FORMATION OF CRITERIA-CORRECTNESS MATHEMATICAL COMPETENCE

Gavrilova M. A.
Penza State University
RUSSIA

Yaremko N. N.
Penza State University
RUSSIA

Dmitriyev D. V.
Penza State University
RUSSIA

ABSTRACT

We use the concept "correctness" as a main idea in the teaching of mathematics at the University. The system of integrated interdisciplinary modules is constructed in this paper. It consists of ten interdisciplinary modules. Mastering of these modules provides the formation of criterion-correctness competence of university students.

Keywords: Concept "correctness", modular technology, teaching of mathematics at the University.

INTRODUCTION

The need to update the system of higher education in Russia is largely caused by increased attention to the developing aspect of training that was reflected in the priority of the competence approach. It should be noted that the concept "competence" was used in Russia as a characteristic of effective professional activity. However, the need for the composition of this concept in the context of professional training has arisen in connection with the integration into the European educational community, (Gavrilova M.A, 2008, 2009).

Competence paradigm of higher education intensified teachers' researches in the field of modeling effective technologies of training and change in the content of professional education on the basis of inter-subject integration (Gavrilova M.A, 2008, 2009). In this context, the idea of allocating a block of questions on the criteria-correctness mathematical teaching, seems to us to actual.

The idea of mathematical correctness can be used as the main principle of professional mathematical education. This idea is proved (Yaremko N.N., Krasnova O.V., 2012):

- metaconcreteness of the concept "correctness",
- its epistemological and ideological potential,
- possibility on its basis the creation of generalized methods for educational, informative and reflex of students' activity.

Let's notice criteria-correctness competence is among key, all-professional, meta-subject competencies and its formation is carried out on subject and inter-subject content, in our case on mathematical content by subject and inter-subject means. The criteria-correctness mathematical training forms the criteria-correctness competence, sets the main purpose to develop students' personality by increasing a role of mathematics in humanization of higher education.

THEORETICAL AND METHODOLOGICAL FOUNDATIONS of integrative and modular technology of professional training in high school were laid in works by V.P. Bespalko, P.Ya. Galperin, E.O. Ivanova, E.N. Kabanova-Meller, I.M. Osmolovskaya, G.K. Selevko, P.A. Yutsavichene (1989), etc. Using theoretical principles of integrative and modular training, we designed the technology of criteria-correctness mathematical training of university students. It represents a system of interconnected modules: local inter-subject integrated modules at the initial stage of learning and further transition to global block and modular technology on special courses. The concept of correctness realizes inter-subject and intra-subject communications, i.e. realizes the idea of an integrity, allows to unite elements of training material of various subjects in one subject and ways of actions on the correctness concept.

Expediency and efficiency of the application of integrative and modular technology is caused by the following reasons:
- there is a single target setting: the formation of a key criteria-correctness competence on mathematical content;
- on the basis of the meta-subject concept "correctness" intra-subject and inter-subject integration of mathematical and related training courses is carried out;
- structured content defined accurate and natural allocation of semantic blocks-modules: correctness of a problem, mathematical model, inference rules, questions-answers, problems' solving, software, etc.;
- inter-subject concept of "correctness" results in need and defines in the general block natural logic association of separate subject knowledge, ways of activity specific to various subjects or disciplines, corresponding to concepts: well-posed mathematical problem, model, question-answer, etc.

SYSTEM OF INTEGRATED INTERDISCIPLINARY MODULES

Practical implementation of criteria-correctness mathematical training of university students in the form of integrative modular system is based on several principles, foremost among them are the principles of interdisciplinary, multidisciplinary integration and modularity. Didactic principles that determined the selection of the mathematical content and the structure of the teaching process are the principles of interdisciplinary and multidisciplinary integration, modularity, focus on the development of the student's personality, vocational guidance, correctness.

The basis of the principles set out to determine the technology and the content, which carries out the formation of criteria-correctness competencies. The content is presented by:
• a basic component of the program - disciplines of mathematical cycle (mathematical analysis, algebra, geometry, mathematical physics, numerical methods, the basic course of computer science);
• integrated special courses and seminars "Well-posed and ill-posed problems of mathematical physics" (A.N. Tikhonov, V.Y. Arsenin, 1977; Yaremko O., Selutin V., Yaremko N., 2014), "Correctness of mathematical definitions and their development", "Logic and culture of mathematical reasoning ", "Extreme problems";
• computing practice;
• pedagogical practice;

• course and diploma projects.

The central link in the developed technology is occupied by the integrated interdisciplinary modules (IIM). Each of Integrated interdisciplinary modules (IIM) is a structural and substantial part of the system of formation of criteria-correctness competence, which is a combination of logical completeness of interdisciplinary theoretical material and generalized practices of using this material in mathematical and related activities, as well as in professional and real life. IIM is enriched with intra- and intersubject content and links, links both across and down.

Each separate module of the system is based on one of the coordinated definitions relating to the concept "mathematical correctness". Each of them represents an independent didactic unit and at the same time modules are interconnected, they interact, enrich each other, form integrity as they correspond to all set of coordinated concepts relating to the general concept "correctness". In view of these circumstances it is possible to conclude that the set of modules forms a functioned and developed system. In the course of assimilation by each student the system passes six stages – the levels from I to VI which characteristic signs are defined in the paper by Yaremko N.N., Krasnova O.V. (2012).

Development of the system of inter-subject modules determined purposeful selection and synthesis of the necessary training material from different subjects, disciplines of a mathematical cycle and the related training courses on the problem of mathematical correctness which is based on integration of intra mathematical (intra subject) and inter-subject, interdisciplinary communications. The content of integrated interdisciplinary modules represents not mechanical connection of several mathematical subjects or disciplines, and is the product of inter-subject, interdisciplinary synthesis. All substantially significant components are implemented in interrelation with each other, training material is built into an integrated system. IIM are focused on the interaction of mathematical courses among themselves and with all-professional, special courses; IIM are designed to standardize, integrate, expand the basic knowledge and develop generalized methods of students' actions which can be applied to professional activities and real life.

Here is a list of IMM - integrated interdisciplinary modules (Yaremko N.N., Krasnova O.V., 2012).
1. The concept of mathematical correctness, its general scheme.
2. Correctness of the mathematical problem.
3. Correctness of the mathematical model.
4. Correctness of the definition of concepts.
5. Correctness of a question and an answer.
6. Method correctness.
7. Correctness of the proof, statement of a material, formulation of a mathematical problem.
8. Correctness of interpretation of results of the experiment, results of supervision.
9. Correctness of Cauchy problem for differential equations.
10. Correctness of computing algorithm, its stability.

The concept of correctness and ability of correctness observance of mathematical actions, identification and correction of incorrectness is formed in the extending system of intersubject

communications, the result of which is universalization, and subsequently on this basis will be the specialization of skills from the area of pedagogical criteria-corrective mathematical preparation or on one of the scientific research or practical areas.

Learning process is spiral-shaped, i.e. we need to go back a number of times at a higher level of knowledge to a known content of the modules - 1, 2, ..., 10, applying a criterion of correctness to a greater number of mathematical objects and objects of the world. At the final stage the acquired concept "correctness" is the device for research of professional and quasiprofessional practical problems.

We give the structure of Modules – 1, 2 as an example. Let's notice that the maintenance of the module develops from the first uncertain level to the finishing autonomous. Following authors of the textbook (Yutsavichene P.A, 1989; Ivanova E.O., Osmolovskaya I.M., 2011) we will define three-component structure of the module:
     1) anticipatory set, 2) theoretical part, 3) practical trainings.

Module - 1. The concept of mathematical correctness, its general scheme

Anticipatory set is based on the fact that the analysis of the objects to establish correctness provides a unified approach to the formation of a universal cognitive and reflexive activity, universal actions. The frequency of usage "correctness": 60 times on 300 million words. In the theoretical part of this module Scheme-1 from the work (Yaremko N.N., Krasnova O.V., 2012) "The scheme of a meta-concepts of correctness" is provided. In the practical part it is offered to discuss nominal and common correctness, to give examples from personal experience, examples of the use of the concept "correctness" from nonmathematical areas. For example, to explain how the following expressions are understood: "incorrect behavior", "ill-posed questions" (Denker John), "incorrect shutdown of Yandex", etc.

Knowledge component: concept of "mathematical correctness", its volume, contents, nominal and common sense, logical characteristic. Activity component is presented by a system of universal actions on the basis of the concept "correctness":
     - existence justification (lack of contradictions);
     - justification of uniqueness or unambiguous definiteness;
     - object analysis on the existence of qualitative characteristics, corrections according to
     internal and external conditions.

Personality-ideological component is based on the philosophical and ideological aspects of the concept of "mathematical correctness".

Module - 2. Correctness of the mathematical problem

As an update it is necessary to inform the students that in life a person constantly have to solve problems that do not always have a mathematical formulation. Solution of practical problems often occur in conditions of uncertainty, redefined or even contradictory input data. Nevertheless, the solution of problems in real life and in any subject area (mathematics, physics, computer science, biology, chemistry, etc.) is subordinated to common laws, it passes the same stages:

from the analysis of data and the requirements of the problem to search of a solution and its implementation and, further, to "look back." What is the methodology for solving problems, what is the structure of the problem and what are the steps to solve it?(G.Polya, 1973; Milgram James R.2005,)

In the theoretical part correct and incorrect problems are defined. A correct problem is a problem that exists, it is unique, stable; and a problem is incorrect if at least one of the three conditions is not satisfied. Stability of the solution means that "small" changes in the data of the problem corresponds to a "small" change in the solution. Any problem consists of four components: data, requirement, decision, basis. The process of solving the problem consists of four steps: clarification of the problem, finding a solution, implementation of the solution, analysis of the received solution. These four stages are invariant activities in solving any problem.

When studying Module-2 we will proceed from the following assumptions:
- the methodology of the theory of ill-posed problems is applied to work with any, both correct and incorrect problems, and also with their separate parts; (The methodology is understood as a conceptual framework of the theory, conceptual provisions, principles, main ideas, methods, means. For the theory of ill-posed problems there are three requirements of correctness, handling (contraction or expansion) of the subject area, splitting tasks into subtasks, regularization, search quasisolutions, building stable algorithms, etc.)
- three correctness requirements: existence of a solution, its uniqueness and stability - are checked parallel implementation of these steps at every stage of solving the problem; such check is an activity invariant according to the decision both correct, and incorrect problems.
In practical trainings problems of the following kind are formulated.
1. To prove that the problem has no solution.
2. To reveal a contradiction in the problem.
3. To prove that the problem has the unique solution.
4. To find the solution of a problem by choosing and to prove its uniqueness.
5. To find solutions of a problem and to prove that all solutions are found and the problem has no other solutions.
6. To investigate a way of problem solution in terms of loss of solutions and acquisition of foreign decisions; to consider a problem of equivalence of the equations, to analyze transitions in the course of equation transformation to its conclusion or to the equivalent equation.
7. To restore the records which have remained on a board. To analyze a uniqueness of the performed problems, to consider all possible variants.
Knowledge component of this module:
a) inter-subject knowledge:
- knowledge about the structure of the problem (data, requirement, solution, basis, subject area);
- knowledge about the stages of solving problem: understanding the conditions and requirements of the problem, solving, implementation of solutions, "look back"(G.Polya, 1973);
- knowledge of activity structure of the solution of a problem (motive, object, purpose, means, methods, operations, result);

b) substantive knowledge of mathematics: concepts, basic statements (theorems, properties, relationships between them);
- knowledge of methods for solving key challenges;
- knowledge of the theory of ill-posed problems, three requirements of a correctness.

Activity component of this module presents universal educational actions of cognitive and estimating character: justification of the unique determination of solving a problem and variation. The operational structure of actions can be developed and presented in more detail.

CONCLUSIONS

Personality-ideological component is related to the facts that
   a) the concept "correctness" is involved in the formation of a coherent picture of the world as well-posed and ill-posed problems give a complete picture of the surrounding reality;
   b) the process of educational and scientific knowledge is limitless, it develops in a spiral and repeatedly passes through the " incorrectness overcoming".
Effectiveness of the offered integrative and modular technology is confirmed experimentally.

REFERENCES

1. Gavrilova M.A. (2008) Effective technologies of teaching mathematics in a system of continuing professional education. *Secondary professional education,* 9, 36-37.
2. Gavrilova M.A. (2009) Formation of a model of professional competence of mathematics teachers on the basis of a university educational complex. *Messenger of the Moscow state regional university. Series pedagogy*, 2, 107-112.
3. Denker John, *How to Deal with Ill-Posed Questions* www.av8n.com/physics/ill-posed.htm
4. Ivanova E.O., Osmolovskaya I.M. (2011) *The tutoring theory in informational society*, Moscow, Education.
5.Milgram James R. (2005). *The Mathematics Pre-Service Teacher Need to Know. Dep.of Math.-Stanford Univ*., California.
6.G.Polya. (1973) *How to solve it. Printed in the United States of America by Princeton University Press*, Princeton, New Jersey, Second Printing.
7. A.N. Tikhonov, V.Y. Arsenin. (1977) *Solutions of Ill-Posed Problems*, Winston, New York.
8. Yutsavichene P.A. (1989) *Theory and practice of modular tutoring*. Kaunas.
9. Yaremko N.N., Krasnova O.V. (2012) Criteria-corrective mathematical training of university students and its implementation. *News from social sciences., 28 , Penza State Pedagogical University*, 1143 - 1147.
10.Yaremko O., Selutin V., Yaremko N. (2014) The Fourier Transform with piecewise trigonometric kernels and its Applications. *WSEAS Transactions on Mathematics,* 13, 615-625.
11.Yaremko O., Yaremko N. (2014) Transformation operators and their applications for modeling in two-layer media. *Science Publishing Corporation*, www.sciencepubco.com/index.php/GJMA doi:10.14419/gjma.v2i3289. Global Journal of Mathematical Analysis, 2(4), 227-234.