v'| + X_{vr}' v' r' + X_r' |r'| + X_{rr}' r' r') \\ Y_H &= -m_Y \dot{v} + \frac{1}{2}\rho L_{PP} d V^2 (Y_v' v' + Y_{vr}' |r'| + Y_r' r' + Y_{vv}' v' |v'| + Y_{rr}' r' |r'|) \\ N_H &= -J_{ZZ} \dot{r} + \frac{1}{2}\rho L_{PP}^2 d V^2 (N_v' v' + N_{vr}' |v'| r' + N_r' r' + N_{vv}' v' |v'| + N_{rr}' r' |r'|) \end{aligned}\right\} \quad (3)$$

Where:

- m_x, m_y : the components of ship's weight by the co-ordinate axes x and y (kgs^2/m);
- J_{ZZ} : the inertia moment by the co-ordinate axe z (kgs^2/m);
- ρ : the density of sea water (kgs^2/m^4)
- S_W : the submerged area (m^2);
- C_1 : the resistance factor of ship's hull (-);
- L_{PP} : the length perpendicular (m);
- d: the ship's drift (m);
- V: the speed quantity ($V = \sqrt{u^2 + v^2}$), (m/s);
- u, v: the components of ship's speed by the co-ordinate axes x and y (m/s);
- v' : the non-dimension coefficient of ship's speed by the co-ordinate axe x ($v' = v/V$);
- r' : the non-dimension coefficient of yaw ($r' = rL_{PP}/V$);
- $X_v', X_{vr}', X_r', X_{vv}', X_{rr}'$: the non-dimension coefficients;
- $Y_v', Y_{vr}', Y_r', Y_{vv}', Y_{rr}'$: the non-dimension coefficients;
- $N_v', N_{vr}', N_r', N_{vv}', N_{rr}'$: The non-dimension coefficients.

The components of the hydro dynamic force produced by ship's propeller by the co-ordinate axe x are:

$$\begin{aligned} X_P = (1 - t_P)\rho n^2 D_P^4 (C_o + C_1\theta_P' + C_2 J_P + C_3\theta_P' J_P + \\ + C_4\theta_P'^2 + C_5 J_P^2 + C_6\theta_P'^2 J_P + C_7\theta_P' J_P^2 + C_8\theta_P'^3 + C_9 J_P^3) \end{aligned} \quad (4)$$

Where:

- t_P: the longitudinal coefficient of dead weight;
- n: the rotation in second (l/s);
- D_P: the diameter of the propeller;
- θ_P' : the angle of the propeller $\theta_P - C_{T0}$ (deg.);

- J_P: the factor of the propeller $J_P = \dfrac{(1-W_P)}{nD_P}u$;
- W_P: the effective coefficient of the propeller;
- C_i: $i = 1,\dots, 9$: the experimental coefficients of the ship's propulsion system (-);

The component of speed is calculated by the formula $u = 2.058 + 0.884\theta_p - 0.014\theta_p^2$.

The forces and moments produced by the rudder's operation are:

$$\left.\begin{aligned} X_R &= -(1-t_R)F_N \sin\delta \\ Y_R &= -(1-a_H)F_N \cos\delta \\ N_R &= -(x_R + a_H x_H)F_N \cos\delta \end{aligned}\right\} \tag{5}$$

Where:

- t_R : the resistance coefficient;
- a_H : the reciprocal impact between the ship's hull and rudder;
- x_H : the co-ordinate of the force of the rudder (m);
- x_R: the co-ordinate of the rudder (m);
- δ : the rudder's angle (rad.);
- F_N : the rudder's force (kg), $F_N = \dfrac{1}{2}\rho A_R f_A U_R^2 \sin\alpha_R$

  Where:
  - A_R : the projection area of rudder (m^2);
  - F_A: the heeling factor;
  - U_R : the water speed effects to rudder;
  - α_R : the angle of current (Rad);

There are:

$$\left.\begin{aligned} U_R &= (\varepsilon - k_x)(1-W_P)u + k_x 0.7\pi D_P n \tan\theta_P \\ \alpha_R &= \delta + \tan^{-1}\left[\frac{\gamma_R(v' + l_R r')}{U_R'}\right]_P \\ \varepsilon &= \frac{1-W_R}{1-W_P} \end{aligned}\right\} \tag{6}$$

Where: W_R, W_P are the private coefficients of the rudder's position (-).

The equations (1) of ship motion in considering of the external factors are written as following:

$$\dot{u} = \frac{1}{m+m_x}\left\{\begin{aligned} & mvr - \frac{1}{2}\rho S_W C_1|u|u + \frac{1}{2}\rho L_{PP} dV^2\begin{pmatrix} X_V'|v'| + X_{vr}'v'r' + X_r'|r'| \\ +X_{vv}'v'v' + X_{rr}'r'r' \end{pmatrix} \\ & +(1-t_P)\rho L_{PP} D_P^2 (C_o + C_1\theta_P' + C_2 J_P + C_3\theta_P' J_P + C_4\theta_P'^2 + C_5 J_P^2 \\ & +C_6\theta_P'^2 J_P^2 + C_7\theta_P^3 J_P^2 + C_8\theta_P'^3 + C_9 J_P^3) - (1-t_R)F_N \sin\delta \end{aligned}\right\} \tag{7}$$

$$\dot{v} = \frac{1}{m+m_Y}\left\{-mur + \frac{1}{2}\rho L_{PP} dV^2\begin{pmatrix} Y_V'v' + Y_{vr}'v'|r'| + Y_r'r' \\ +Y_{vv}'v'|v'| + Y_{rr}'r'|r'| \end{pmatrix} + T_B\frac{B_T}{10} + T_S\frac{S_T}{10} - (1+a_H)F_N\cos\delta\right\} \tag{8}$$

$$\dot{r}=\frac{1}{I_{ZZ}+J_{ZZ}}\left\{\begin{array}{l}\frac{1}{2}\rho_{PP}^{2}dV^{2}\left(N_{v}^{'}v'+N_{vr}^{'}\left|v'\right|r'+N_{r}^{'}r'+N_{vv}^{'}v'\left|v'\right|+N_{rr}^{'}r'\left|r'\right|\right)\\+T_{B}x_{B}\frac{B_{T}}{10}+T_{S}x_{S}\frac{S_{T}}{10}-(x_{R}+a_{H}x_{H})F_{N}\cos\delta\end{array}\right\} \quad (9)$$

$$\left.\begin{array}{l}\dot{\delta}=\frac{(\delta'-\delta)}{(|\delta'-\delta|T_{RUD}+a)}\\ \dot{\theta}=\frac{(\theta_{P}'-\theta_{P})}{(|\theta_{P}'-\theta_{P}|T_{FP}+a)}\end{array}\right\} \quad (10)$$

The float force:

$$m(\dot{u}_a-v_a r)=X_H+X_P+X_R+X_W \quad (11)$$

The slide force:

$$m(\dot{v}_a+u_a r)=Y_H+Y_T+Y_R+Y_W \quad (12)$$

The roll force:

$$I_{ZZ}\dot{r}=N_H+N_T+N_R+N_W \quad (13)$$

Where:

- u_a, v_a: the components of the relative speed (m/s);
- X_W, Y_W, N_W: The components of wind force by the co-ordinate axes *x, y* and the respective moments of the wind pressure by the co-ordinate axis *z* (-).

The components of wind force are written:

$$\left.\begin{array}{l}X_W=\frac{1}{2}\rho_A A_{of} U_W^2 C_X(\alpha_W)\\ Y_W=\frac{1}{2}\rho_A A_{os} U_W^2 C_Y(\alpha_W)\\ N_W=\frac{1}{2}\rho_A A_{os} L_{PP} C_Y(\alpha_W)\end{array}\right\} \quad (14)$$

Where:

- C_X, C_Y, C_N: The components of wind force by the co-ordinate axes *x, y* and the respective moments of the wind pressure by the co-ordinate axis *z* (-).
- A_{of}, A_{os} : the projection of ship's hull that is pressed by wind pressure (m^2);
- ρ_A: The density (kg/m^3).

Now, it's considered the relation for evaluation of current effect for the ship motion as following (see figure 1):

$$\left.\begin{array}{c}u=u_a-u_c\\ u_c=X_{cc}\cos\psi+Y_{cc}\sin\psi\\ v=v_a-v_c\\ v_c=-X_{cc}\sin\psi+Y_{cc}\cos\psi\end{array}\right\} \quad (15)$$

Where: X_{cc}, Y_{cc} - the components of the current speed, then the following relation will be right:

$$\left.\begin{array}{l}\dot{u}=\dot{u}_a-rv_c\\ \dot{v}=\dot{v}_a-ru_c\end{array}\right\} \quad (16)$$

It is considered the different cases of the linearization of the general model:
In case of ship sailing with the sea speed (4-8kts):

$$\dot{u}=\frac{1}{m+m_x}\left\{\begin{array}{l} mvr-\frac{1}{2}\rho S_W C_1 u^2+\frac{1}{2}\rho L_{PP} dU^2(u,v)\left(\begin{array}{l}X'_{|v|}|v'|+X'_{|r|}|r'|+X'_{vr}v'r'+\\+X'_{vv}v'v'+X'_{rr}r'r'\end{array}\right)\\ +(1-t_P)\rho n^2 D_P^2(C_o+C_1\theta'_P+C_2J_P+C_3\theta'_P J_P+C_4\theta'^2_P+C_5J_P^2+C_6\theta'^2_P J_P^2\\ +C_7\theta'_P J_P^2+C_8\theta'^3_P+C_9J_P^3)-(1-t_R)F_{N.medium}(u,v,r,\delta,\theta_P)\sin\delta \end{array}\right\} \quad (17)$$

$$\dot{v}=\frac{1}{m+m_Y}\left\{\begin{array}{l}-mur+\frac{1}{2}\rho L_{PP}dU^2\left(Y'_v v'+Y'_r r'+Y'_{vvr}v'^2r'+Y'_{vrr}v'r'^2+Y'_{vvv}v'^3+Y'_{rrr}r'^3\right)\\ -(1-a_H)F_{N.medium}(u,v,r,\delta,\theta_P)\cos\delta\end{array}\right\} \quad (18)$$

$$\dot{r}=\frac{1}{I_{ZZ}+J_{ZZ}}\left\{\begin{array}{l}\frac{1}{2}\rho L_{PP}dU^2(u,v)\left(N'_v v'+N'_r r'+N'_{vvr}v'^2r'+N'_{vrr}v'r'^2+N'_{vvv}v'^3+N'_{rrr}r'^3\right)\\ -(x_R-a_Hx_H)F_{N.medium}(u,v,r,\delta,\theta)\cos\delta\end{array}\right\} \quad (19)$$

It's difficult to find the experimental coefficient $F_{N.medium}$.
In case of ship sailing with slow speed (1.5-3kts):

$$\dot{u}=\frac{1}{m+m_x}\left\{\begin{array}{l} mvr-\frac{1}{2}\rho S_W C_1|u|u+\frac{1}{2}\rho L_{PP}d\left(X_{|v|}|v'|+X_{|r|}|r|+X_{vr}vr++X_{vv}v^2+X_{rr}r^2\right)\\ +(1-t_P)\rho n^2D_P^2(C_o+C_1\theta'_P+C_2J_P+C_3\theta'_PJ_P+C_4\theta'^2_P\\ +C_5J_P^2+C_6\theta'^2_PJ_P^2+C_7\theta'_PJ_P^2+C_8\theta'^3_P+C_9J_P^3)\end{array}\right\} (20)$$

$$\dot{v}=\frac{1}{m+m_Y}\left\{\begin{array}{l}-mur+\frac{1}{2}\rho L_{PP}d\left(Y_v v+Y_r r+Y_{v|v|}v|v|+Y_{r|r|}r|r|+Y_{|v|r}v|r|\right)\\ +T_B(\overline{\theta}_{P,b,th})+T_{S,th}S_{th}\end{array}\right\} \quad (21)$$

$$\dot{r}=\frac{1}{I_{ZZ}+J_{ZZ}}\left\{\begin{array}{l}\frac{1}{2}\rho L_{PP}d\left(N_v c+N_r r+N_{v|v|}v|v|+N_{r|r|}r|r|+N_{|v|r}|v|r+N_{v|r|}v|r|\right)\\ +T_B(\overline{\theta}_{P,b,th})x_B+T_{S,th}S_{th}x_B\end{array}\right\} \quad (22)$$

Where: $T_B(\overline{\theta}_{P,b,th})=26.651\overline{\theta}_{P,b,th}+0.157\theta^3_{P,b,th}$ *and* $T_{s,th}=60s$

One of the most common models is Nomoto model (it's so called T, K model). This model describes fairly the ship motion in maneuvering.
It's supposed that: instead of using $\tau\dot{r}+r=k\delta$, it's used $\tau\dot{r}+H(r)=k\delta$ ($H(r)$ is the nonlinear function).
Where:

- $H(r)=\alpha_3r^3+\alpha_2r^2+\alpha_1r+\alpha_0$
- α_0,α_2 : the symmetric coefficients of ship;
- α_1,α_3 : The coefficients in showing of relative dependence of *r* on *δ.*

It's supposed that the ship's hull is symmetric, so:

$$\left.\begin{array}{c}H(r)=\alpha_3r^3+\alpha_1r\\ \dfrac{r(s)}{\delta(s)}=\dfrac{k}{S\tau+1}\end{array}\right\} \quad (23)$$

The Mathematical Model of the Auto Pilot Regulator

The exact controlling of ship requires considering the above said problem from many larger field, but not the design of tuning unit. Generally, the ship's auto pilot consists of two blocks, as: the block of course selection and the block of steering as the given course. Here, this is showed the basic solution of ship control that the ship sails. The components of this manoeuver are showed as figure 2 [1].

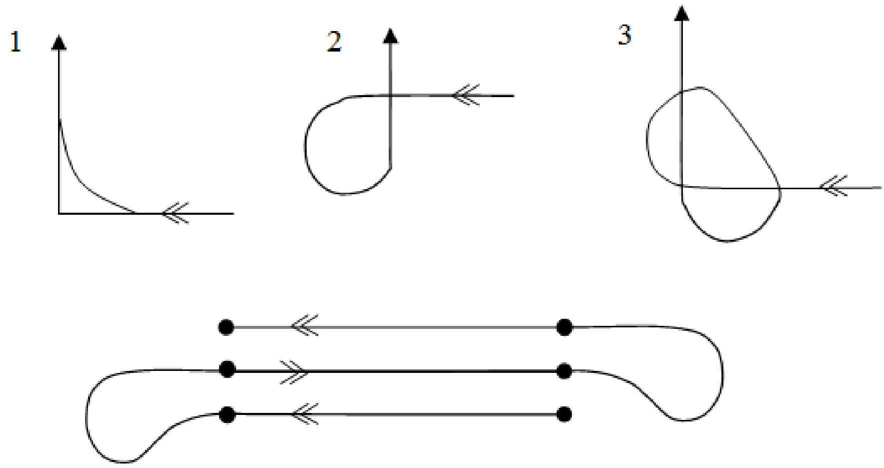

Figure 2: the main components of the ship manoeuver

The ship's course is the desired trajectory. In order to synthesize the ship tuning regulator, it requires developing its mathematical model. In the structure role of mathematical model, it's accepted the linear model as form [2, 3]:

$$\begin{pmatrix} \dot{r}(t) \\ \dot{v}(t) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} r(t) \\ v(t) \end{pmatrix} + \begin{pmatrix} b_{11}(t) \\ b_{12}(t) \end{pmatrix} \delta(t) \tag{24}$$

The ship model can be developed as the figure 3.

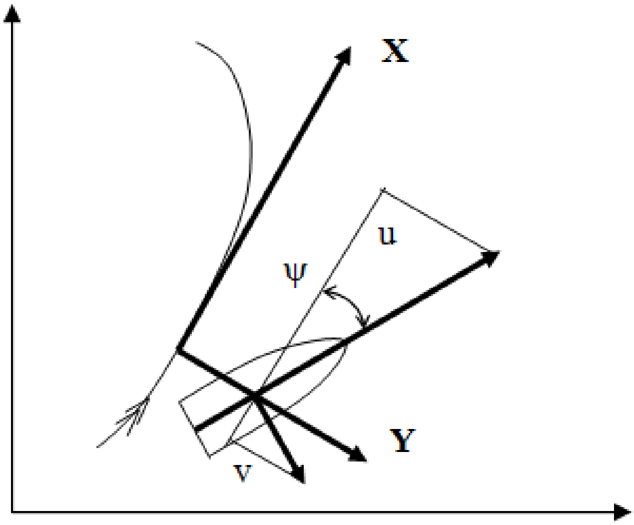

Figure 3: the ship model moving in the co-ordinate axes XY

The parameters of this model are found as the following:

$$\dot{\psi} = r \tag{25}$$

$$\dot{x} = U\cos\psi + v\sin\psi + d_x \tag{26}$$

$$\dot{y} = U\sin\psi + v\cos\psi + d_y \tag{27}$$

In the trajectories with the small angle ψ, the equations (25), (26) can be linearized.
When developing the preliminary mathematical model of ship, it can be accepted $a_{12}(t) = a_{21}(t) = 0$ and the model of the disturbances is the form:

$$\dot{r}_b = \omega_2 \tag{28}$$

$$\dot{d}_x = \omega_3 \tag{29}$$

$$\dot{d}_y = \omega_4 \tag{30}$$

Where: $\omega(t)$ - the process of "white interference" form.

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In the preliminary model of ship:

$$\dot{r} = -\frac{1}{T}r + \frac{K}{T}\delta + r_b \quad (31)$$

$$\begin{pmatrix} r \\ v \end{pmatrix} = \begin{pmatrix} 1 \\ \alpha \end{pmatrix} r \quad (32)$$

The values of the parameters T and K will be influence of the ship's characteristic as well as the unmeasured indefinite disturbances. But, relying on the adjust aimed for these parameters, it can be optimized the mathematical model and the auto pilot regulator of ship.
In order to optimize the system it's used the criteria of the quadratic as the form:

$$J = E\left(\lambda_y y^2 + \lambda_\psi \psi^2 + \delta^2\right) \quad (33)$$

Where: λ_y, λ_ψ - the weight coefficients.

Because the measurements of the ship's trajectory are carried out on the background noises, beside of the diagram of structure of the ship control as the given trajectory it requires the observer. The observer's duty in the linear model is Kalman filter that corresponds to the nonlinear discrete model.

$$x_{t+1} = f\left(x_t\right) + Gu_t + \omega_t \quad (34)$$

$$z_t = Hx_t + v_t \quad (35)$$

Where: v_t, ω_t - the unmeasured processes, "white interference" form. This tuning dynamic regulator (the observer and the tuning unit) is described as the following expression:

$$\left.\begin{array}{c} x^*_{t+1} = f(x_t) + G\underline{u}_t + K(z_t - Hx_t) \\ u_t = L(x_t - x^*_t) \end{array}\right\} \quad (36)$$

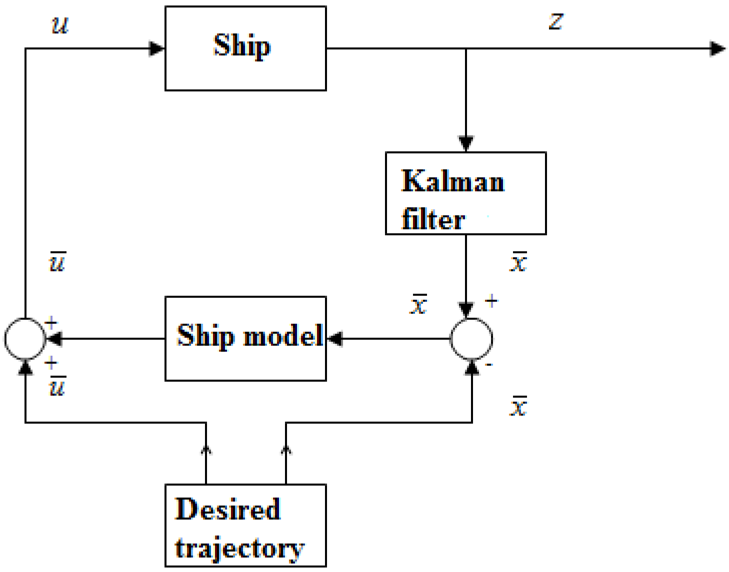

Figure 4: Diagram of Structure of Ship Control Regulator

In order to calculate the value of the elements in the matrix *K*, it should be solve the Rickati equation. To solve that problem, it requires the knowledge about the interference intensity *v* and *ω*. It is normally supposed that the interferences are stationary and the operations are carried out by the fully defined values of the parameters in the ship model and those matrixes. However, the dynamic tuning regulator with the parameters obtained by the analytical method when controlling the ship in the unstable conditions will not often assure the given demands on the control accuracies. As the experimental result, when the ship controlled by the optimized average dynamic control regulator, with the speed of 3m/s as the trajectory in the figure 1 with the oscillation nearly 3m is assured to sail with the approximated accuracy of 30m [4, 5].

Therefore, the ship auto pilot in the condition of varied impacts (wind pressure, waves, current) that is used the unchanged mathematical model will not assure the given parameters in regarding to the accuracy of the ship motion as the given course and the standard maneuvers. That condition is required to add the series of the self-tuning circuit to the basic structure of the ship control system (figure 3). The duty of that circuit is the optimization of the ship auto pilot in correspondent with the required accumulable information that improve the accuracy and safety of the ship control. The auto dynamic control is normally operated in two basic modes, as following:

- The mode of the ship's course maintenance;
- The maneuvering mode.

Prior to develop the self-adjusted algorithms, it is considered the characteristics of the disturbance impacts. The most negative impact for the ship control quality is the wave impact that is influence of the height and the period of wave. The wind pressure with nonlinear property is mostly depended on the wind force, the shape of ship's superstructure and the wind direction. The statistic of wind pressure plays an importance role in regarding to keep stable for ship's course. The spectrum density of the focus stable process of wave is described as the following expression:

$$S(\omega) = \frac{1}{\pi}\int_0^\infty R(\tau)\cos\omega\tau d\tau \tag{37}$$

In order to establish the spectrum density experiment, it should be used the below expression:

$$S_r(\omega) = \frac{c}{2}\omega^{-6} e^{-\frac{2g^2}{\omega^2 v^2}} \tag{38}$$

Where:

- c: the constant;
- g: the accelerator of wind pressure;
- ω: the frequency of wind;
- v: the speed of wind.

The correlative function of wave is described by the following expression:

$$R_r(\tau) = D_r e^{-\alpha t}\cos\beta\tau \tag{39}$$

Where:

- D_r: the dispersity of wave;
- α: the parameter described the infrequency of wave;
- β: the specific coefficient of the spectrum band of wave impact.

The spectrum density of the process and the correlative function (39) may be written as the form:

$$S_r(\omega) = \frac{D_r\alpha}{\pi}.\frac{\omega^2+\omega_0^2}{(\omega^2-\omega_0^2)+4\alpha^2\omega^2} \tag{40}$$

Where: $$\omega^2 = \alpha^2 + \beta^2 \tag{41}$$

Basing of the experimental data, the dependent relation of the parameters α and β may be defined by the relation:

$$\frac{\alpha}{\beta} = 0.21 \tag{42}$$

The relation between the dispersity and the height of wave is described by the experimental formula, as:

$$D_r = \frac{1}{k^2}\left(\frac{h}{2}\right)^2 \tag{43}$$

In the function described the impact of wave to ship's hull, the following parameters are important: r – angle of wave effected on the ship's hull, α_0 – the inclination of wave. The relation between these parameters is described by the following expression:

$$\alpha_0 = \frac{2\pi r}{\lambda} = \frac{r\omega^2}{g} \tag{44}$$

Where:

- λ: the height pf wave;
- ω: the frequency.
- r : angle of wave affected on the ship's hull.

The stationary effect on the ship's hull is considered the process formed from the set of the harmonized processes. The function of the ship motion that is used in the synthesized process of the tuning unit and the self-tuning circuit can be established by mean of analyzing of figure 5 [6, 7 and 8].

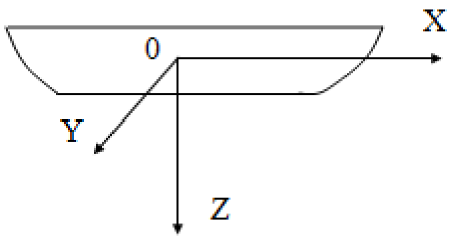

Figure 5: the Ship Model in the 3-D Co-ordinate

The functions of forces and moments effected to ship are described by the following expression:

The function of forces effected to ship by the OY axis (the drift function of wind pressure):

$$A_1(p)\beta + B_1(p)\theta + C_1(p)\varphi = D_1(p)\delta + E_1(p)u + Y_b \tag{45}$$

The function of moments by the OX axis:

$$A_2(p)\beta + B_2(p)\theta + C_2(p)\varphi = D_2(p)\delta + E_2(p)u + L_{xb} \tag{46}$$

The function of moments by the OZ axis:

$$A_3(p)\beta + B_3(p)\theta + C_3(p)\varphi = D_3(p)\delta + E_3(p)u + L_{zb} \tag{47}$$

Where:

- $p = \frac{d}{dt}$
- L_b : the sum of external forces effected longitudinally by OY axis;
- L_{xb} : the sum of external forces effected longitudinally by OX axis;
- L_{zb} : The sum of external forces affected longitudinally by OZ axis.

If the impact of the yaw and heave forces are small, it can be skipped, then there is:

$$B_1(p) = B_3(p) = D_1(p) = D_3(p) = 0 \tag{48}$$

In this case, the equations (45), (46) and (47) can be rewritten as the following forms:

$$\left.\begin{array}{c} A_1(p)\beta + C_1(p)\varphi = E_1(p)u + Y_b \\ A_2(p)\beta + B_2(p)\theta + C_2(p)\varphi = D_2(p)\delta + E_2(p)u + L_{xb} \\ A_3(p)\beta + C_3(p)\varphi = E_3(p)u + L_{zb} \end{array}\right\} \tag{49}$$

In the equations (49), the first and third equation depends on the second one. In order to research the controllable ability of the system in the stable course, it can be used the following equations:

$$\left.\begin{array}{l} A_1(p)\beta + C_1(p)\varphi = E_1(p)u + Y_b \\ A_3(p)\beta + C_3(p)\varphi = E_3(p)u + L_{zb} \end{array}\right\} \tag{50}$$

The above-said relations allow developing the transfer matrix of ship for the different processes. For example, the transfer function of yaw angle and rudder angle has the form:

$$W_{\phi u}(p) = \frac{\phi(p)}{u(p)} = \frac{A_1E_3 - A_3E_1}{A_1C_3 - A_3C_1} \tag{51}$$

In corresponding to each impact of disturbance, it can be found the transfer function as:

$$W_{\phi Y_B}(p) = \frac{\phi(p)}{Y_B(p)} = \frac{A_3}{A_1C_3 - A_3C_1} \tag{52}$$

$$W_{\phi L_{ZB}}(p) = \frac{\phi(p)}{L_{ZB}(p)} = \frac{A_3}{A_1C_3 - A_3C_1} \tag{53}$$

If all the impacts of disturbance can be presented by the relation:

$$f_\varphi = A_1L_{zb} - A_3Y_b \tag{54}$$

Then the transfer function of yaw angle and the disturbance forces in equation (53) can be presented as the form:

$$W_{\phi f}(p) = \frac{\phi(p)}{f_\varphi(p)} = \frac{1}{A_1C_3 - A_3C_1} \tag{55}$$

If the frequent impact of wave can be present as the form of sinusoid complex, then:

$$\left.\begin{array}{l} Y_b = Y_{bm}\sin\omega_k t, \\ L_{xb} = L_{bm}\sin\omega_k t, \\ L_{zn} = L_{zbm}\cos\omega_k t, \end{array}\right\} \tag{56}$$

Where Y_{bm}, L_{bm}, L_{bz} - the functions of wave intensity and the angle φ_b created by the ship motion under the wave impact.

In many cases, the transfer function of yaw angle and rudder angle can be also present as the form:

$$W_{\phi u}(p) = \frac{k_{ou}}{p(Tp+1)} \tag{57}$$

And

$$W_{\phi f}(p) = \frac{k_{of}}{p(Tp+1)} \tag{58}$$

Where k_{ou}, k_{of} - the brake coefficients in corresponding to the impact of rudder and the above-said disturbance force; T is the time constant. In order to find the transfer function in referent to the sway and heel of ship, it’s necessary to consider the fact that heel is not influence of the ship displacement but also its drift. The transfer function of heel will have the form:

$$W_{\theta\delta}(p) = \frac{\theta(p)}{\delta(p)} = \frac{D_2(p)}{B_2(p)} \tag{59}$$

$$W_{\theta Y_B}(p) = \frac{\theta(p)}{Y_B(p)} = \frac{A_3C_2 - A_2C_3}{B_2(A_1C_3 - A_3C_1)} \tag{60}$$

$$W_{\theta L_{XB}}(p) = \frac{\theta(p)}{L_{XB}(p)} = \frac{1}{B_2} \tag{61}$$

$$W_{\theta L_{ZB}}(p) = \frac{\theta(p)}{L_{ZB}(p)} = \frac{A_2C_1 - A_1C_2}{B_2(A_1C_3 - A_3C_1)} \tag{62}$$

The above transfer functions are used to establish and analyze the controllable motion of the ship under the stable disturbance impacts.As the above said mention, the dynamic disturbances have always static property. Therefore, the auto pilot regulator should have the ability to optimize its operation that adapts with the received information.

When solving of the optimization problem of parameter of the auto pilot regulator, it can be divided into two sub problems. In the first problem, it should be synthesized the tuning unit exactly for each number and in the second one, it should be established the optimization algorithm of the parameters of the auto pilot. Supposedly, the basic structure of considered object is described by the transfer functions (57), (58), the transfer function of the feedback relation has the form:

$$W_{goc}(p) = S_1 p + S_0 \tag{63}$$

The transfer function of the tuning unit PI has the form:

$$W_{reg}(p) = \frac{u(p)}{u_1(p)} = \frac{k_{reg}}{T_{reg}\,p + 1} \tag{64}$$

Where:

$$k_{reg} = \frac{1}{k_{oc}}, T_{reg} = \frac{1}{k_{oc}k} \tag{65}$$

And the equation will be presented by the expression:

$$\Delta u = (\Delta\phi_{task} - \Delta\phi W_{g_{oc}})\frac{k_{reg}}{T_{reg}\,p + 1} \tag{66}$$

At the stable mode $\Delta_{\varphi task} = 0$, then:

$$\Delta u = \Delta\phi(S_1 p + S_0)\frac{k_{reg}}{T_{reg}\,p + 1} = \frac{S_0 \frac{1}{k_{oc}}}{\frac{1}{kk_{oc}}p + 1}\Delta\phi - \frac{S_1 \frac{1}{k_{oc}}}{\frac{1}{kk_{oc}}p + 1}p\Delta\phi \tag{67}$$

With understanding that, the quality of control is mainly influence of the time constant value of the tuning unit T_{reg}. The optimized parameters are $S_0, S_1, k \text{ and } k_{oc}$. The transfer function of the tuning unit at $T_{reg} \approx 0.5\sec$ has the form:

$$\Delta u = -S_0\frac{1}{k_{oc}}\Delta\phi - S_1\frac{1}{k_{oc}}p\Delta\phi \tag{68}$$

The relation between the parameters $S_0, S_1, k \text{ and } k_{oc}$ is defined by the following relation:

$$S_0 = \frac{1}{4Tk_p k_{ou}}(1 + k_p k_{ou} S_1)^2 \tag{69}$$

With playing the role of the selected standard for choosing of the optimized algorithm of the parameters of the tuning unit, it's often selected the expression that consists of the following format:

- Minimization of the defined errors;
- Minimization of the differential error or total.

However, the criteria may be different for each other at the different mode of the ship control. The adjust of the parameters can be carried out by the computer program or automated fine-tune (lookup).

The computer program of the parameter changing requires the knowledge of the varied rule of the external parameters as well as the varied rule of the objective parameters. In this case, it can be synthesized the self-adjusted optimized algorithm in the meaning of any given criteria. When the dynamic disturbance has the unstable and directly unmeasurable property, it could not be adjusted the parameters as the above program.

In general case, the problem will be solved by the method of homogenizing of the object and disturbance. The received information will be used to adjust the parameters of the tuning unit. Beside the problem of control of the ship's trajectory in the condition of many stable disturbances, it will be considered the harmonized problem of rolling of ship motion. In three types of oscillation (pitching, rolling and heaving), the rolling is focused above all other, because of its most intensity. In fact, one of the most effective methods of anti-rolling is applied the self-tuning unit to control the special fins attached in the both side of the ship. In order to carry out this tuning unit, it requires the equipment as the accelerometer and the gyro meter. The transfer function of that harmonizing system (by the port and starboard side) will have the form [9, 10 and 11]:

$$W_{\theta\delta}(p) = \frac{\theta(p)}{\delta(p)} = \frac{k_{ou}}{T_2 p_2 + 2\gamma Tp + 1} \qquad (70)$$

$$W_{\theta f}(p) = \frac{\theta(p)}{f(p)} = \frac{k_{of}}{T_2 p_2 + 2\gamma Tp + 1} \qquad (71)$$

Where: k_{ou} *and* k_{of} are acceleration coefficients, γ is the anti-rolling coefficient, T is the time constant. The transfer function of the tuning unit of the anti-rolling has the form:

$$W_{per}(p) = \frac{\frac{k}{p}}{1 + \frac{kk_{0c}}{p}} = \frac{k}{p + kk_{0c}} = \frac{\frac{1}{k_{0c}}}{\frac{1}{k_{0c}} p + 1} = \frac{k_{per}}{T_{per} p + 1} \qquad (72)$$

Where:

$$k_{per} = \frac{1}{k_{0c}}; T_{per} = \frac{1}{kk_{0c}} \qquad (73)$$

Now, it is going to consider the subjects of the analytical design for the optimized control system of the ship. Each case will be considered separately, these are the control with enough objective information and the control with lacking of objective information.
Firstly, it's supposed that the model of objective control will be:

$$\begin{Bmatrix} \dot{x}(t) = f(x,u,t) \\ x(t_0) = x_0 \end{Bmatrix} \qquad (74)$$

Where: $x(t) \in R^n, u(t) \in U \subset R^m, U$ - the areas of the permitted values of the control parameters. It's necessary to change the object from the initial state $x(t_0) = x_0$ to the last state $x(T) = 0$, by the mean of optimizing the following norm:

$$\Im(x,u) = F(x(T),T) + \int_{t_o}^{T} L(x,u,t)dt \qquad (75)$$

Let's enter the Hamiltonian:

$$H(x,u) = L(x,u,t) + \lambda^T f(x,u,t) \qquad (76)$$

And reckoning with the ungiven time T, the functions $f_i(x,u,t), \frac{\partial f_i(x,u,t)}{\partial x}, i = 1..n$ are continuous.

Then, the solving of the problem is not only finding the function $u(t) \in U$, but also changing the object to the state $x(T) = 0$ and assuring to achieve the minimization of the quality function:
The necessary condition of the optimization:

$$\left.\begin{aligned} \dot{x}(t) &= f(x(t),u(t)) \\ x(t_o) &= x_o,\, x(T) = 0 \\ \dot{\lambda}(t) &= -\left\{\frac{\partial H(x,\lambda,u)}{\partial x(t)}\right\}^T \\ \lambda(T) &= \frac{\partial F(x(T),T)}{\partial x(T)} \end{aligned}\right\} \tag{77}$$

$$\frac{\partial H(x(t),\lambda(t),u(t))}{\partial u(t)} = 0 \tag{78}$$

$$u(t) = \varphi(x,\lambda)$$

Where:

$$\lambda^o(t) = \lambda(t) + \alpha(t) \tag{79}$$

And the $\lambda(t)$ - The solution of the following equation:

$$\dot{\hat{\lambda}}(t) = -\left\{\frac{\partial H(x,u,\lambda,\alpha)}{\partial x(t)}\right\}^T \tag{80}$$

The basic relations: $\lambda(t_0) = \lambda_0 \lambda_0$ - the first approximated operation; $\alpha(t)$ - the optimized vector:

$$\dot{\alpha}(t) = \beta(t) H(x,\hat{\lambda},\alpha,u),\ \alpha(t_0) = 0 \tag{81}$$

The block diagram of the optimized tuning unit is presented as the following:

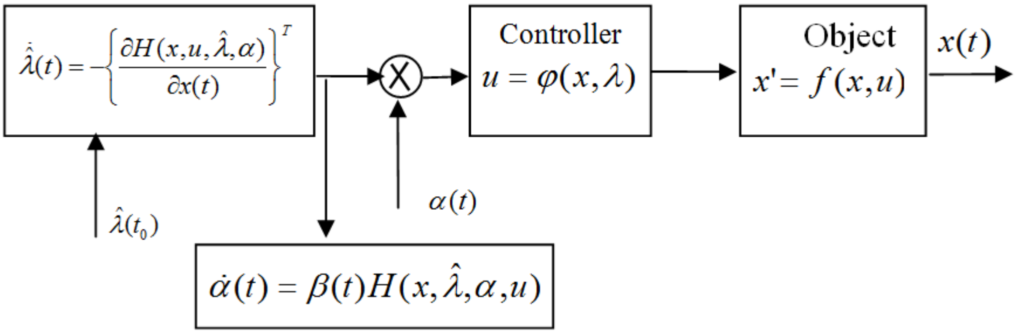

Figure 6: the Diagram of the Optimized Tuning Unit

Consequently, it is considered the subjects of the design of the optimized tuning unit in the condition of lacking of the objective information.

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \tag{82}$$

Where: A, B - the matrixes, $A(t) = A^0 + a(t),\ B(t) = B^0 + b(t)$ disturbed by the parameters $a(t), b(t)$.
Similarly, there is the quality function as the following form:

$$\Im(x,u) = \frac{1}{2} x^T(T) F x(T) + \frac{1}{2}\int_{t_0}^{T} \left\{ x^T(t) Q x(t) + u^T(t) R u(t) \right\} dt \tag{83}$$

The general diagram of the tuning unit in that case of lacking of information will has the following form: $\dot{x}(t) = A^0 x(t) + B^0 u(t),\ x(t_0) = x_0$

In which: $$u(t) = -R^{-1}(B^0)^T \lambda^0(t),\ \lambda^0(t) = S^0 x(t) \quad (84)$$

Where: $S^0(t)$ - the solution of the Rickati equation:

$$\dot{S}^0(t) = -S^0(t)A^0 - (A^0)^T S^0(t) + S^0(t)BR^{-1}B^T S^0(t) - Q, \quad S^0(T) = F \quad (85)$$

And in this case, the Hamiltonian will have the form:

$$H^0(x,u,\lambda^0) = \frac{1}{2}x^T Qx(t) + \frac{1}{2}u^T(t)Ru(t) + (\lambda^0(t))^T (A^0 x(t) + B^0 u(t)) \quad (86)$$

For the norm in the quadratic form, the structure of the tuning unit has the following form (figure 7):

$$\Im(x,u) = \frac{1}{2}\int_{t_0}^{T} \left\{ x^T(t)Qx(t) + u^T(t)Ru(t) \right\} dt$$

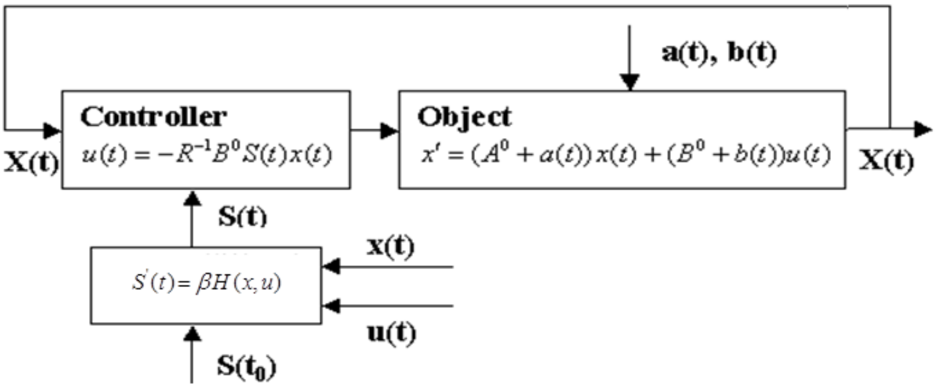

Figure 7: the Diagram of the Optimized Tuning Unit

In the case of solving of the problem of ship motion in the programed trajectory, it should be entered $\varepsilon = x - x_M$, where x_M is the expected trajectory. The differential equations:

$$\begin{pmatrix} \dot{x}(t) \\ \dot{\lambda}(t) \end{pmatrix} = \begin{pmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{pmatrix} \begin{pmatrix} x(t) \\ \lambda(t) \end{pmatrix} + \begin{pmatrix} 0 \\ Q \end{pmatrix} x_M \quad (87)$$

$$\lambda(t) = S(t)x(t) - K(t)x_M(t) \quad (88)$$

Where $s(t)$ - the symmetrical matrix $S(t) = S^T(t)$.

The control function has the form: $u(t) = -R^{-1}B^T(S(t)x(t) - K(t)x_M(t))$

The quality function:

$$\Im(x,u) = \frac{1}{2}\varepsilon^T(T)F\varepsilon(T) + \frac{1}{2}\int_{t_0}^{T} \left\{ \varepsilon^T(t)Q(t)\varepsilon(t) + u^T(t)R(t)u(t) \right\} dt$$

$$\varepsilon^T Q\varepsilon = (x - x_M)^T Q(x - x_M) = x^T Qx + (x_M)^T Qx_M - 2(x_M)^T Qx$$

And the Hamiltonian:

$$H = \frac{1}{2}x^T Qx - x^T Qx_M + \frac{1}{2}x_M^T Qx_M - x^T Qx_M + \frac{1}{2}u^T Ru + \lambda^T Ax + \lambda^T Bu$$

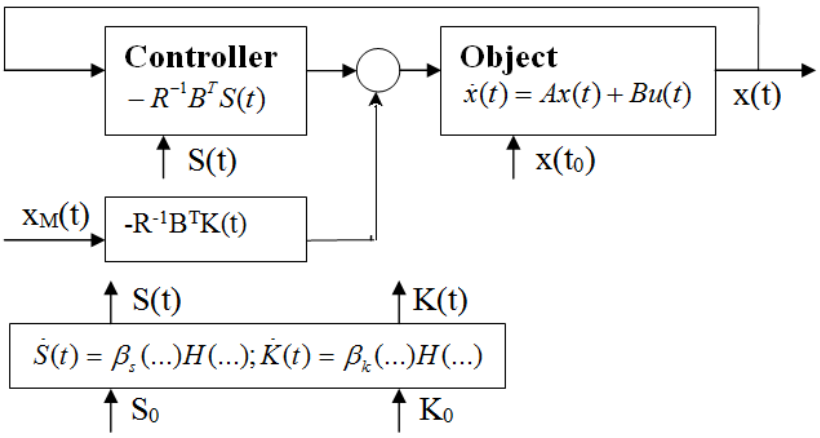

Figure 8: the Optimization Block of the Tuning Unit

The figure 8 is presented the structure of the optimized tuning unit in the connection with the ship control problem in the programed trajectory in case of the objective information shortage. Basing on the fuzzy logic and neuron network technology, it'll be subsequently considered two basic subjects, as following:

- To keep the ship route stably as the expected trajectory (programed);
- To synthesize the ship control to the expected trajectory (given).

It's presented the ship motion by the model:

$$\left.\begin{aligned} \dot{V}_y(t) = a_{11}^x(t)V_y(t) = a_{12}^x(t)\gamma(t) = b_1^x(t)u(t) \\ \dot{\gamma}(t) = a_{21}^x(t)V_y(t) + a_{22}^x(t)\gamma(t) + b_2^x(t)u(t) \\ \dot{\psi} = \gamma \end{aligned}\right\} \quad (89)$$

Where V – the speed vector; V_x, V_y - the speed component V by the axes x, y; ψ, γ - the corresponding angles.

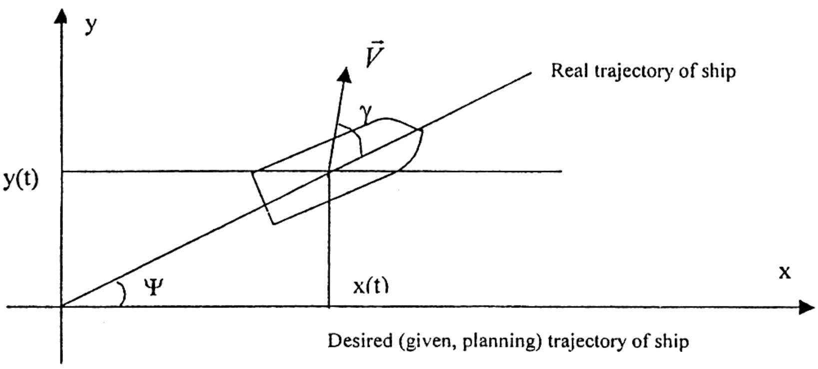

Figure 9: The Model of the Ship Motion

$$\left.\begin{aligned} a_{11}^x = V_x\frac{a_{11}}{L},\ a_{12}^x = V_x a_{12},\ a_{21}^x = V_x\frac{a_{21}}{L^2} \\ a_{22}^x = V_x\frac{a_{22}}{L},\ b_1^x = V_x^2\frac{b_1}{L^2},\ b_2^x = V_x^2\frac{b_2}{L^2} \end{aligned}\right\} \quad (90)$$

Where: L – ship's length; $a_{11}, a_{12}, a_{21}, a_{22}, b_1, b_2$ - the given coefficients.

By this mean, it's obtained the standard establishment of the synthetic problem of the optimized equation, as:

$$\dot{x}(t) = f(x,u,t),\ x_0 = x(t_0),\ u(t) \in U \subset R^m,\ x(t) \in R^n \quad (91)$$

With the quality function:

$$J(x,u) = F\left[x(T),T\right] + \int_{t_0}^{T} L(x,u,t)dt \quad (92)$$

Synthetic of the Ship Control Algorithms Based on the Approached Fuzzy Logic Method

It should be entered here the angle φ- the angle between the real course and the expected course (figure 9), and established the continuous control $u(t)=\varphi(t)$ with the help of the approximated discrete diagram, as [12, 13]:

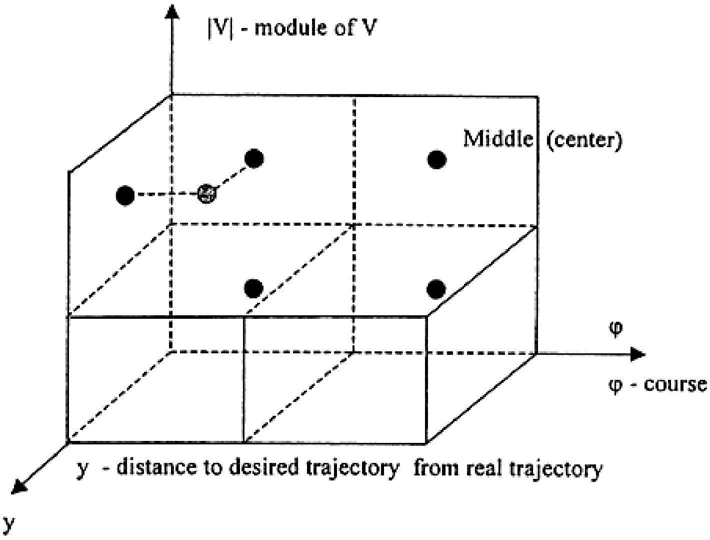

Figure 10: the Approximated Discrete Diagram

By using of chain of quantitative changes of $\varphi(t)$:

$$u_0 = \varphi(t_0),\ u_1 = \varphi(t_1), ...,\ u_k = \varphi(t_k) \quad (93)$$

It will be established the required control. The ship sails in the condition with the contingent dynamic disturbances ξ as:

$$\dot{x} = f(x,u,t) + \xi \quad (94)$$

After applying of the physical operation, there will be the 3-D space of the discrete control (the discrete set of control mode) $\{y,\varphi,|V|\}$ and the corresponding set of the 3-D cubes $\{y_i,\varphi_i,|V_i|\}$. Subsequently, it's considered that the point inside the cubes is not subdivided, i.e. they are only defined by their numeric value center. The varied linguistic parameters of the system will be connected with the co-ordinate of the center point of the cubes.

RESULTS

The diagram of the algorithm control is:

1. By the navigation equipment, it will be obtained the ship position and calculated $\{y,\varphi,|V|\}$. Thence, it is defined the cube position of present ship position.
2. Solve the problem of the ship control: $\dot{z}(t) = f(z,u,t),\ \ z(t_0) = z_0$

Where: $z(t) \in R^n$, $u(t) \in U \subset R^m$; U - the set of the control functions with $z = (x,y,\psi,r)$, z_0 - the quantity of center point of cube where the object is.
In corresponding to the quality function of control, it should be established the control as:

$$u = -R^{-1}B^{T}Sz$$
$$u(t) = -R^{-1}B^{0}S(t)z(t)$$
$$u(t) = -R^{-1}B^{T}\left[S(t)z(t) - K(t)z_{M}(t)\right]$$

The control $u(t)$ in the discrete rounded operation will be established by chain $\varphi_1 = \varphi(t_1),\ \varphi_2 = \varphi(t_2), ...,\ \varphi_k = \varphi(t_k)$. This chain presents the moment by time and the yaw of the ship trajectory. After t_k, it should be kept value $\varphi(t_k)$ unchanged. The subsequent application of the above method may be repeated, if the ship is continuously deviated far from the desired value in comparison with the desired trajectory. It can be illustrated the difference between the normal control and the fuzzy logic control by the result of math model.

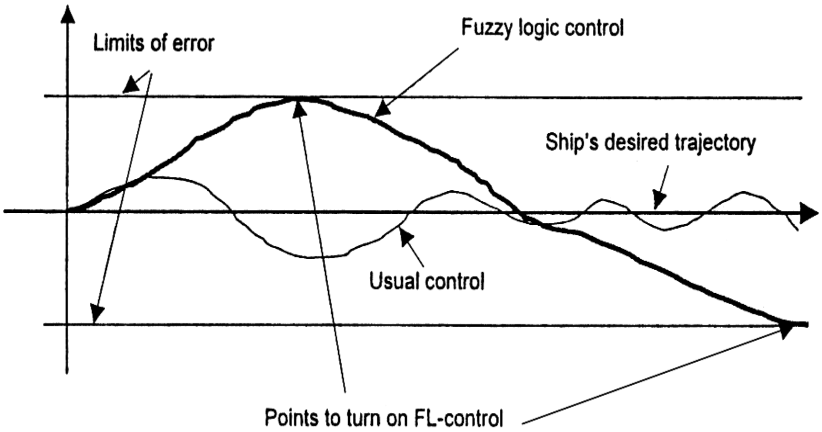

Figure 11: The Diagram of the Neuron – Network Projects

DISCUSSION

It's recognized that the weakness of Nomoto model is to use the secondary term of Taylor expansion. In fact, this only can be established the model for ship motion with the small angle of the rudder. The core significance of the fuzzy logic application is to obtain the following subjects:

- Minimize the running time of engine in the top state, i.e. more consumptive state;
- Synthesize the ship control mode that is maximized the behavioral simulation of the seaman in order to obtain the algorithm control as simple as possible.

CONCLUSIONS

The mathematical model proved the advanced property of the fuzzy logic control in many cases, i.e. to present the dynamic disturbance. Knowingly, in corresponding to the diagram of neuron network projects, chain $\{\varphi_1(.), ..., \varphi_k(.)\}$ should be primarily established. The adaptive algorithms of the ship control are based on the fuzzy logic that can be able to prevent the negative contingent impacts leading to ship's deviation in condition of bad weather. During navigation, the operation time of engine that encounters the over load state is reduced considerably and also maintained the desired trajectory.

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