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PROGRAMS FOR THE CONTROL OF SHIPS IN MEETING MOTIONS

Nguyen Xuan Phuong
Ho Chi Minh City University of Transport
VIETNAM

ABSTRACT

In this paper, the author presents the algorithms and numerical procedures for the control of going vessels to ensure the implementation of the processes of oncoming traffic. The development of these algorithms performed so that the class limitations are imposed on the control action and phase position can be easily expanded to restrictions on the rate of variable control or the acceleration, with the general form of this algorithm is fully maintained. This property allows them to use these algorithms for the synthesis of control systems and controlling the power plant complex in order to ensure safe navigation and obtaining economical control modes. In some cases, it is sufficient to purely qualitative assessment of the proposed algorithm to generate the correct control. But most require specific, that is, numerical solutions. This led to the development of such control uses the principle of maximum of LS. Pontryagina. The paper devotes the algorithm for ship control as: meeting movement; the predetermined moving area; the environmental variation of a time function.

Keywords: Ship **c**ontrol in a predetermined moving area, Ship control in meeting motion, Ship control in environment variation of time function.

INTRODUCTION

In maritime practice of Vietnam, when the ships sail through the river in the Mekong delta, Vung tau, Saigon, it is necessary the task of forming system of ships that is the task of the meeting of movements. Establishing of the system is started with a leader who is placed at the top of the convoy. The task of the remaining ships (mobile systems) is to take their place and accepting on the parameters of motion with the leader ship. In this situation, it should be carried out assessment of the dynamics of mobile systems and the initial values of the phase coordinates. Also the question is how to control the ship approach the leader ship soonest.

LITERATURE REVIEW

In this subject, there are researches of authors such as Krasovsky A.A (1999), Peshekhonov V.G (2000), Kolesnikov A.A (2002), Astana Y.M (2002), V.S Medvedev et al. (2005), Hecht-Nielsen r. (2007), Stone M. (2009), Weierstrass K (2010). Their works are based on the classical methods of construction of automatic control systems and in particular the ship's course allows classifying the type of techniques used by the mathematical model of the vessel, processed information, methods of adaptation, design features. In this paper, the author suggests the algorithm for ship control as: meeting movement; the predetermined moving area; the environmental variation of a time function.

METHODOLOGY

It's considered the way of decisions based on the Mayer's approach in the minimum principle that allows bringing the interesting analytical solutions to numerical values. It's used the system of equations:

$$\left.\begin{aligned}\frac{dl}{dt} &= v_c - v_l \\ \frac{dv_c}{dt} &= -a_c v_c + k_c \omega_c \\ \frac{dv_l}{dt} &= -a_l v_l + k_l v_l\end{aligned}\right\} \qquad (1)$$

Where: *l*-the distance between the leader and the vessel;
v_c - speed of own ship;
v_l - the speed of the leader;
ω_l - Manipulated leader (speed of diesel engine);
ω_c - Manipulated vessel (speed of diesel engine);
a_c, a_l, k_c, k_l - Constant coefficients.
The impact of restrictions are applied on the ship control

$$|\omega_l| \le \Omega_{l\max};\ |\omega_c| \le \Omega_{c\max} \qquad (2)$$

The value ω_l is determined by a fixed subsystem based on a predetermined motion technology of ships. The control ω_c is selected by the captain of vessels. It must be such that the time interval $t = 0 \div t_k$ the following conditions are met the requirement $v_l = v_c,\ l \cong 0$
Here are t_k - the finishing time of controlled motion of ships.
Found a control law for speed of ship's diesel engine, which provides a minimum of time, during which there will be equality $v_l = v_c$ speeds and the distance between the "leader" and the following ship will be $l \cong 0$. The characteristics for the development of a mathematical model of the ship fuel consumption power plant.
With a decrease in the degree of automation that controls ship skipper on the basis of the program generated a stationary system, the dimension of the problem can be reduced by excluding from the general equation of a diesel engine, and the time constant is small in comparison with the time constant of the vessel.
It's introduced a new coordinate:

$$T = x \qquad (3)$$

Moreover, $T \,\square\, t_k$. Criterion-based control (3) has the form: $\Delta G = x\Big|_0^T \to \min$

The Hamiltonian:

$$H = (v_c - v_l)\psi_1 + (k_c\omega_c - a_c v_c)\psi_2 + (k_l\omega_l - a_l v_l)\psi_3 + \psi_4 \qquad (4)$$

The equations to determine the auxiliary variation:

$$\left.\begin{aligned}\frac{d\psi_1}{dt} = 0,\ \frac{d\psi_2}{dt} = -\psi_1 + a_c\psi_2, \\ \frac{d\psi_3}{dt} = \psi_1 + a_л\psi_3,\ \frac{d\psi_4}{dt} = 0.\end{aligned}\right\} \qquad (5)$$

The transversal condition is written:

$$[\delta x + H\delta + \psi_1\delta l + \psi_2\delta v_c + \psi_3\delta v_l + \psi_4\delta x]_0^T = 0$$

This problem has a first integral. Due to the fact that *T* - available at the right end and at *t = 0*, *v = constant*, $v_c = 0$, $l = 0$, and $t = T$, $v_l = v_c$, $l = l_k$ in equation (4) is possible when:

$$\psi_4 = -1, \quad H_T = 0. \qquad (6)$$

But *H = constant*, the section $t = 0 \div T$, so the first task is integral

$$(v_c - v_l)\psi_1 + (\kappa_c\omega_c - a_c v_c)\psi_2 + (\kappa_l\omega_l - a_l v_l)\psi_3 + \psi_4 = 0 \qquad (7)$$

The switching function of the manipulated variation is determined from such an expression:

$$\psi_2 = c_1 e^{a_c t} + \frac{1}{a_c}\psi_1 \qquad (8)$$

As follows from the system (5) $\psi_1 = const$, in the area $t = 0 \div T$

It's found constants c_1 and ψ_1 in equation (8). To do this, write the first integral for the time $t = t_n$, taking into account (6), where t_n - the time when the function ψ_2 changes the sign, that is, it switches control actions ω_c:

$$\psi_1(v_c - v_l) - 1 = 0$$

From whence

$$\psi_1 = \frac{1}{(v_c - v_l)_{t=t_n}} \tag{9}$$

At time $t = t_n$, $v_c = v_{c_{\max}}$.

Write the first integral for the time $t = 0$.

$$\kappa_c \omega_{c_0} \psi_{20} - v_l \psi_{10} - 1 = 0 \tag{10}$$

From equation (10) the initial value of ψ_{20} is found:

$$\psi_{20} = \frac{1 + v_l \psi_{10}}{\kappa_c \omega_{c_0}} \tag{11}$$

With the help of (8, 9, and 10), a constant c_1 is defined:

$$c_1 = \frac{v_{c_{\max}} - \kappa_c \omega_{c_0}}{a_c (v_{c_{\max}} - v_l) \kappa_c \omega_{c_0}} \tag{12}$$

The first integral equation and formula (8) is used to determine the control time $t = T$ and torque switching control action $t = t_n$. At the point in time, there is $t = T$.

$$\psi_{2T} = c_1 e^{a_c T} + \frac{1}{a_c} \psi_{1T} \tag{13}$$

Using the expression (7) for $t = T$, it's obtained:

$$\psi_{2T} = \frac{1}{\kappa_c \omega_{cT} - a_c v_c} \tag{14}$$

It's substituted equations (13), (9), (12) and (14) and solved the resulting expression for the unknown *T*.

$$T = \frac{1}{a_c} \ln \frac{q - d}{b} \tag{15}$$

In the formula (15) the following notations are used:

$$q = \frac{1}{\kappa_c \omega_{cT} - a_c v_c}, \quad b = \frac{v_{c_{\max}} - \kappa_c \omega_{c_0}}{a_c (v_{c_{\max}} - v_l) \kappa_c \omega_{c_0}}, \quad d = \frac{1}{a_c (v_{c_{\max}} - v_l)}. \tag{16}$$

Define the time switch control action ω_c. To do this, it's written equation (8) for the time $t = t_n$

$$\psi_{2_{t_n}} = c_1 e^{a_c t_n} + \frac{1}{a_c} \psi_{1_{t_n}}$$

But $t = t_n$, $\psi_2 = 0$ so:

$$t_n = \frac{1}{a_c} \ln \gamma \tag{17}$$

Where:

$$\gamma = \frac{\kappa_c \omega_{c_0} (v_{c_{\max}} - v_l)}{(v_c - v_l)(v_{c_{\max}} - \kappa_c \omega_{c_0})} \tag{18}$$

It can be found the value of the path, which must pass the ship to enter the control zone. From the system of equations (1), it's obtained:

$$l = \int_0^T (v_c - v_l)dt \pm l_n \tag{19}$$

l_n - the initial distance between the ship and the "leader".
It's considered the example. Using the obtained expression, it's found the numerical values of T, t_n and l, given the following factors:

$$a_c = 0.01733 \text{ l/s}, \ \kappa_c = 0.00284 \text{ m/s}$$

The initial values of the phase coordinate from the constraints (2) are:

$$t = 0, v_c = 0, v_l = 2.5 \text{ m/s}, \ \omega_c = 36.6 \text{ l/s}, \ l_n = 0.$$

The final values of the phase coordinate are:

$$t = T, \quad v_c = 2.5 \text{ m/s} \ v_l = 2.5 \text{ m/s}, \ \omega_c = 15.4 \text{ l/s}$$

The maximum value of the velocity of longitudinal movement of the vessel:

$$v_{c_{\max}} = 5.92 \text{ m/s}$$

The values of the vector components of ψ in considering (11) at time $t = 0$ are:

$$\psi_{10} = 0.2924, \ \psi_{20} = 16.66$$

The velocity of the "leader" takes constant time interval equal to $t = 0 \div T$ and $v_l = 2.5$ m/s
Doing calculations using formulas (15), (16), (17), (18), (19), it can be obtained the following variables values of interest:

$$T = 272.6 \text{ s}, \ t_n = 253.1 \text{ s}, \ l = 1117.2 \text{ m}.$$

Control of the Vessel's Side at a Meeting Movement

These controls allow the required lateral displacement of the vessel in a mooring of a vessel to another, or when approaching to a predetermined point mooring, or fuel during filling of the tanker vessel. Availability of this algorithm in a stationary system, such as the CPU controller of navigable channel to determine location on the track committed data processing steps and their duration in time, properly consider constraints on the phase position and the geometrical dimensions of the fairway.

Stationary system in these conditions can take on a coordinating role in solving these problems and thereby increase the effectiveness of their implementation in the real world.
Figure 1 presented a plan speeds ship in longitudinal and transverse directions. It follows from this plan, the rate of lateral movement will be determined as

$$v_h = v_0 \sin|(\alpha_0 - \alpha_1)| \tag{20}$$

Equation (20) can easily be simplified, considering that the angle increment rate $\phi = \alpha_0 - \alpha_1$ is sufficiently small and the conditions of the river channel do not exceed 5^0 - 7^0. Then it can be obtained the following:

$$v_h = v_0 \phi \tag{21}$$

The magnitude of lateral movement is determined by the following equation:

$$h = \int_{t_1}^{t_2} v_0 \phi dt \tag{22}$$

If we take the rate of longitudinal motion constant, the differential equation for the lateral movement will be linear:

$$\frac{dh}{dt} = v_0 \phi \tag{23}$$

Given the level of control the initial system of equations is a lateral movement:

$$\left.\begin{aligned} &\frac{dh}{dt} = v_0\phi,\ \frac{d\phi}{dt} = \Omega, \\ &\frac{d\Omega}{dt} = -a_{31}\Omega + \kappa^{\beta}\beta_p. \end{aligned}\right\} \tag{24}$$

Where $a_{31} = 1/\Omega,\quad \kappa^{\beta} = \kappa_{\Omega}^{\beta}/T_{\Omega}$

We find $\beta_p = \beta_p(t)$ control which takes an object from the initial state: $t = 0, h = 0, \phi = \phi_0, \Omega = 0,$

To the final state:

$$t = T, h = h_T, \Delta G = h_T - h = \min \tag{25}$$

And $T, \phi,\ \Omega$ - are free

Functional select in this form:

$$J = \phi_T \tag{26}$$

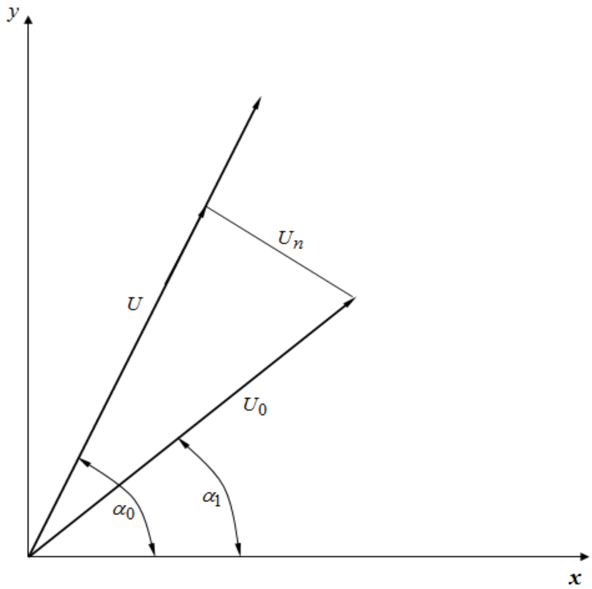

Figure 1. Terms of speed of the vessel in the longitudinal and transverse directions

It's assumed that the curves h, ϕ, Ω are smooth functions (due to the large inertia of the object, this conclusion is true), and the function β_p is piecewise continuous. Basing on those functions the functional (26) is defined.

Among the acceptable curves it's wanted to find the one that minimizes the functional (26). Physically, this means the need for a given lateral movement at the lowest-possible changes azimuth vessel. This is necessary when the ship navigates through a narrow fairway to avoid grounding in the shallows and colliding to the coast and slopes.

The control action in the process control will become:

$$-\beta_{p_{\max}} \le \beta_p \le \beta_{p_{\max}} \tag{27}$$

It's formed the Hamiltonian *H*:

$$H = v_0\phi\alpha_1 + \Omega\alpha_2 + (\kappa^{\beta}\beta_p - a_{31}\Omega)\alpha_3 \tag{28}$$

Euler-Lagrange equations:

$$\left.\begin{aligned} &\frac{d\alpha_1}{dt} = 0,\ \frac{d\alpha_2}{dt} = -v_0\alpha_1, \\ &\frac{d\alpha_3}{dt} = -\alpha_2 - a_{31}\alpha_3 \end{aligned}\right\} \tag{29}$$

The equations of the problem are the first integral:

$$v_0\phi\alpha_1+\Omega\alpha_2+(\kappa^{\beta}\beta_p-a_{31}\Omega)\alpha_3=c \tag{30}$$

Terms of transversal tasks:

$$[\delta\phi-cdt+\alpha_1\delta h+\alpha_2\delta\phi+\alpha_3\delta\Omega]_0^T=0 \tag{31}$$

Since the variable h at time T is fixed, then the conditions (31) can be written as

$$[(1+\alpha_2)\delta\phi+c\delta t+\alpha_3\delta\Omega]_0^T=0 \tag{32}$$

Equation (32) in mind the rules of selection is possible if only when

$$\alpha_{2T}=-1,\quad c=0,\quad \alpha_{3T}=0. \tag{33}$$

It's found the control law:

$$\frac{\partial H}{\partial\beta_p}=\kappa^{\beta}\alpha_3=0 \tag{34}$$

On the basis of (34), it's obtained:

$$\left.\begin{array}{l}\beta_{\rm p}=\beta_{\rm p_{max}}\ \text{ at }\ \kappa^{\beta}\alpha_3>0,\\ \beta_{\rm p}=-\beta_{\rm p_{max}}\ \text{ at }\ \kappa^{\beta}\alpha_3<0.\end{array}\right\} \tag{35}$$

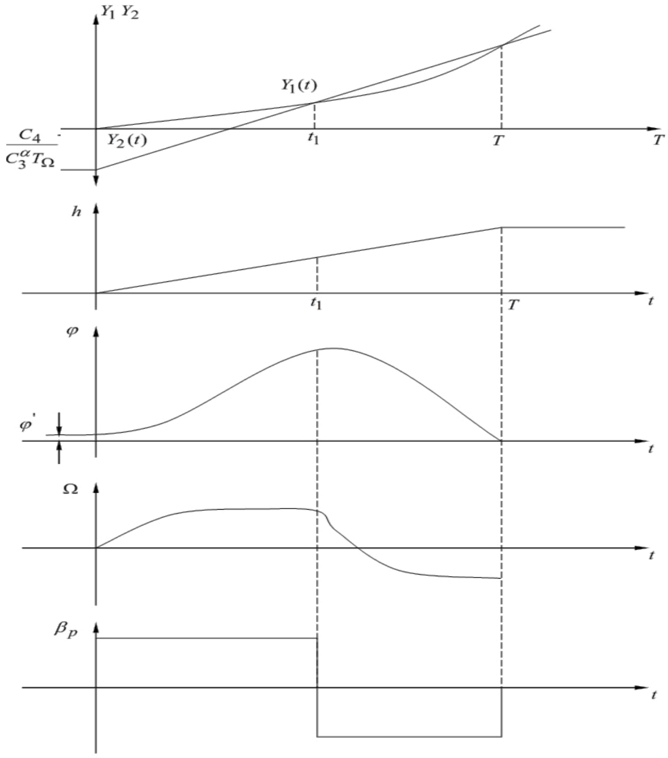

Figure 2. The phase position of the lateral movement of ship

The function of switching control action is investigated, it is transformed the equations (29) to the form:

$$\frac{d^2\alpha_3}{dt^2}-a_{31}\frac{d\alpha_3}{dt}-v_0c_1^{\alpha}=0 \tag{36}$$

The solution α_3 from the integral equation (36) is

$$\alpha_3=c_3^2e^{a_{31}t}-\frac{v_0}{a_{31}}c_1^{\alpha}t-\frac{v_0c_1+a_{31}c_2}{a_{31}^2} \tag{37}$$

It's found the roots of the equation (37), provided that at the time of sign change of control action $\alpha_3=0$. To do this, it's transformed (37) as:

$$\left.\begin{aligned} c_3^2 T_\Omega e^{a_{31}t} - v_0 c_1^\alpha t + c_4 = 0, \\ e^{a_{31}t} = \frac{v_0 c_1^\alpha}{c_3^\alpha T_\Omega} t - \frac{c_4}{c_3^\alpha T_\Omega} \end{aligned}\right\} \quad (38)$$

The notation is given:

$$y_1 = e^{a_{31}t}; \quad y_2 = \frac{v_0 c_1^\alpha}{c_3^\alpha T_\Omega} t - \frac{c_4}{c_3^\alpha T_\Omega}$$

Figure 2 is graphically constructed according to $y_1 = y_1(t)$ and $y_2 = y_2(t)$. The points of intersection of these curves define the moments of switching control from $+\beta_{p_{\max}}$ to $-\beta_{p_{\max}}$.

Ship Control to Sail in a Predetermined Moving Area

The control task ensures that there is a safe environment differences in cramped vessels sailing conditions, when moving along a predetermined path, for example, to pass the obstacles encountered with known coordinates in advance or obstacles that appear randomly on the fairway. As restrictions on river conditions may be the coast, the width of the fairway, counter and passing ships, waterworks.

Create a model of divergence of two ships *P* and *E*.

$$D = v_p \cos\psi + v_E \cos\phi \quad (39)$$

Where

D - the distance between the ships

v_p - speed of the vessel *P;*

v_E - speed of the vessel *E;*

ψ, ϕ - course angle.

It's assumed that the discrepancy has occurred at time *T* at a distance of perpendicular D_T if, starting from a certain point in time $T + \Delta t$, the distance *D* begins to increase. And Δt can be arbitrarily small.

The first problem is solved without variation inertia of bearings ψ and ϕ. Let

$$U = \cos\psi, \quad V = \cos\phi \quad (40)$$

Place here the restrictions on control

$$|U| \le 1, \quad |V| \le 1$$

It's assumed the initial and final conditions for the solution of problem

$$\left.\begin{aligned} at \ \ t = 0 \quad D = D_0, \\ at \ \ t = T \quad D = D_t \end{aligned}\right\} \quad (41)$$

It will be solved the quadratic functional

$$J = \int_0^T \left[(D + D_T)^2 + cU^2 + qV^2 \right] dt \quad (42)$$

In order to find the controls of *U* and *V* that minimizes the function (42), it should be used the minimum principle. The Hamiltonian is written:

$$H = \left[(D + D_T)^2 + cU^2 + qV^2 \right] x_0 + (v_p U + v_E V) x_2 \quad (43)$$

From this the control function is obtained

$$U = -\frac{v_p}{2c} x_1, \quad V = -\frac{v_E^2}{2q} x_1 \quad (44)$$

The equations for determining the components of the vector $x = x(x_0, x_1)$ will be:

$$\left.\begin{aligned} \dot{x} &= 0, \\ \dot{x}_1 &= -(2D + 2D_T)x_0 \end{aligned}\right\} \quad (45)$$

Accept $x_0 = 1$ and write the original system of differential equations:

$$\left.\begin{aligned} \dot{D} &= -\left(\frac{v_p^2}{2c} + \frac{v_E^2}{2q}\right)x_1, \\ \dot{x}_1 &= -2D - 2D_T \end{aligned}\right\} \quad (46)$$

It's introduced by the variation law of the distance *D*, for which it must be excluded x_1 from the system of equations (46):

$$\ddot{D} = \left(\frac{v_p^2}{c} - \frac{v_E^2}{q}\right)D + \left(\frac{v_p^2}{c} + \frac{v_E^2}{q}\right)D_T \quad (47)$$

Let

$$a = \frac{1}{v_p^2/c - v_E^2/q} \quad (48)$$

Then equation (47) is as follows:

$$a\ddot{D} - D = D_T \quad (49)$$

It's obtained the roots of the characteristic equation:

$$P_{1,2} = \pm 1/\sqrt{a} \quad (50)$$

In order to ensure the stability of the control on the basis of the positive root, it's excluded:

$$D = c_1 e^{(-1/\sqrt{a})t} + D_T \quad (51)$$

Considering the initial conditions, the constant c_1 is found:

$$c_1 = D_0 - D_T$$

Then a solution of (51) will look like this:

$$D = (D_0 - D_T)c_1 e^{(-1/\sqrt{a})t} + D_T \quad (52)$$

Using the equation (52) to find the solution x_1

$$\dot{x}_1 = -2(D_0 - D_T)e^{(-1/\sqrt{a})t} - 4D_T, \quad (53)$$

$$x_1 = 2(D_0 - D_T)\sqrt{a}\, e^{(-1/\sqrt{a})t} - 4D_T + c_2 \quad (54)$$

The control (44) in view of (54) will be as follows:

$$\left.\begin{aligned} U &= -\frac{v_p}{c}[(D_0 - D_T)\sqrt{a}\, e^{(-1/\sqrt{a})t} - 2D_T t + c_2/2, \\ V &= -\frac{v_E}{q}[(D_0 - D_T)\sqrt{a}\, e^{(-1/\sqrt{a})t} - 2D_T t + c_2/2 \end{aligned}\right\} \quad (55)$$

Speed v_p and v_E depends on the mode of the power plant which are determined by speed characteristics.

The controls of the two vessels in avoidance are programmed. It's required the initial information of the initial status D_0, the final perpendicular distance D_T and speed of the first and second ships or their power plant.

The advantage of these controls is unnecessary for continuous measurements of the velocities v_p and v_E.

The second task is to find the control that takes into account the variable inertia of the angles ψ and ϕ. Differential equations of the ships in avoidance write as:

$$\left.\begin{aligned}\dot{D}=v_p\cos\psi+v_E\cos\phi,\\ \dot{\psi}=U,\ \dot{\phi}=V.\end{aligned}\right\}\tag{56}$$

The solution is considered the function (42). The initial condition is:

If $$t=0,\quad \psi=\psi_0,\quad \phi=\phi_0,\quad D=D_0.\tag{57}$$

And the last condition is

If $$t=T,\quad \psi=\frac{\pi}{2},\quad \phi=\frac{\pi}{2},\quad D=D_T.\tag{58}$$

Limits on control:

$$|U|\le U_{\max},\quad |V|\le V_{\max}\tag{59}$$

The Hamiltonian:

$$H=x_0\left[(D+D_T)^2+cU^2+qV^2\right]+x_1(v_p\cos\psi+v_E\cos\phi)+x_2U+x_3V.\tag{60}$$

It's found the controls *U* and *V*, which deliver the maximum of *H*:

$$\left.\begin{aligned}\frac{\partial H}{\partial U}=2cUx_0+x_2=0,\quad U=-x_2/2cx_0,\\ \frac{\partial H}{\partial V}=2qVx_0+x_3=0,\quad V=-x_3/2qx_0.\end{aligned}\right\}\tag{61}$$

The equation of finding x_i is written:

$$\left.\begin{aligned}\dot{x}_0=0,\ \ \dot{x}_1=-(2D-2D_T)x_0,\\ \dot{x}_2=v_px_1\sin\psi,\ \ \dot{x}_3=v_Ex_1\sin\phi.\end{aligned}\right\}\tag{62}$$

Based on the fact that the vector $x=x(x_0,\ldots,x_3)$ does not depend on x_0, it's assumed $x_0=-1$. Using equations of (56, 61, and 62), it's obtained:

$$\left.\begin{aligned}\dot{D}=v_p\cos\psi+v_E\cos\phi,\ \ \dot{\psi}=1/2cx_2,\ \ \dot{\phi}=1/2q\,x_3,\\ \dot{x}_1=2D-2D_T,\ \ \dot{x}_2=v_px_1\sin\psi,\ \ \dot{x}_3=v_Ex_1\sin\phi.\end{aligned}\right\}\tag{63}$$

The system of equations (63) can be integrated in elementary functions. After integrating, it's obtained the control function of time. In order to implement, it's required to know the initial and final conditions for solving the problem, the speed of ships and their corresponding modes of Main Engine.

The second task gives the opportunity to get a control that provides discrepancy with greater precision, since the dimension of the problem is higher than the first.

The Ship Control Mode in the Head-On Navigation with Environmental Changes as a Function of Time

Considering the ship dynamic as a mobile system is especially necessary when it comes to solving the above tasks. The exact implementation of the timetable is often significantly more cost-effective than the longitudinal motion control modes, which is intended to fuel economy. The control task of the dynamics of longitudinal motion of the mobile system was considered in. However, the problems of control are poorly understood and especially the problem of dynamics of energy in motion. No quantitative estimates of control is dynamic object, the qualitative side of the solutions obtained is too general, does not allow to use these results in the synthesis of both mobile and stationary control systems.

It's put the problem of finding the control mode of (Diesel Engine Unit) DEU's operation for longitudinal movement of the ship to the complex conditions of some uncertainty in the initial information about the environment. As shown, the characteristics are determined by the generalized environmental parameter. It's assumed that this parameter is a function of time. This corresponds to the action on the set of disturbances or wind. Figure 3 shows the dependence of $\frac{dG}{dt}$ on ω and *h*. The value can be represented as:

$$\frac{dG_m}{dt} = \kappa_q^{\omega}\omega^2 \quad (64)$$

and the value of ω:

$$\omega = \kappa_q^h h \quad (65)$$

The equation can be written in fuel consumption:

$$\frac{dG_m}{dt} = \kappa_g^{\omega}\kappa_g^h h\omega \quad (66)$$

where *h* - the movement of rail diesel injection pumps.

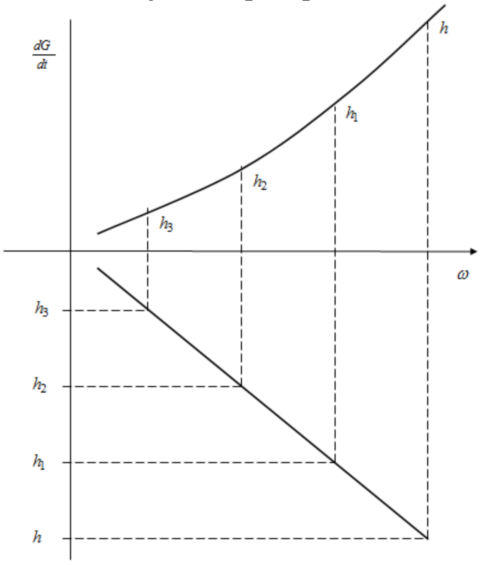

Figure. 3. Specifications for mathematical model of Ship's fuel power plant.

The equations governing the motion of the ship's complex dynamics DEU considering equations (66), are as follows:

$$\left.\begin{aligned} \frac{dv}{dt} &= -\frac{1}{T}v + \frac{\kappa_c^{\omega}}{T_c}\omega - \frac{\kappa_c^{\psi}}{T_c}\psi(t), \\ \frac{d\omega}{dt} &= -\frac{1}{T_g}\omega + \frac{\kappa_q^h}{T_g}h + \frac{\kappa_q^v}{T_g}v, \\ \frac{dG_m}{dt} &= \kappa_g^{\omega}\kappa_g^h h\omega, \quad \frac{ds}{dt} = v. \end{aligned}\right\} \quad (67)$$

It can be shown that the equation (67) is provided by equations with variable parameters that are non-stationary. To do this, it'd do the following formal conversion in the first equation of the system (67):

$$\frac{1}{\psi(t)}\frac{dv}{dt} + \frac{1}{T_c\psi(t)}v = \frac{1}{T_c}\left(\frac{\kappa_c^{\omega}}{\psi(t)}\omega - \kappa_c^{\psi}\right) \quad (68)$$

The equation (68) confirms the conclusion that the system (67) is non-stationary. It's posed the problem of finding *h* at the controls as follows: finding $h(0 \le h \le h_{\max})$, which provides for a given travel time *T*, a minimum fuel consumption over the period of time $0 \div T$. The problem is formulated as a problem of Mayer and the role of function is the dependent relation:

$$\Delta G_m = G_m(v,t) \tag{69}$$

The boundary conditions are

$$\left.\begin{array}{r} \text{at the left end } t=0, v_0=\omega_0=G_{m_0}=S_0=0 \\ \text{on the right end } t=T, v_T, \omega_T \; - free \;\; S=S_T \end{array}\right\} \tag{70}$$

It's introduced a new variation $Z=t$, and then the equation (67) will be:

$$\left.\begin{array}{l} \frac{dv}{dt}=-\frac{1}{T_c}v+\frac{\kappa_c^{\omega}}{T_c}\omega-\frac{\kappa_c^{\psi}}{T_c}\psi(t), \\ \frac{d\omega}{dt}=-\frac{1}{T_g}\omega+\frac{\kappa_q^h}{T_g}h+\frac{\kappa_q^v}{T_g}v, \\ \frac{dG_m}{dt}=\kappa_g^{\omega}\kappa_g^h\omega h, \; \frac{ds}{dt}=v, \; \frac{dz}{dt}=1. \end{array}\right\} \tag{71}$$

Due to the fact that it is necessary to check whether there is a control called special. It's writing for the system (71), the Hamiltonian:

$$H=\left[-\frac{1}{T_c}v+\frac{\kappa_c^{\omega}}{T_c}\omega-\frac{\kappa_c^{\psi}}{T_c}\psi(t)\right]\alpha_1+\left[-\frac{1}{T_g}\omega+\frac{\kappa_q^h}{T_g}h+\frac{\kappa_g^h}{T_g}v\right]\alpha_2+\kappa_g^{\omega}\kappa_g^h\omega h\alpha_3+v\alpha_4+\alpha_5 \tag{72}$$

The equations for the vector α:

$$\left.\begin{array}{l} \frac{d\alpha_1}{dt}=\frac{1}{T_c}\alpha_1-\frac{\kappa_g^v}{T_g}\alpha_2-\alpha_4, \\ \frac{d\alpha_2}{dt}=-\frac{\kappa_c^{\omega}}{T_c}\alpha_1+\frac{1}{T_g}\alpha_2-\kappa_g^{\omega}\kappa_g^h h\alpha_3, \\ \frac{d\alpha_3}{dt}=0, \; \frac{d\alpha_4}{dt}=0, \; \frac{d\alpha_5}{dt}=\frac{\kappa_c^{\psi}}{T_c}\frac{d\psi(t)}{dt}\alpha_1. \end{array}\right\} \tag{73}$$

The transversal conditions are:

$$[(1+\alpha_3)\delta G_m - c\delta t+\alpha_1\delta v+\alpha_2\delta\omega+\delta s\alpha_4+\alpha_5\delta t]_0^T=0 \tag{74}$$

These terms and conditions are written on the assumption that the problem is the first integral:

$$\left[-\frac{1}{T_c}v+\frac{\kappa_c^{\omega}}{T_c}\omega-\frac{\kappa_c^{\psi}}{T_c}\psi(t)\right]\alpha_1+\left[-\frac{1}{T_g}\omega+\frac{\kappa_g^h}{T_g}h+\frac{\kappa_g^v}{T_g}v\right]\alpha_2+\kappa_g^{\omega}\kappa_g^h h\omega\alpha_3+v\alpha_4+\alpha_5=c \tag{75}$$

In accordance with the boundary conditions, the transversal condition (74) will be

$$[(1+\alpha_3)\delta G_m+(\alpha_5+c)\delta t+\alpha_1\delta v+\alpha_2\delta\omega]_0^T=0 \tag{76}$$

Due to the contingent variations of $\delta G_m, \delta v, \delta\omega$ in equality (76), it's only possible when $\alpha_{4T}=0, \alpha_{5T}=c=0, \alpha_{3T}=-1, \alpha_{2T}=0, \alpha_{1T}=0$ (according to the condition of the maximum at T - fixed). It will be located the control by h. The maximum condition H in h is

$$\frac{\partial H}{\partial h}=\frac{\kappa_g^h}{T_g}\alpha_2+\kappa_g^{\omega}\kappa_g^h\omega\alpha_3=0 \tag{77}$$

Hence it can be argued that the function will reach to minimum in the control that varies as the following law:

$$\left.\begin{array}{l} h=h_{\max} \; at \; \frac{\kappa_g^h}{T_g}\alpha_2+\kappa_g^{\omega}\kappa_g^h\omega\alpha_3>0, \\ h=0 \; at \; \frac{\kappa_g^h}{T_g}\alpha_2+\kappa_g^{\omega}\kappa_g^h\omega\alpha_3<0, \end{array}\right\} \tag{78}$$

The transversal conditions make it possible to find the solutions for all the components of the vector α and, in particular, α_2 and α_3.
It's investigating the possibility of singular controls in systems of equations (71) and (72), which transform the Hamiltonian

$$H=\left[-\frac{1}{T_c}v\alpha_1+\frac{\kappa_c^{\omega}}{T_c}\omega\alpha_1-\frac{\kappa_c^{\psi}}{T_c}\psi(t)\alpha_1-\frac{1}{T_g}\omega\alpha_2+\frac{\kappa_q^{v}}{T_g}v\alpha_2+v\alpha_4+\alpha_5\right]+h\left[\frac{\kappa_g^{h}}{T_g}\alpha_2+\kappa_g^{\omega}\kappa_g^{h}\omega\alpha_3\right] \tag{79}$$

Let

$$\left.\begin{aligned}H_0&=-\frac{1}{T_c}v\alpha_1+\frac{\kappa_c^{\omega}}{T_c}\omega\alpha_1-\frac{\kappa_c^{\psi}}{T_c}\psi(t)\alpha_1-\frac{1}{T_g}\omega\alpha_2+\frac{\kappa_g^{v}}{T_g}v\alpha_2+v\alpha_4+\alpha_5,\\ H_1&=\frac{\kappa_g^{h}}{T_g}\alpha_2+\kappa_g^{\omega}\kappa_g^{h}\omega\alpha_3.\end{aligned}\right\} \tag{80}$$

Using the Poisson brackets for finding specific controls, it's obtained:

$$\frac{d}{dt}H_1=\{H_1,H\}=0 \tag{81}$$

The Poisson bracket (81) is expanded:

$$\{H_1,H\}=\frac{\kappa_g^{h}}{T_g}\frac{d\alpha_2}{dt}+\kappa_g^{\omega}\kappa_g^{h}\frac{d\omega}{dt}\alpha_3+\kappa_g^{\omega}\kappa_g^{h}\omega\frac{d\alpha_3}{dt}=0$$

The special control can appear only in terms of even order derivative, so find:

$$\frac{d^2H_1}{dt^2}=-\{\{H_0,H_1\},H\}=\frac{\kappa_g^{h}}{T_g}\frac{d^2\alpha_2}{dt^2}+\kappa_g^{\omega}\kappa_g^{h}\frac{d^2\omega}{dt^2}\alpha_3+\kappa_g^{\omega}\kappa_g^{h}\frac{d\omega}{dt}\frac{d\alpha_3}{dt}=0$$

As $\frac{d\alpha_3}{dt}=0$, it's obtained

$$\frac{d^2\alpha_2}{dt^2}=-\kappa_g^{h}/T_g\frac{d^2\alpha_2}{dt^2}+\kappa_g^{\omega}\kappa_g^{h}\frac{d^2\omega}{dt^2}\alpha_3=0 \tag{82}$$

In which:

$$\left.\begin{aligned}\frac{d^2\alpha_2}{dt^2}&=-\frac{\kappa_c^{\omega}}{T_c}\frac{d\alpha_1}{dt}+\frac{1}{T_g}\frac{d\alpha_2}{dt}-\frac{\kappa_g^{\omega}}{T_g}\frac{dv}{dt},\\ \frac{d^2\omega}{dt^2}&=-1/T_g\frac{d\omega}{dt}+\kappa_g^{h}/T_g\frac{dh}{dt}+\kappa_g^{v}/T_g\frac{dv}{dt}\end{aligned}\right\} \tag{83}$$

Substituted (83) corresponding to the equation (71) and (73), it's obtained the following system:

$$\left.\begin{aligned}\frac{d^2\alpha_2}{dt^2}&=-\left(\frac{\kappa_c^{\omega}}{T_c^2}-\frac{\kappa_c^{\omega}}{T_gT_c}\right)\alpha_1+\left(\frac{\kappa_g^{v}\kappa_c^{\omega}}{T_gT_c}+\frac{1}{T_g^2}\right)\alpha_2+\kappa_c^{\omega}/T_c\,\alpha_4-\kappa_g^{\omega}\kappa_g^{h}\frac{dh}{dt}\alpha_3-\frac{\kappa_g^{\omega}\kappa_g^{h}\alpha_3}{T_g}h\\ \frac{d^2\omega}{dt^2}&=-\left(\frac{\kappa_g^{v}}{T_g^2}-\frac{\kappa_g^{v}}{T_gT_c}\right)v+\left(\frac{1}{T_g^2}+\frac{\kappa_c^{\omega}\kappa_g^{v}}{T_gT_c}\right)\omega-\frac{\kappa_c^{\psi}\kappa_g^{v}}{T_gT_c}\psi(t)-\frac{\kappa_g^{h}}{T_g^2}h+\frac{\kappa_g^{h}}{T_g}\frac{dh}{dt}\end{aligned}\right\} \tag{84}$$

As $\alpha_4=c$ in all section control with the transversal effect condition is leading to $\alpha_{4T}=0$, it's obtained:

$$\frac{d^2H_1}{dt^2}=b_1^H\alpha_1+b_2^H\alpha_2+c_1^Hv-c_3^H\psi(t)+(c_5^H-b_3^H)\frac{dh}{dt}-b_4^H+c_4^H)h=0 \tag{85}$$

In which

$$\left.\begin{aligned} b_1^H &= \frac{\kappa_g^h}{T_g}\left(-\frac{\kappa_c^\omega}{T_c^2}-\frac{\kappa_c^\omega}{T_g T_c}\right);\ b_4^H = \alpha_3 \kappa_g^\omega \kappa_g^h \frac{\kappa_g^h}{T_g^2};\ b_3^H = \frac{\kappa_g^h}{T_g}\kappa_g^\omega \kappa_g^h \alpha_3; \\ b_4^h &= \alpha_3 \kappa_g^\omega \kappa_g^h \frac{\kappa_g^h}{T_g^2};\ c_1^H = \kappa_g^\omega \kappa_g^h \alpha_3 \left(-\frac{\kappa_g^v}{T_g^2}-\frac{\kappa_g^v}{T_g T_c}\right);\ c_2^H = \kappa_g^h \kappa_g^\omega \left(\frac{1}{T_g^2}+\frac{\kappa_c^\omega \kappa_g^v}{T_g T_c}\right)\alpha_3; \\ c_3^H &= \kappa_g^h \kappa_g^\omega \alpha_3 \frac{\kappa_c^\psi \kappa_g^v}{T_g T_c};\ c_4^H = \kappa_g^h \kappa_g^\omega \alpha_3 \frac{\kappa_g^h}{T_g^2};\ c_5^H = \kappa_g^\omega \kappa_g^h \frac{\kappa_g^h}{T_g}\alpha_3;\ c_5^H = b_3^H. \end{aligned}\right\} \tag{86}$$

From equation (85), it is finding the special control:

$$h = -\frac{b_1^H \alpha_1 + b_2^H \alpha_2 + c_1^H v + c_2^H \omega - c_3^H \psi}{b_4^H + c_4^H} \tag{87}$$

It should be examined the specific control at G. Kelly's optimal condition. If the inequality (88) is performed the specific control will be optimized:

$$\frac{\partial}{\partial H}\frac{d^2}{dt^2}\frac{\partial H}{\partial h} \ge 0 \tag{88}$$

From (77), (78) and $\frac{\partial H}{\partial h} = H_1$, corresponding to (85) it is obtained:

$$\frac{d^2}{dt^2}\frac{\partial H}{\partial h} = \frac{\partial^2}{\partial t^2} H_1 \tag{89}$$

Then

$$\frac{\partial}{\partial h}\frac{d^2}{dt^2} H_1 = -(b_4^H + c_4^H) \tag{90}$$

From equation (86), it's obtained

$$b_4^H = \kappa_g^\omega \kappa_g^h \alpha_3 \frac{\kappa_g^h}{T_g^2},\quad c_4^H = \kappa_g^h \kappa_g^\omega \alpha_3 \frac{\kappa_g^h}{T_g^2}$$

In accordance with (73) and the transversal condition (76), it's $\alpha_3 = -1$ so

$$b_4^H + c_4^H > 0 \tag{91}$$

It means that the G. Kelly's condition is satisfied and the specific control is optimal.

With a decrease in the level of automation, where the ship is controlled by the program generated from a stationary system, the dimension of the problem can be reduced by excluding from the system (71) of a diesel engine that its time constant is too small in comparison with the time constant of the vessel.

Statement of the problem search controls that minimize the function (69) will be considered similar. As a control action is used the frequency rotation of diesel. The initial system of equations is:

$$\left.\begin{aligned} \frac{dv}{dt} &= -\frac{1}{T_c}v + \frac{\kappa_c^\omega}{T_c}\omega - \frac{\kappa_c^\psi}{T_c}\psi(t), \\ \frac{dG_m}{dt} &= \kappa_g^\omega \omega^2,\ \frac{ds}{dt} = v,\ \frac{dz}{dt} = 1. \end{aligned}\right\} \tag{92}$$

The limited condition applied for the control is following:

$$0 \le \omega \le \omega_{\max} \tag{93}$$

The transversal condition:

$$[(1+\alpha_2)\delta G_m - c\delta t + \alpha_1 \delta v + \delta s \alpha_3 + \delta t \alpha_4]_0^T = 0 \tag{94}$$

The Hamiltonian system (92) is the following:

$$H = \left[-\frac{1}{T_c}v + \frac{\kappa_c^\omega}{T_c}\omega - \frac{\kappa_c^\psi}{T_c}\psi(t)\right]\alpha_1 + \kappa_g^\omega \omega^2 \alpha_2 + v\alpha_3 + \alpha_4 \tag{95}$$

Condition (94) is written on the assumption that the problem is the first integral:

$$\left[-\frac{1}{T_c}v+\frac{\kappa_c^{\omega}}{T_c}\omega-\frac{\kappa_c^{\psi}}{T_c}\psi(t)\right]\alpha_1+\kappa_g^{\omega}\omega^2\alpha_2+v\alpha_3+\alpha_4=c \qquad (96)$$

In accordance with the boundary conditions of dependence (94) will be the:

$$[(1+\alpha_2)\delta G_m+(\alpha_4-c)\delta t+\alpha_1\delta v+\delta s\alpha_3]_0^T=0 \qquad (97)$$

Due to the contingent variations of $\delta G_m, \delta v$, equality (97) is only possible when $\alpha_{3T}=0, \alpha_{1T}=0, \alpha_{2T}=-1, \alpha_{4T}=c=0$. The equations for determining the components of the vector will be:

$$\left.\begin{aligned}\frac{d\alpha_1}{dt}=\frac{1}{T_c}\alpha_1-\alpha_3,\ \frac{d\alpha_2}{dt}=0,\\ \frac{d\alpha_3}{dt}=0,\ \frac{d\alpha_4}{dt}=\frac{\kappa_c^{\psi}}{T_c}\frac{d\psi(t)}{dt}\alpha_1\end{aligned}\right\} \qquad (98)$$

It's found the control by ω. The maximum condition H by ω will be:

$$\frac{\partial H}{\partial\omega}=\frac{\kappa_c^{\omega}}{T_c}\alpha_1+2\kappa_g^{\omega}\omega\alpha_2=0 \qquad (99)$$

Hence it's obtained the variation law of ω

$$\left.\begin{aligned}\omega=\omega_{\max}\ \textit{when}\ \frac{\kappa_c^{\omega}}{T_c}\alpha_1+2\kappa_q^{\omega}\omega\alpha_2>0,\\ \omega=0\ \textit{when}\ \frac{\kappa_c^{\omega}}{T_c}\alpha_1+2\kappa_g^{\omega}\omega\alpha_2<0.\end{aligned}\right\} \qquad (100)$$

Relying on the transversal condition, it will be found the solution for all components of vector α

It's investigated the system (92), (98) and (100) on the possibility of specific controls. The Hamiltonian is written as:

$$H=\left[-\frac{1}{T_c}v\alpha_1-\frac{\kappa_c^{\psi}}{T_c}\psi(t)\alpha_1+v\alpha_3+\alpha_4\right]+\left[\alpha_1\kappa_c^{\psi}/T_c+\kappa_g^{\omega}\omega\alpha_2\right]\omega \qquad (101)$$

Let

$$\left.\begin{aligned}H_0=-\frac{1}{T_c}v\alpha_1-\frac{\kappa_c^{\psi}}{T_c}\psi(t)\alpha_1+v\alpha_3+\alpha_4,\\ H_1=\frac{\kappa_c^{\omega}}{T_c}\alpha_1+\kappa_g^{\omega}\omega\alpha_2.\end{aligned}\right\} \qquad (102)$$

According to equations (98) $\alpha_2=c_2^{\alpha}$, but as the transversal condition implies that $\alpha_{2T}=-1$, so

$$c_2^{\alpha}=-1 \qquad (103)$$

On the basis of equation (103), the second equation (102) is

$$H_1=\frac{\kappa_c^{\omega}}{T_c}\alpha_1-\kappa_g^{\omega}\omega \qquad (104)$$

It'd be found

$$\frac{d}{dt}H_1=\frac{\kappa_c^{\omega}}{T_c}\frac{d\alpha_1}{dt}-\kappa_g^{\omega}\frac{d\omega}{dt}=0 \qquad (105)$$

And

$$\frac{d^2}{dt^2}H_1=\frac{\kappa_c^{\omega}}{T_c}\frac{d^2\alpha_1}{dt^2}-\kappa_g^{\omega}\frac{d^2\omega}{dt^2}=0 \qquad (106)$$

RESULTS

Using the equation (98), the expression (106) can be written in the form:

$$\frac{d^2}{dt^2}H_1 = \kappa_g^{\omega}\frac{d^2\omega}{dt^2} - \frac{\kappa_c^{\omega}}{T_c^3}\alpha_3 = 0 \qquad (107)$$

It's taken into account the conditions (94) and written the solution of the first equation of (98):

$$\alpha_1 = c_1^{\alpha} e^{t/T_c} \qquad (108)$$

Then, a special control is defined as

$$\omega = \frac{\kappa_c^{\omega} c_1^{\alpha}}{T_c^5 \kappa_g^{\omega}} e^{t/T_c} + c_2 t + c_3 \qquad (109)$$

Where c_1, c_2, c_3 are constant of integration.
In this case, it's investigated the optimal control G. Kelly's condition:

$$\frac{d^2}{dt^2}\frac{\partial H}{\partial \omega} = \kappa_g^{\omega}\frac{d^2\omega}{dt^2} - \frac{\kappa_c^{\omega}}{T_c^3}\alpha_1 \qquad (110)$$

and itself the G. Kelly's condition

$$\frac{\partial}{\partial \omega}\frac{d^2}{dt^2}H_1 = 0 \qquad (111)$$

Special controls on the basis of (111), defined by (109) will be optimal.

DISCUSSION

In order to control the motion of ship in complex system such as coastal navigation, there should be a number of algorithms to control the ship's power plant and steering gear system. The above-said algorithms allow the mariner evaluates the navigational situation and makes the best solutions. The same programs will help the mariner forecasts the ship motion in following the real time. These algorithm controls are considered evaluated, supported models that obtained the proper programs. They are applied to ensure the ship passing the emergency situation.

CONCLUSIONS

The research has obtained the results:
Proposed the algorithm control of minimum principle on the basis of selection the transversal conditions;
Obtained the algorithm controls of ship's power plant system that allows approaching the leader ship;
Investigated the programed control of ship's steering gear that ensures the ship in head-on navigation;
Established the programed control of ship's power plant system that effected by the variable non-linear parameters.

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