ANALYSIS OF VIBRATION OF EULER-BERNOULLI CLAMPED LAMINATED BEAM WITH NON-UNIFORM PRESSURE DISTRIBUTION AT THE INTERFACES

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ABSTRACT

The increase in the utilization of high performance equipments has necessitated the use of laminated beams in aerospace engine, machine structures and electronic devices in order to dampen vibration and reduce noise. In such equipments, two laminates are pressed together and the occurrence of micro interfacial slip between these two laminates helps to effectively dissipate any unwanted vibration or noise. Also, when such structure are subjected to either static or dynamic loading, non-uniformity in interfacial pressure have significant effect on both the energy dissipated and the logarithmic damping decrement associated with the mechanism of slip damping. Thus, laminated beams can be effectively used to increase the level of damping available in such a mechanism. Hence, in this work, with the aid of Finite Difference Method, the effects of laminates on the energy dissipation due to frictional damping between the two laminates are investigated, so also the effects of the material properties on the dynamic behaviour and energy dissipation are also analysed.

Keywords: Laminates; Dynamic deflection; Energy dissipation; Finite difference method.

INTRODUCTION

The mechanism of damping as a means of controlling undesirable effect of vibration and noise has received considerable research attention in the past three decades. This has significantly resulted in considerably intense research in the dynamic behaviour of laminates. In aerospace engine, machine structures and also in electronic devices, laminated beams are used in the damping of vibration and the reduction of noise in electronic systems and devices. In such equipments, two laminates are pressed together and the occurrence of micro interfacial slip between these two laminates helps to effectively dissipate any unwanted vibration or noise. Also, when such structure are subjected to either static or dynamic loading, non-uniformity in interfacial pressure have significant effect on both the energy dissipation and the logarithmic damping decrement associated with the mechanism of slip damping, thereby damping response caused by the lamination of these beams is of great importance. Modeling plays a crucial role in the controlling and optimizing of engineering applications by providing means of better understanding of the involved phenomena with reduce cost of investigation, and on improving the capacity of predicting the dynamic response.

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Hence, finite difference method was used in the modelling of effect of laminates on the energy dissipation due to frictional damping between the laminates, so also the effect of the material properties on the dynamic behaviour.

LITERATURE REVIEW

One of the pioneer works of Goodman and Klumpp(1952) was on the analysis of slip damping with reference to turbine blade vibration, and in the previous work Masuko (1973), Gould and Mikic (1972), Motosh (1975), M., Nishiwaki et al (1978, 1980), Ziada and Abd (1980) several studies were carried out on the damping capacity and pressure distribution of jointed structures under uniform or constant intensity of pressure distribution at the interface. Nanda and Behera (1999, 2000, 2006) studied on damping in layered and jointed structures and relaxing the restriction of uniform interfacial pressure to allow for more realistic pressure profiles that are encountered in practice.

The studies of structural damping in laminated beams due to interfacial slip had earlier been carried out by Hansen and Spies (1997), while Kapuria and Alam (2006) presented efficient layerwise finite element model for dynamic analysis of laminated piezoelectric beams.

Damisa (2007, 2008) Olunloyo (2007) presented the static and dynamic analysis of slip damping in clamped layered beams with non-uniform pressure distribution at the interface. In particular, whereas the investigation in Damisa, Olunloyo, Osheku, and Oyediran [1, 3] was limited to the case of linear pressure profile, the static analysis in in Damisa, Olunloyo, Osheku, and Oyediran [2] included other forms of interfacial pressure distributions such as polynomial or hyperbolic representations and the results obtained demonstrated that the effects of such distributions in comparison with the linear profile were largely incremental in nature and no fundamental differences were found. The results of the analysis of the cantibeam in [1–3] revealed that when the beam laminates are of the same material and thickness, non-uniformity in interface pressure can for example have significant effect on the mechanism of slip damping for static load while the energy dissipation and the logarithmic damping decrement associated with dynamic loads are significantly influenced by the nature of the interfacial pressure profile between the laminates. Bassiouni et al. (1999) used a finite element model to obtain the natural frequencies and mode shapes of laminated composite beams. A formulation for the exact dynamic stiffness matrix for symmetric and unsymmetrically laminated beams has been derived using the exact shape functions for the deflection and bending slope of composite laminated beam elements (Abramovich et al., 1995; Eisenberger et al., 1995).

Later, Khdeir and Reddy (1997) used the state-space concept in conjunction with the Jordan canonical form to solve the governing equations for the bending of cross-ply laminated beams. Kam and Chang (1992) studied the bending and free vibration behavior of laminated composite beams using First-order Shear Deformation Theory (FSDT) and Higher-order Shear Deformation Theory (HSDT).

Kadivar and Mohebpour (1998) studied the finite element dynamic response of an unsymmetrically laminated composite beam subject to moving loads. Cho and Averill (1997) developed a beam finite element based on a new discrete layer laminated beam theory with sublaminate first-order zig-zag kinematic assumptions for both thin and thick laminated beams.

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Robaldo (2006) and Ballhause (2005)separately investigated the ESL theories and layerwise approaches for the analysis of multilayered plates integrated with piezoelectric elements. Marurb and Rae (1998) used higher order refined theory analytical solution to the dynamic analysis of laminated beams while Rahman and Alam (2012) used zigzag theory to study the dynamic analysis of laminated smart beams.

In this paper, investigation was carried out on the effect of laminates on energy dissipation due to frictional damping between the two laminates, and also on the effect of material properties on the dynamic behavior and energy dissipation.

MATHEMATICAL MODEL ASSUMPTIONS

The following assumptions have been made for deriving the equations:

- That both layers are assumed to bend according to Bernoulli-Euler's theory that a plane cross-section originally plane remains plain and normal to the longitudinal fibres of the beam after bending.
- The Shear effect in the layers are neglected and only bending and extensional effects are considered.
- The transverse displacement at a section is assumed to be constant along the thickness.
- A continuity of displacements at the interfaces is assumed.
- All displacements are assumed small, as in linear elasticity.
- The material of the visco-elastic layer is assumed to be linear, that is, properties are strain independent.

FREE BODY DIAGRAMS

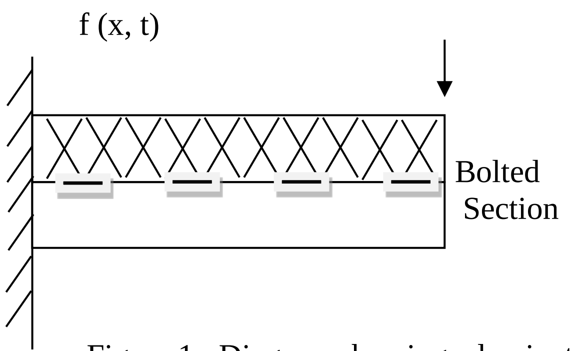
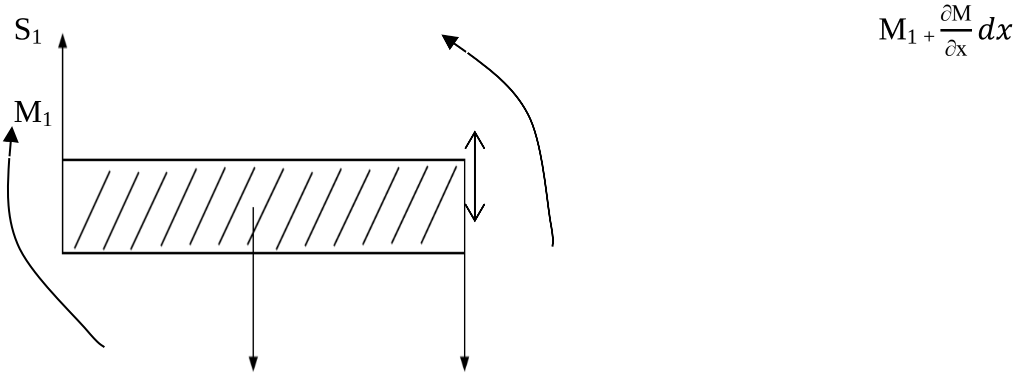


Figure 1: Diagram showing a laminated Beam

I Upper Laminate



$\rho_1 bh_1 \frac{\partial^2 w1}{\partial t} S_1 + \frac{\partial s1}{\partial x} dx$

Figure 2: Free body diagram showing the Upper Laminate

II Lower Laminate

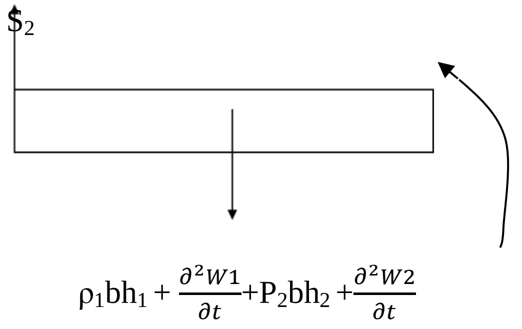


Figure 3: Free body diagram showing the lower laminate

III Laminated Beam

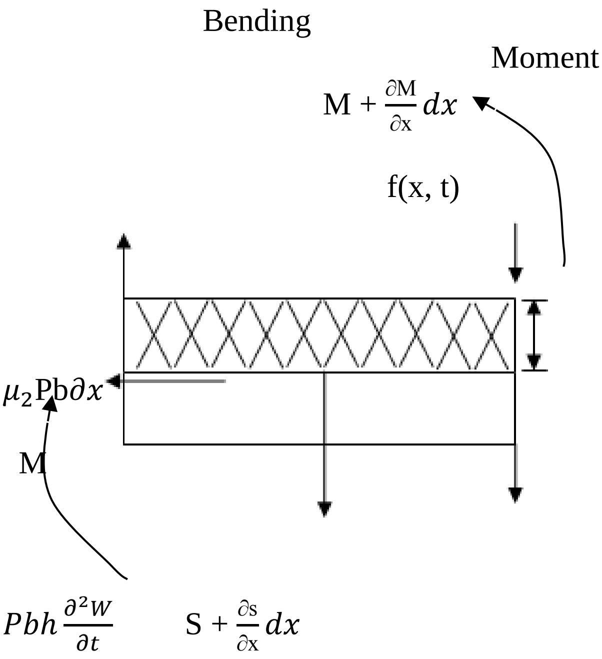


Figure 4: Free body diagram of Laminated Beam

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GOVERNING EQUATIONS

Upper Laminate

The governing equation of motion

$$E_1 I_1 \frac{\partial^2 W_1}{\partial x^4} + \rho_1 bh_1 + \frac{\partial^2 W1}{\partial t^2} - \mu_{21} b \frac{h_1}{2} \frac{\partial P_{21}}{\partial x} = 0 \ldots\ldots\ldots (1)$$

Lower Laminate

The governing equation of motion

$$E_2 I_2 \frac{\partial^4 W_{2*}}{\delta x^4} + p_2 bh_2 \frac{\partial^2 W_1}{\partial t^2} - \mu_{21} b \frac{h_2}{2} \frac{\partial P_{21}}{\partial x} = \ldots\ldots\ldots(2)$$

The initial conditions of the laminates are

$$t = 0,\ W_1 = W_2 = 0,\ \frac{\partial W_1}{\partial t} = \frac{\partial W_2}{\partial t} = 0 \ldots\ldots\ldots\ldots\ldots (3)$$

The boundary conditions for the laminates

$$x = 0\ W_1 = W_2 = 0,\ \frac{\partial W_1}{\partial x} = \frac{\partial W_2}{\partial x} = 0 \ldots\ldots\ldots\ldots\ldots (4)$$

$$x = L, \frac{\partial^2 W_1}{\partial x} = \frac{\partial^2 W_2}{\partial x} = 0, \frac{\partial^3 W_1}{\partial x^3} = \frac{\partial^3 W_2}{\partial x^3} = 0 \ldots\ldots\ldots(5)$$

Using the following Non-dimensional parameters

$$W = \frac{w}{\overline{W}},\quad X = \frac{x}{\overline{X}},\ \tau = \sqrt{\frac{PbhX^4}{EI}} t,\ P = \frac{\mu bhX^4}{2EI\overline{W}} P \ldots\ \ldots\ldots(6)$$

and assuming a linear pressure variation,

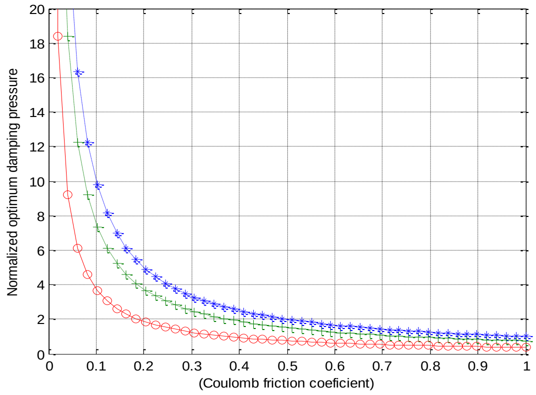
$$P = P_0(1 + \frac{x}{L}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Using finite difference method for equation (1)

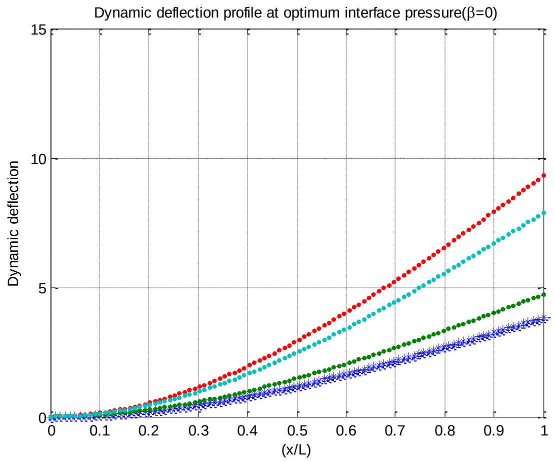
$$E_1 I_1 [\frac{(W1_{i+2}^n) - 4(W1_{i+1}^n) + 6(W1_i^n) - 4(W1_{i-1}^n) + (W1_{i-2}^n)}{(\Delta x^4)}] + \rho_1 bh_1 [\frac{(W1_i^{n+1}) - 2(W1_i^n) + (W2_i^{n-1})}{\Delta t^2}] - \frac{\mu bh_1}{2}[\frac{p_0}{L}] = 0 \ldots\ldots\ldots\ldots (8)$$

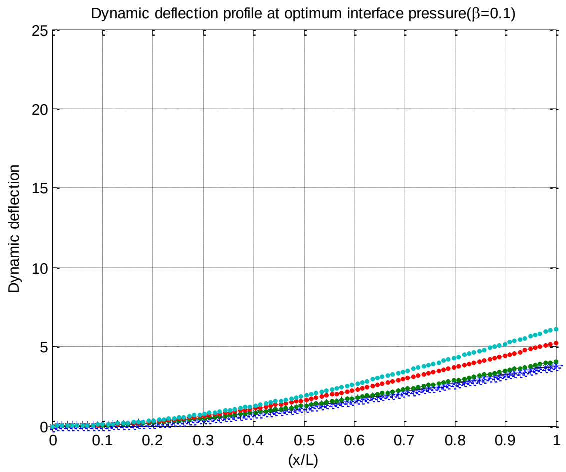
Similarly, for equation (2)

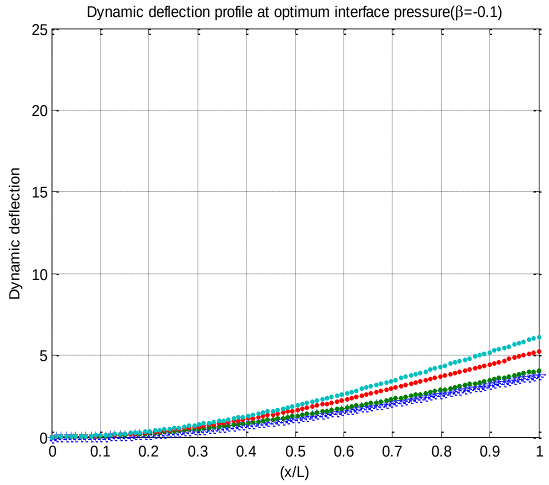
$$E_2 I_2 [\frac{(W2_{i+2}^n) - 4(W2_{i+1}^n) + 6w2_i^n - 4(W2_{i-1}^n) + (W2_{i-2}^n)}{(\Delta x^4)}] + \rho_2 bh_2 [\frac{(W2_i^{n+1}) - 2(W2_i^n) + (W2_i^{n-1})}{\Delta t^2}] - \frac{\mu bh_2}{2}[\frac{p_0}{L}] = 0 \ldots\ldots\ldots\ldots (9)$$

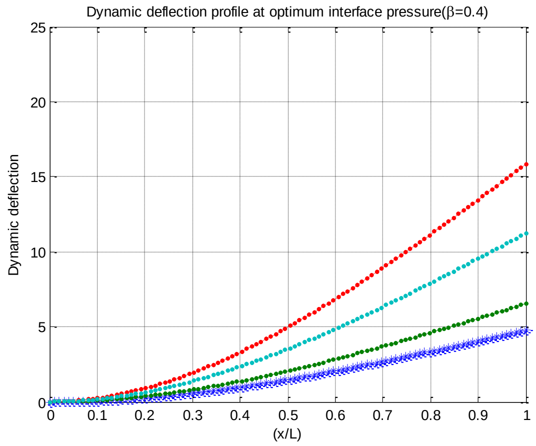


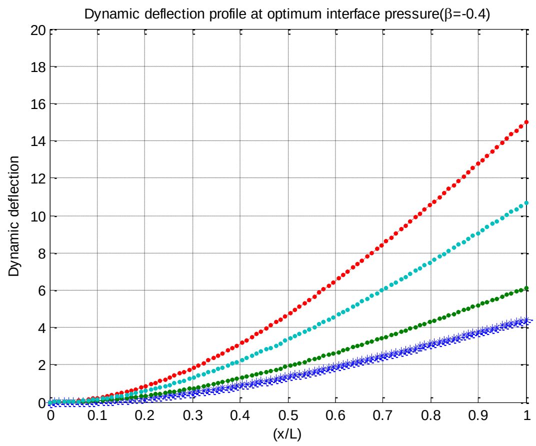
RESULTS AND DISCUSSIONS

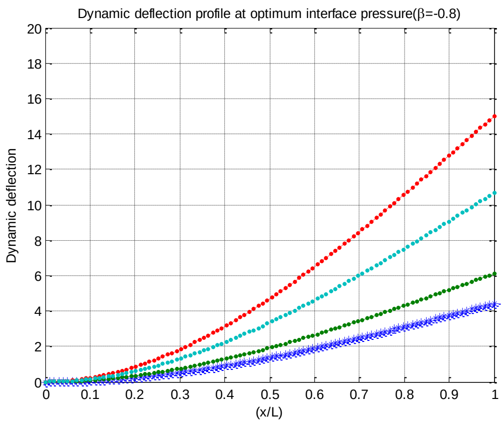


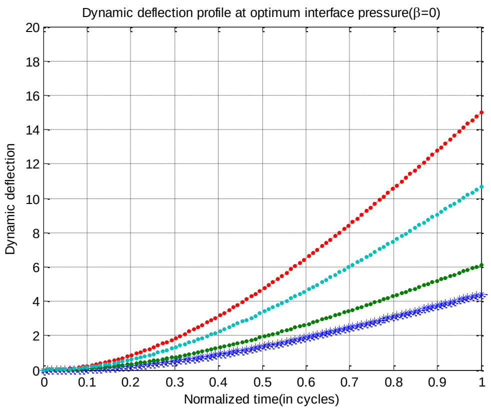


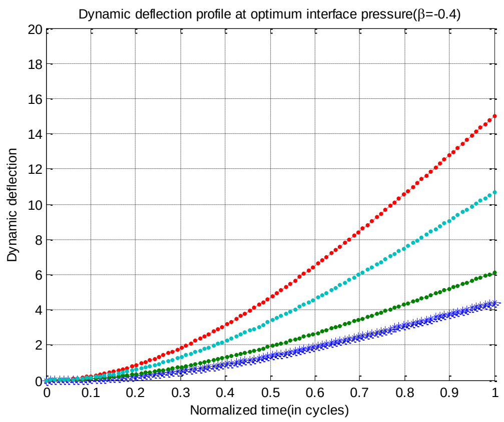

Dynamic deflection profile at optimum interface pressure(β=-0.1)
Dynamic deflection
(x/L)

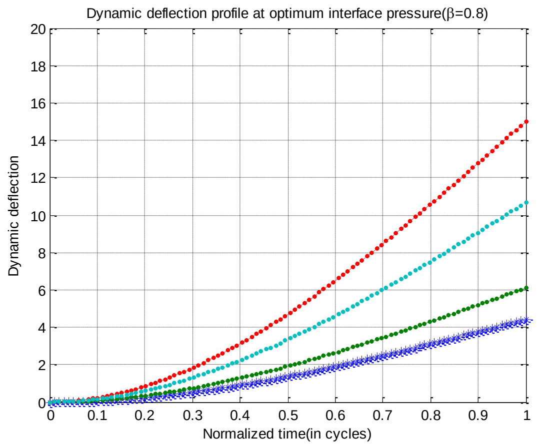

Dynamic deflection profile at optimum interface pressure(β=0.4)
Dynamic deflection
(x/L)

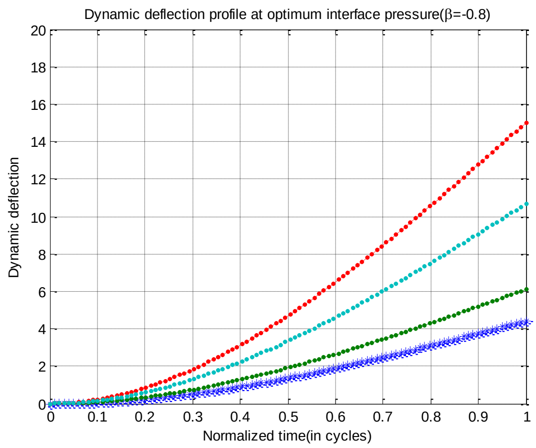

Dynamic deflection profile at optimum interface pressure(β=-0.4)
Dynamic deflection
(x/L)


Dynamic deflection profile at optimum interface pressure(β=-0.8)
Dynamic deflection
(x/L)









From the results shown above, the damping reduces with increase in the coefficient of friction, while the dynamic deflection increases with the length of the laminated beam for different interfacial pressure. It was well known that a negative pressure gradient in a cantilever beam tends to increase the level of energy dissipation whereas an enhanced frequency ratio of the driving load tends to reduce the amount of energy dissipation that can be arranged via slip at the laminate interface. These observations however assume that both upper and lower laminates are of the same thickness and are made from the same material. When such restrictions are removed, two new effects arise and are the subject of this paper. Our findings in fact confirm that each of these factors can independently be exploited to enhance the level of energy dissipation that can be arranged. In other words such increases can be arranged either by using different materials for the upper and lower laminates in a prescribed fashion or by retaining the same material for both laminates but varying the individual ratios of the laminate thicknesses in a defined manner. Another deduction from the present work is that for effective energy dissipation, it is better to simultaneously play with choice of the laminate materials and their thickness ratios rather than thinker with any one of them by itself. In fact there are many instances when choice of material alone eclipses whatever gains can be made from playing with interfacial pressure gradient. This underscores the quest in the search for composites in the construction of such laminates. The strategy here is to exploit the advantage of composite structures to dissipate vibration energy via slip damping especially in aerodynamic structures where the effect of weight of structural member becomes significant.

CONCLUSION AND FUTURE WORK

In this paper the problem of using a layered structural member as a mechanism for dissipating unwanted vibration or noise has been revisited, be it in an aerodynamic or machine structure. Earlier work had established that some of the factors influencing the level of energy dissipation include the nature of the pressure distribution profile at the interface of the laminates as well as the nature of the external force to which the structure is subjected. The conclusion therefore, is that for maximum energy dissipation, laminates of different materials and of different thicknesses is required. This makes the use of composites beams inevitable. These results can be positively exploited in the design of aerodynamic and machine structures.Hence, it will assist the product designer consider the use of laminated metal material in place of traditional sheet metal thereby enabling various practical modeling techniques to be used both as a damping prediction and design optimization tool. This complexity offers more design flexibility as the thickness and type of the damping core as well as the constraining layers can be altered to optimize effectiveness of the laminated metal product.

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