THE NUMERICAL SOLUTION OF THE NEWTONIAN FLUIDS FLOW DUE TO A STRETCHING CYLINDER BY SOR ITERATIVE PROCEDURE

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ABSTRACT

The numerical solution has been obtained of the governing equations for the steady, incompressible fluid flow due to a stretching cylinder. The numerical results are calculated, by using SOR method and Simpson's (1/3) rule, for the range 0.1 to 100 of the parameter *R*. The accuracy of the results is checked very carefully by performing calculations on three different grid sizes and comparing them with the know results.

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INTRODUCTION

The fluid dynamics due to a stretching surface is important in extrusion processes. Crane [1], discussed a closed form exact solution of the Navier-Stokes equations subject to two dimensional stretching of a flat surface. Brady and Acrivos [2], examined the exact similarity in solutions of a flow inside a stretching channel and inside a stretching cylinder. Crane [3], again found the boundary layer solution outside a stretching cylinder. Kuiken [4] and Banks [5], studied the two-dimensional boundary layer due to non-uniform stretching. Wang [6] considered the two-dimensional stretching of a surface in a rotating fluid. The three-dimensional flow subject to a stretching flat surface was studied by Wang [7]. Also, Wang [8] solved the problem of the exterior fluid flow due to the extrusion of hollow tubes.

In the present paper, Wang's [8] work is extended to large values of the Reynold number *R* using different numerical techniques. The numerical techniques used in the present work are straightforward and easy to program. Wang [8] has used the Runge-Kutta method, which is quite laborious and not straightforward for solving the boundary value problems. The basic analysis of this problem is presented. The finite difference equations are obtained and solved by using SOR iterative procedure and the Simpson's (1/3) rule, subject to the appropriate boundary conditions. The results are given in tabular and graphical form, and compared with previous results.

BASIC ANALYSIS

The Navier-Stokes equation and continuity equation for steady and incompressible flow in the absence of the body force are given by

$$-\frac{1}{\rho}\nabla p = -\upsilon\nabla^2\underline{V} + (\underline{V}\cdot\nabla)\underline{V}\,, \qquad (1)$$

$$\nabla\cdot\underline{V} = 0\,, \qquad (2)$$

where ρ, υ and $\underline{V}$ are density, kinematics viscosity and velocity of the fluid respectively.

Wang [8] obtained the equations of motion for the flow of fluid due to a stretching cylinder. He used the cylindrical coordinate system (r,θ,z) such that the cylinder is described with the radius $r = a$.

Let u and w are the velocity components in the r and z directions, respectively then the equation (1) and (2) take the following form:

$$-\frac{1}{\rho}p_r = -\upsilon\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right) + \left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) \tag{3}$$

$$-\frac{1}{\rho}p_z = -\upsilon\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) + \left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) \tag{4}$$

with continuity equation

$$\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} = 0 \tag{5}$$

where the subscripts denote the partial differentiation with respect to space coordinates, ρ is the density, p the pressure and υ the coefficient of kinematics viscosity.

The equations (3) to (5) are to be solved subject to the following boundary conditions:

when $r= a$, $u=0$, $w=2kz$ when $r=\infty$, $u=0$ (6)

where k is a positive constant of dimension [1/*Time*].

We now use the similarity transformations to make the equations of motion in dimensionless form as follows.

$$u= -ka(f(\eta)/\sqrt{\eta}) \quad \text{and} \quad w= 2kf'(\eta)z, \tag{7}$$

where $\eta=(r/a)^2$ is the dimensionless variable.

The equation (5) is satisfied and the equation (3) and (4) by using (7) become:

$$\eta f''' + f'' = R(f'^2 - ff''), \tag{8}$$

$$\frac{p}{\rho} = \frac{p_\infty}{\rho} - 2k\upsilon f' - \frac{k^2a^2f^2}{2\eta}. \tag{9}$$

Here $R = \frac{ka^2}{2\upsilon}$ is the Reynolds number, where a is the radius of the cylinder, υ the coefficient of kinematics viscosity and k the given constant.

In view of (7), the boundary conditions (6) take the form:

$\eta=1$: $f = 0$, $f' = 1$
$\eta\to\infty$: $f' = 0$. (10)

In order to solve numerically, it is convenient to reformulate the problem by using the following transformation:

$$\eta = e^x \tag{11}$$

Thus the equation (8) and boundary conditions (10), due to (11), become

$$f''' - 2f'' + f' = R(f'^2 - ff'' + ff') \tag{12}$$

and

$x=0$: $f = 0$, $f' = 1$

$$x \to \infty: \qquad f' = 0, \tag{13}$$

where here the prime denotes differentiation with respect to x.

In order to treat the equation (12) numerically we write it into two equations as follows:

$$f' = p, \tag{14}$$

$$p'' - 2p' + p = R(p^2 - fp' + fp) \tag{15}$$

The boundary conditions (13) now become:

$$\left.\begin{array}{ll} x=0: & f = 0, \quad p = 1 \\ x \to \infty: & p = 0. \end{array}\right\} \tag{16}$$

Now, if we approximate the equation (15) by central difference approximation at a typical point x = x_n of the interval [0,∞), we obtain

$$(1 - h + \frac{Rhf_n}{2})p_{n+1} + (1 + h - \frac{Rhf_n}{2})p_{n-1} - (2 - h^2 + Rh^2 f_n + Rh^2 p_n)p_n = 0 \tag{17}$$

where h denotes a grid size. For computational purposes, we shall replace the interval [0,∞) by [0, β), where β is a sufficiently large.

We now solve numerically the first order ordinary differential equations (14) and the finite difference equation (17) at each interior grid point of the interval. The equation (14) is integrated by the Simpson's (1/3) rule, with the formula given in Milne, whereas the equation (17) is solved by using SOR iterative procedure, subject to the appropriate conditions. The pressure p can be calculated by integrating (9).

DISCUSSION ON RESULTS

Calculations have been carried out to obtain numerical solutions of the equations (14) and (17) by using Simpson's (1/3) Rule and SOR iterative procedure. In order to check the accuracy of the numerical results, they have been calculated on three different grid sizes. Also, the results have been compared with the previous results by Wang [8] and are found to be in good agreement.

The effects of the flow parameters namely R and Pr have been examined for the velocity and temperature profiles. The results have been presented in tabular as well graphical forms. The comparison of the present results with the previous results by Wang [8] is given in Table 1 to Table 3. The Table 1 shows that all the values of $f''(1)$ are negative that means, the fluid is under the action of a dragging force due to stretching surface. Figure 2 demonstrate $f(\eta)$ for various values of R. Figure 1 show $f'(\eta)$ for different values of R. The velocity gradient also increases for increasing values of R as can be seen in Figure 1. It is worth mentioning that the velocity field is not affected by Prandtle number Pr.

The Figure 3 and Figure 4 demonstrate the temperature distributions for $\Pr = 0.7$ (such as air) and $\Pr = 7$ (such as water) for a fixed value of R. In both the figures $\theta(\eta)$ decreases and then becomes zero at a large η in both the cases. This situation causes the increase in wall temperature gradient and thus the surface heat transfer rate is increased.

Table 1: Comparison of $f(\infty)$ for possible values of Reynolds number *R*.

γ	*R*=2.0		*R*=5.0		*R*=10.0	
	Present	Wang	Present	Wang	Present	Wang
-1.2	1.055091	---	0.497766	---	0.282937	---
-1.0	1.050510	---	0.496125	---	0.284980	---
-0.5	1.053649	1.0277	0.508461	0.5027	0.293924	0.2999
0.0	1.105823	1.0983	0.592566	0.5933	0.385362	0.3857
0.5	1.252686	1.2524	0.828878	0.8291	0.675645	0.6757
1.0	1.516071	---	1.201347	---	1.100102	---
1.2	1.649482	---	1.371547	---	1.284416	---

Table 2: Comparison of $-f''(1)$ for possible values of Reynolds number *R*.

γ	*R*=2.0		*R*=5.0		*R*=10.0	
	Present	Wang	Present	Wang	Present	Wang
-1.2	0.798736	---	0.838972	---	0.843940	---
-1.0	0.890127	---	0.971818	---	0.995908	---
-0.5	1.184350	1.1810	1.485243	1.4811	1.682201	1.6776
0.0	1.595297	1.5941	2.410499	2.4175	3.318044	3.3445
0.5	2.141963	2.1468	3.882652	3.9308	6.435548	6.6222
1.0	2.818490	---	5.774575	---	10.371458	---
1.2	3.118564	---	6.591984	---	11.996690	---

Table 3: Compression of $-\theta'(1)$ for possible value of *R*=10.

Pr	Present	Wang
0.1	1.050543	---
0.7	1.702547	1.5683
2.0	3.026962	3.0360
7.0	6.155753	6.1592
10.0	7.462454	7.4668
15.0	9.258509	---

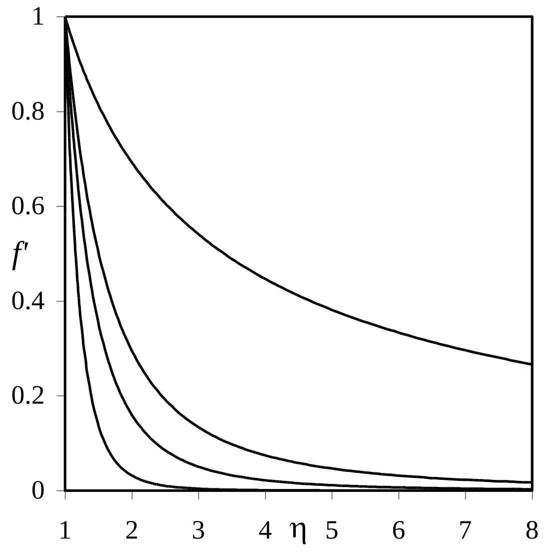

Figure 1: The velocity profile $f'(\eta)$ for R=0.1, 2, 5 and 20 from top to bottom.

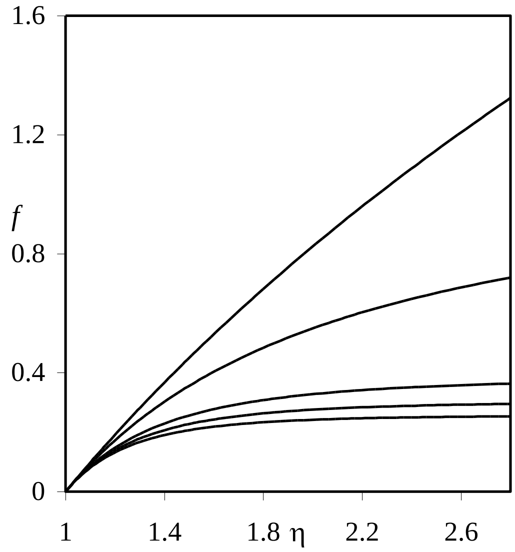

Figure 2: The similarity profile $f(\eta)$ for R=0.1, 2, 5, 10 and 20 from top to bottom.

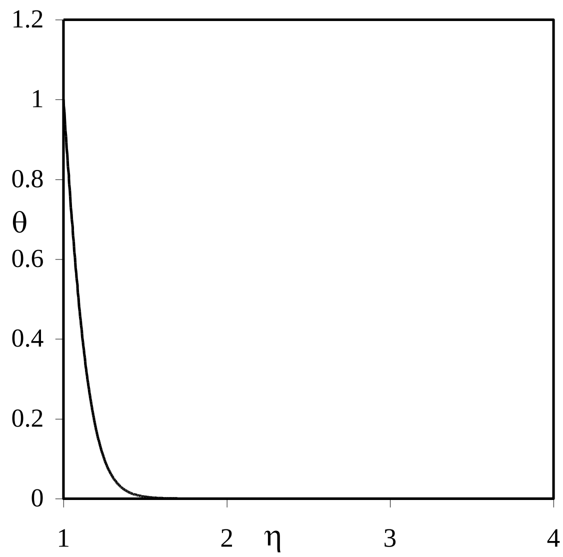

Figure 3: Temperature profile $\theta(\eta)$ for R=10, Pr=7

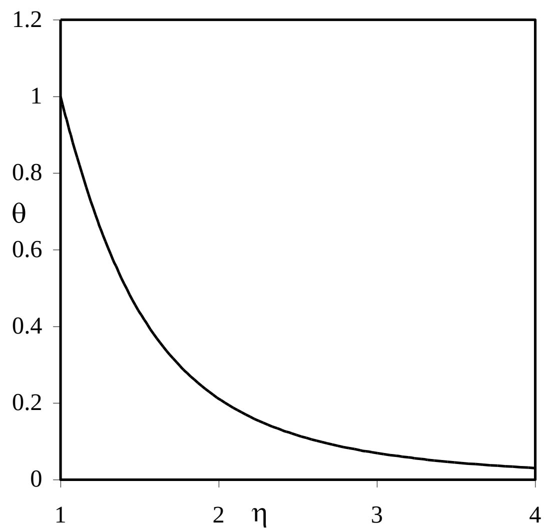

Figure 4: Temperature profile $\theta(\eta)$ for R=10, Pr=0.7.

REFERENCE

[1] I.J. Crane, Zeit. Angew. Math. Phys., 21, pp645, (1970).
[2] J.F. Bardy and A. Acrivos, J.Fluid Mech., 112, pp127, (1981).
[3] I.J. Crane, Zeit. Angew. Math. Phys., 26, pp619, (1975).
[4] H.K. Kuiken, IMA J. Appl. Math., 27, pp387, (1981).
[5] W.H.H. Banks, J. Mech. Theor. Appl., 2, pp375, (1983).
[6] C.Y.Wang, Zeit. Angew. Math. Phys., 39, pp117, (1988).
[7] C.Y.Wang, J. Phys. Fluids, 27(8), pp1915, (1984).
[8] C.Y.Wang, J. Phys. Fluids, 31(3), pp466, (1988).
[9] R.L. Burden, Numerical Analysis, Prindle, Weber & Schmidt, Boston (1985).